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Pension reform and wealth inequality: evidence from Denmark*

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Abstract

A growing literature explores reasons for rising wealth inequality, but disregards the role of pension systems despite their well-understood influence on life-cycle saving. In theory and according to available evidence, both pay-as-you-go (PAYG) and fully-funded (FF) pension schemes crowd out voluntary retirement saving. They differ because aggregate savings decrease in the former but increase under the latter system. Unlike most nations, Denmark has seen a decline in wealth inequality in recent decades. This paper studies a calibrated life-cycle model of Denmark and employs unique registry data to argue that a Danish pension system transition, from a mostly PAYG to a dominant, mandated FF scheme, explains much of this decline.

Keywords: Wealth inequality, pension systems, crowding out, life-cycle savings.

JEL Classifications: D31, E01, E21, H55, G51, J32.

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1 Introduction

In recent decades, wealth inequality has displayed an upward trend in many countries (Zucman, 2019). The list of contributory factors includes increasing inequality in labor earnings, intergenerational transfers, heterogeneity in preferences and investment returns, and the role of different asset holdings across the wealth distribution. Doubtless, these drivers are important. Nevertheless, somewhat surprisingly, the extant literature has been mostly silent on the role of the pension system in explaining wealth inequality, especially among the retired. This paper is an attempt to fill this void.

The World Bank (Holzmann and Hinz, 2005) distinguishes three pillars in earnings-related contributory pensions. The first pillar is a publicly managed, mandated pension plan; the second involves mandatory, private pension plans. Voluntary private plans make up the third pillar. Traditionally, pension systems have had a dominant pillar: either the first pillar, a pay-as-you-go (PAYG) pension plan, or the second pillar, a fully-funded (FF) scheme. In a typical PAYG scheme, the working generations contribute, and the receipts are distributed forthwith among current retirees. In a typical FF scheme, workers contribute to their individual account, and those savings are invested in the market by professionals, delivering a return post retirement.[1] In recent decades, a transition to a multi-pillar architecture with an increased role for private pension funds is underway in almost all OECD countries (OECD, 2019). This paper studies the implications of such a transition from a mainly public PAYG unfunded system to a hybrid system where a private FF system plays a large role. In particular, we establish an important role of the pension system for asset accumulation and wealth distribution.

Why might the nature of the pension system be salient to the understanding of wealth accumulation and inequality? Pension schemes are ways to smooth life-cycle consumption, aimed mainly at ensuring adequate living standards during retirement. It seems straightforward that implementing either a PAYG or a mandated FF scheme will crowd out voluntary retirement saving. However, the two schemes differ in their implications for wealth accumulation. A PAYG scheme is part of an intergenerational social compact in which the state does *not* accumulate assets in anticipation of future pension claims but can tax the *then* working population to fund the pensions of the retired generation (Barr and Diamond, 2006). In contrast, a FF scheme is a method of accumulating financial assets (with interest and dividends) to be exchanged for goods upon retirement. In a FF scheme, because mandated saving rises, total retirement saving increases (see e.g. Andersen and Bhattacharya, 2011, 2021) – this last observation has substantial empirical support.[2]

[1] At the outset, it should be made clear that participation in PAYG schemes is generally and universally mandated with no opt-out option; by contrast, FF schemes may either be voluntary or mandated. If participation in a FF scheme is voluntary, then it is, in principle, no different than any other vehicle for retirement saving.

[2] The empirical literature tends to find that the crowding out of FF pensions is incomplete, much less than one for one (Gale, 1998; Attanasio et al., 2007; Engelhardt and Kumar, 2011; Chetty et al., 2014). This means private pension funds increase asset holdings. Indeed, the net savings rate in Denmark has increased considerably in the past three decades. Despite some evidence for increased indebtedness related to the increase in pension assets, Andersen et al. (2021) find that mandatory contributions lead to a significant increase in net wealth.

A transition from a mainly PAYG-based to a hybrid system has broader implications for the distribution of wealth. Increasing the dominance of the FF pillar allows participants to benefit from returns on financial investments. In particular, through the pension funds, it allows low-and middle-income (wealth) households access to financial markets, and thus return possibilities, they may not have if their pensions were a pure government transfer (Fagereng et al., 2020). Furthermore, workers, *de facto*, become (part) owners of capital. This is relevant in the context of declining labor shares and increasing returns to capital (Piketty, 2014). Strikingly, these last two effects entangle the level *and* the distribution of wealth. This happens because mandated FF plans convert low income people, often low savers, to asset owners, thereby increasing net saving. While pension reform affects the whole distribution, the response is heterogeneous across the income distribution. Wealthy people rely relatively less on the pension system and they offset the mandated savings more actively.[3] That is why in a funded system, we would expect the wealth distribution to be different. The seminal World Bank Report (World Bank, 1994), a cornerstone for pension reform discussions worldwide, had stressed that a shift from a PAYG to a FF funded scheme would have significant effects on capital and wealth accumulation. However, the implied link to wealth inequality, the aim of this paper, has not been explored in the literature.

Our laboratory is Denmark. There are several reasons for this. First, over the period 1993-2009, Danish households became increasingly less reliant on PAYG pensions and more dependent on occupational pension schemes, which are fully-funded, defined-contribution schemes created and managed under collective agreements. Starting from meager participation in the mid-80s, some 85% of the employed workforce is currently contributing to such schemes indicating that Denmark, by now, has a well-established FF pillar. Second, Denmark has shown a downward trend in wealth inequality: in our data, the Gini coefficient (including individual pension assets) for all Danish households in 1992 was 0.86, while in 2017, the corresponding number was 0.70, a 19% decline.[4] Note that the changes in the Danish pension system were introduced to ensure adequate living standards post-retirement and were not aimed at reducing wealth inequality. Finally, Danish administrative wealth registry data is of high quality and very valuable because one can directly observe the market value of most wealth components including pension wealth for the entire population. This last fact allows us to avoid many complications that researchers face when working in other laboratories, such as France (Garbinti et al., 2021).

We start by studying how PAYG and FF pension schemes affect individual and aggregate savings, thus wealth accumulation and its redistribution, in a simple two-period life-cycle model.

[3] Chetty et al. (2014) estimate at least 85% of Danes respond *little* to the higher return of FF schemes, contrary to what textbook intuition suggests. Such people ("passive savers") see increases in retirement wealth when confronted with a mandated FF scheme. In a similar vein, Andersen (2018) demonstrates that merely 23% of Danes actively rebalance their savings in response to a change in tax incentives related to pension savings. Moreover, Andersen et al. (2021) show that increased mandatory pension savings lead to increased net wealth.

[4] See Table A1. While we work with the data from 1986 to 2017, we pay particular attention to years 1992 and 2017. 1992 is a benchmark pre-reform year and is chosen because from 1993, contribution rates for private sector employees started to increase, see Figure 2a. 2017 is the last year in the sample, and by this year contribution rates have reached their final level.

Next, we present a series of new stylized facts about the evolution of wealth inequality in Denmark over the past three decades. We follow the by-now standard definitions used in Roine and Waldenström (2015), Alvaredo et al. (2018), Zucman (2019), and others. Household net wealth includes "all the non-financial assets—real estate, land, buildings, etc.—and financial assets—equities, bonds, bank deposits, life insurance, pensions funds, etc.—over which households can enforce ownership rights and that provide economic benefits to their owners, net of any debts" (Zucman, 2019: 4), each valued at prevailing market prices.

Lastly, the paper develops a structural life-cycle model, calibrated to pre-transition Denmark, that is used to quantify the impact of the pension transition on Danish near-retirement wealth inequality. The model features a small open economy with long-lived overlapping generations, an exogenous labor income process, preference heterogeneity, uncertain lifespans and a borrowing constraint. We include preference heterogeneity because prior work, e.g. Hendricks (2007), Suen (2014), and Carroll et al. (2017), find discount factor-heterogeneity to be salient in the evolution of wealth[5] – also, unlike other drivers of wealth inequality such as bequest inequality or return heterogeneity, it is likely invariant to policy changes.[6]

Our results indicate that the pension system was significantly responsible for the wealth inequality declines in Denmark during the 1992-2017 period. Our calibrated model explains the data almost fully. The decline in wealth inequality is driven mainly by an increase in savings by poor households. Despite being forced to save a larger share of labor income than desired, they do not face strong adverse consequences for consumption. Consumption inequality in retirement even declines.

Most work on the drivers of wealth inequality is focused on the U.S. experience. De Nardi and Fella (2017) survey the different determinants: transmission of bequests, entrepreneurship, medical expense risk, preference heterogeneity, rich earnings processes and heterogeneity in rates of return. Studying these through the lens of a Bewley-Huggett-Aiyagari model, they find bequests, entrepreneurship and medical-expense risk to be particularly important in the U.S. The extant literature is, however, silent on the role played by the pension system.[7] Hubmer et al. (2020), in a similar model set-up with heterogeneous, infinitely-lived agents, find that the drop in tax progressivity in the U.S. since the 1970s is the most important driver of the increase in wealth inequality. Hendricks (2007) show that discount rate heterogeneity can increase the Gini coefficient of wealth to levels close to the data and can explain the large wealth inequality observed among households

[5] In this context, a valuable recent contribution is Epper et al. (2020). They collate data from incentivized preference-elicitation experiments with Danish administrative data for roughly 3,600 mid-life Danes to document time-discounting heterogeneity and its association with the wealth distribution. Charles and Hurst (2003) find significant parent-child similarities in savings behavior, indicative of preference-instilling behavior by parents. More recently, Cronqvist and Siegel (2015) using the Swedish Twin Registry data report 39% of the cross-sectional variation in wealth accumulated up to retirement is explained by genetic variation which includes preferences.

[6] Krusell and Smith (1998) were among the first to add heterogeneity in discount rates to a Bewley model. They find that small differences in patience across agents can create large heterogeneity in wealth.

[7] One possible reason could be the focus on Bewley-type models to study wealth inequality; in such models, saving is entirely for precautionary purposes. An exception is Kaymak and Poschke (2016), who introduce dynastic elements into a Bewley model.

with similar lifetime earnings. These findings have recently been confirmed by Carroll et al. (2017). Benhabib et al. (2019) estimate a macroeconomic model of the distribution of wealth in the U.S. with special emphasis on matching the upper tail. It is noteworthy that the mechanisms highlighted in U.S.-focused research are unable to explain wealth inequality in Denmark. For one, medical expenses in Denmark are mostly covered by the state, and tax reforms have not played the same role as in the U.S. Also, wealth inequality in Denmark, like in Sweden (Domeij and Klein, 2002), is driven mainly by the large fraction of households with zero or near-zero net wealth rather than by high wealth of the top 1% or top 0.1%. Indeed, Jakobsen et al. (2020) document that, over the past three decades in Denmark, the wealth shares of the top 10%, top 1%, and top 0.1% have stayed relatively stable.[8]

Finally, a few remarks surrounding the definition of wealth are in order. In a recent paper, Greenwald et al. (2021) distinguishes between financial wealth (net wealth according to our definition) and total wealth, the sum of financial and human wealth (including PAYG pensions). Since total wealth represents the entirety of claims on lifetime consumption, it is relevant for welfare considerations. For our focus on wealth accumulation and its distribution, the narrower definition (financial wealth) is more appropriate. After all, contributions to PAYG schemes are not tied to any accumulation of saving since contributions are not held in (or backed by) assets: the return to said contributions is entirely reliant on the fulfillment of future government obligations, not on asset returns (Zucman, 2019).[9] That is not to say that PAYG pension payouts are irrelevant: as the model in Section 2 shows, such benefits enter the intertemporal budget constraint of agents, and influence their own retirement wealth-accumulation decisions. This is also the case in the calibrated model.

The paper is organized as follows. Section 2 presents the basic analytical model and develops intuition for the different implications of both pension schemes for wealth accumulation and its distribution. The structure and transition of the Danish pension system are reviewed in Section 3. Section 4 documents the development in Danish wealth inequality over the period 1986-2017. Section 5 develops a rich life-cycle model calibrated to Denmark, and explores how the shift towards more funded pensions affects savings for different cohorts during the transition and the implications for wealth inequality. Section 6 offers a few concluding remarks.

[8] Their paper is a significant recent contribution to an emerging literature that studies the effects of wealth taxes on taxable wealth. They explore a reform of the Danish wealth tax and focus is on estimating short- to long-term behavioral responses (the elasticity of wealth w.r.t. a wealth tax). They report that the implied long-run elasticity of taxable wealth with respect to the after-tax rate of return equals 0.77 for the moderately wealthy and 1.15 for the very wealthy.

[9] The issue of what constitutes wealth in the context of pensions goes back to Feldstein (1976) who shows how marketable wealth, excluding Social Security, is more unevenly distributed than total wealth. More recently, Catherine et al. (2020) shows, contrary to popular views about widening U.S. wealth inequality over recent decades, once Social Security "wealth" is incorporated, wealth inequality has not moved much. Indeed, it stands to reason were other transfers such as Medicare included in definitions of wealth, U.S. wealth inequality would be considered even lower.

2 Pension systems, saving and the retirement wealth distribution – the basics

To set the scene, we present a stylized, overlapping generations model with no population growth and no bequests in which agents (indexed by i) are two period lived. They receive an endowment or earned income, w_i, when young (no endowment when old), and save for their retirement using a single asset with gross return R. Following the literature, dynamic efficiency is assumed ($R > 1$). Assume $w_i \in [\underline{w}, \overline{w}]$, with a cumulative distribution function $\mathcal{F}(w)$, and associated density function $f(w)$; also, $w_i < w_j$ for $i < j$, that is, the lower the index, the poorer the agent. Preferences are given by $\Omega_i = u(c_i^y) + \beta_i u(c_i^o)$, where c_i^y and c_i^o is consumption of agent i when young and old respectively, $u(\cdot)$ has standard properties, and β_i is the subjective discount factor. Agents are ordered such that $\beta_k < \beta_m$ for $k < m$, that is, the lower the index the more impatient the household. Subjective discount rates $\beta_i \in \left[\underline{\beta}, \overline{\beta}\right]$, $\overline{\beta} \leq 1$, with a cumulative distribution function $\Phi(\beta)$, and associated density function $g(\beta)$. It is possible to allow for correlations between β_i and w_i, but this is not done here.

In a world without any policy intervention, i.e., laissez faire, let s_i denote voluntary retirement saving by agent i; then, $c_i^y = w_i - s_i$, $c_i^o = Rs_i$, and $\left(c_i^y, c_i^o\right) \in \Re_{++}^2$. Denote optimal saving by $\widetilde{s}(w_i, \beta_i)$; then $-u_{c^y}(w_i - \widetilde{s}(w_i, \beta_i)) + R\beta_i u_{c^o}(R\widetilde{s}(w_i, \beta_i)) \equiv 0$. First note, in the absence of income in the second period, standard assumptions on preferences imply everyone saves: $\widetilde{s}(w_i, \beta_i) \in (0, w_i)$. Second, optimal saving rises with income and patience, $\frac{\partial \widetilde{s}(w_i,\beta_i)}{\partial w_i} > 0$ and $\frac{\partial \widetilde{s}(w_i,\beta_i)}{\partial \beta_i} > 0$. And finally, the distributions of income or patience induce a distribution in retirement wealth ($\chi_i \equiv R\widetilde{s}(w_i, \beta_i)$). In this context, notice $\frac{\partial \widetilde{s}(w_i)}{\partial w_i} > 0 \Leftrightarrow \frac{\partial \chi_i(w_i,\beta_i)}{\partial w_i} > 0$ implying high income groups also hold more retirement wealth. The same is true for patience: more patient households possess more retirement wealth.

As noted in the introduction, the main driver of wealth heterogeneity we study is via β assuming w is invariant to i. The case assuming identical β but person-specific w is virtually the same and is, therefore, not analyzed. The calibrated model will feature uncorrelated heterogeneity in both w and β simultaneously. Importantly, in the calibrated model, age and random effects on income are included, while heterogeneity in β is permanent and inherent to agents.

2.1 PAYG pension system

Consider a PAYG pension system in which each young agent contributes a lump-sum amount (τ) and the proceeds are transferred forthwith to each of the retired as a lump-sum pension (b).[10] Given a fixed population, the budget of the scheme is simply, $\tau = b$. The agent budget constraints are reformulated as: $c_i^y = w - s_i - \tau$, $c_i^o = Rs_i + b$. Let $s_P(w, R, \beta_i, \tau)$ denote optimal retirement saving (the sub-P refers to the PAYG regime); here on, the notation is reduced to $s_P(\beta_i, \tau)$. Clearly $-u_{c^y}(w - s_P(\beta_i, \tau) - \tau) + R\beta_i u_{c^o}(Rs_P(\beta_i, \tau) + b) \equiv 0$.

[10]Since both the contribution and the transfer are lump sum, there is no redistribution via the pension system.

After imposing $\tau = b$, it follows straightforwardly that $\partial s_P(\beta_i, \tau)/\partial\tau < 0$ implying the PAYG pension crowds out voluntary saving. An immediate implication of crowding-out is that there exists some i for whom $s_P(\beta_i, \widehat{\tau}) \equiv 0$, and any increase in τ beyond $\widehat{\tau}$ would increase the critical value and increase the number of households desiring to borrow against her pension. In this context, few points deserve attention. First, $s_P(\beta_i, \widehat{\tau}) = 0$ simply means agent i's voluntary *retirement* saving is driven to the zero corner. This, in no way, precludes agent i for saving for other reasons (such as precautionary or liquidity reasons that are absent here, but included in the calibrated model). Second, for a variety of reasons, explicit borrowing against one's pension (or using pension payouts as collateral on loans) is not allowed in most countries. Relatedly, as is common in the literature, we impose a "borrowing constraint": $s_P(\beta_i, \tau) \geq 0 \; \forall i$. The condition binds for whom $u_{c^y}(w - \tau) > R\beta_i u_{c^o}(\tau)$. Define β_P by $u_{c^y}(w - \tau) - R\beta_P u_{c^o}(\tau) \equiv 0 \Leftrightarrow \beta_P = \frac{u_{c^y}(w-\tau)}{R u_{c^o}(\tau)}$. It follows voluntary retirement saving for person i is

$$s_P\left(\beta_i, \tau\right) = \begin{cases} 0 \text{ for } \beta_i \leq \beta_P(\tau) \\ > 0 \text{ for } \beta_i > \beta_P(\tau) \end{cases} .$$

where

$$s_P\left(\beta_i, \tau\right) < \widetilde{s}(\beta_i) \; \forall \; i.$$

For future use, define the set of i for whom $s_P = 0$ as "passive" savers: they do not respond to an increase in pension generosity, or equivalently, τ. The rest are "active" savers.

It is easily shown that $\partial\beta_P\left(\tau\right)/\partial\tau > 0$. Two important implications are immediate. As the system becomes more generous, a) the cut-off β below which agents stop voluntary retirement saving rises: all else same, the mass of passive savers increases, and only sufficiently high-β individuals are active savers; and b) since $\partial s_P(\beta_i, \tau)/\partial\tau < 0$, those above the cut-off, β_P, respond by *further* reducing their voluntary retirement saving. A welfare rationale (Andersen and Bhattacharya, 2011) for an optimal b may emerge for households ($\beta_i \leq \beta_P(\tau)$) at the zero-savings corner: an increase in b increases consumption one-to-one since $c_i^o = b$. Define aggregate voluntary retirement saving by $S_P(\tau) \equiv \int_{\beta_P(\tau)}^{\overline{\beta}} s_P(\beta_i, \tau) g(\beta) d\beta$. It follows, a more generous PAYG pension unambiguously decreases total saving:

$$\frac{\partial S_P(\tau)}{\partial\tau} = \int_{\beta^P(\tau)}^{\overline{\beta}} \frac{\partial s_P(\beta_i, \tau)}{\partial\tau} g(\beta) d\beta - s_P(\beta_P, \tau) g(\beta_P) \frac{\partial\beta_P(\tau)}{\partial\tau} < 0.$$

A comparison with laissez faire is in order. Under laissez faire, retirement wealth for i is $\chi_i = R\widetilde{s}(\beta_i)$ and retirement saving is $\widetilde{s}(\beta_i) > 0 \; \forall i$. Under a PAYG system, retirement wealth[11] is given

[11]Note that the pension benefit, b, enters the intertemporal budget constraint although it is not explicit retirement wealth (as in Catherine et al., 2020). The PAYG pension is not explicit wealth because the individual does not have any tradeable

by

$$\chi_{i,P} = \begin{cases} 0 \text{ for } \beta_i \leq \beta_P(\tau) \\ Rs_P(\beta_i, \tau) \text{ for } \beta_i > \beta_P(\tau) \end{cases}.$$

Note given the absence of other savings motives than for the retirement, in our stylized model, retirement wealth (retirement saving) and wealth (saving) are synonymous and we use these terms interchangeably henceforth. At the same time, in the presence of a borrowing constraint, wealth and net wealth are equivalent. Wealth for agents satisfying $\beta_i \leq \beta_P(\tau)$ (the "passive" savers) is lower under the PAYG scheme; and although "active" agents still save, savings is lower than in laissez faire, satisfying $\beta_i > \beta_P(\tau)$ since $R\widetilde{s}(\beta_i) > Rs_P(\beta_i, \tau)$. Combined, wealth inequality rises with a PAYG scheme relative to laissez faire.[12]

2.2 FF pension system

For easy comparison, next consider a fully-funded pension scheme which *mandates* the young to contribute τ (the same as in the PAYG scheme) and returns a pension $R\tau$ to the same person when old. In this case, own retirement saving is determined by $-u_{c^y}(w - s_F(\beta_i, \tau) - \tau) + R\beta_i u_{c^o}(R(s_F(\beta_i, \tau) + \tau)) \equiv 0$, (the sub-$F$ refers to FF). If $s_F(\beta_i, \tau) > 0$, it follows $\partial s_F(\beta_i, \tau)/\partial\tau = -1$: increasing the mandated contribution to the FF scheme reduces voluntary saving one-for-one since the two forms of retirement saving, mandated and voluntary, are perfect substitutes – they offer the same return. Total retirement saving is, therefore, unchanged. The mandate is, thus, ineffective in affecting total saving if $s_{i,F} > 0$; an effect arises when the borrowing constraint is binding.

property rights on it. It cannot be used in any trade: it cannot be used as collateral or seized in the event of a bankruptcy, cannot be bequeathed, and so on.

[12]Define the wealth share for agent i under PAYG by $\phi_P(\beta_i) \equiv \frac{s_P(\beta_i,\tau)}{s_P(\tau)}$. The correspondent wealth share under laissez faire is denoted as $\widetilde{\phi}(\beta_i)$. Our conclusion follows using

$$\int_{\underline{\beta}}^{\overline{\beta}} \phi_P(\beta_i) g(\beta) d\beta = \int_{\underline{\beta}}^{\beta_P(\tau)} \phi_P(\beta_i) g(\beta) d\beta + \int_{\beta_P(\tau)}^{\overline{\beta}} \phi_P(\beta_i) g(\beta) d\beta = 1$$

and hence,

$$\int_{\underline{\beta}}^{\overline{\beta}} \widetilde{\phi}(\beta_i) = \int_{\underline{\beta}}^{\beta_P(\tau)} \widetilde{\phi}(\beta_i) g(\beta) d\beta + \int_{\beta_P(\tau)}^{\overline{\beta}} \widetilde{\phi}(\beta_i) g(\beta) d\beta = 1.$$

Since $\int_{\underline{\beta}}^{\beta_P(\tau)} \phi_P(\beta_i) g(\beta) d\beta = 0$ and $\int_{\underline{\beta}}^{\beta_P(\tau)} \widetilde{\phi}(\beta_i) g(\beta) d\beta > 0$ it follows

$$\int_{\beta_P(\tau)}^{\overline{\beta}} \phi_P(\beta_i) g(\beta) d\beta > \int_{\beta_P(\tau)}^{\overline{\beta}} \widetilde{\phi}(\beta_i) g(\beta) d\beta.$$

Define $\beta_F(\tau)$ by $-u_{c^y}(w-\tau)+R\beta_F u_{c^o}(R\tau)\equiv 0$. Then, as before,

$$s_{i,F}=\begin{cases} 0 \text{ for } \beta_i\le\beta_F(\tau) \\ >0 \text{ for } \beta_i>\beta_F(\tau) \end{cases},$$

Total retirement saving is

$$s_{i,F}+\tau=\begin{cases} \tau \text{ for } \beta_i\le\beta_F(\tau) \\ \widetilde{s}(\beta_i) \text{ for } \beta_i>\beta_F(\tau) \end{cases},$$

and aggregate retirement saving is $S_F(\tau)$,

$$S_F(\tau)=\int_{\underline{\beta}}^{\beta_F(\tau)}\tau g(\beta)d\beta+\int_{\beta_F(\tau)}^{\overline{\beta}}\widetilde{s}(\beta_i)g(\beta)d\beta.$$

Aggregate saving is increasing in the contribution rate

$$\frac{\partial S_F(\tau)}{\partial\tau}=\int_{\underline{\beta}}^{\beta_F(\tau)}g(\beta)d\beta>0.$$

Thus, under a FF system, wealth is given by:

$$\chi_{i,F}=\begin{cases} R\tau \text{ for } \beta_i\le\beta_F(\tau) \\ R\widetilde{s}(\beta_i) \text{ for } \beta_i>\beta_F(\tau) \end{cases}.$$

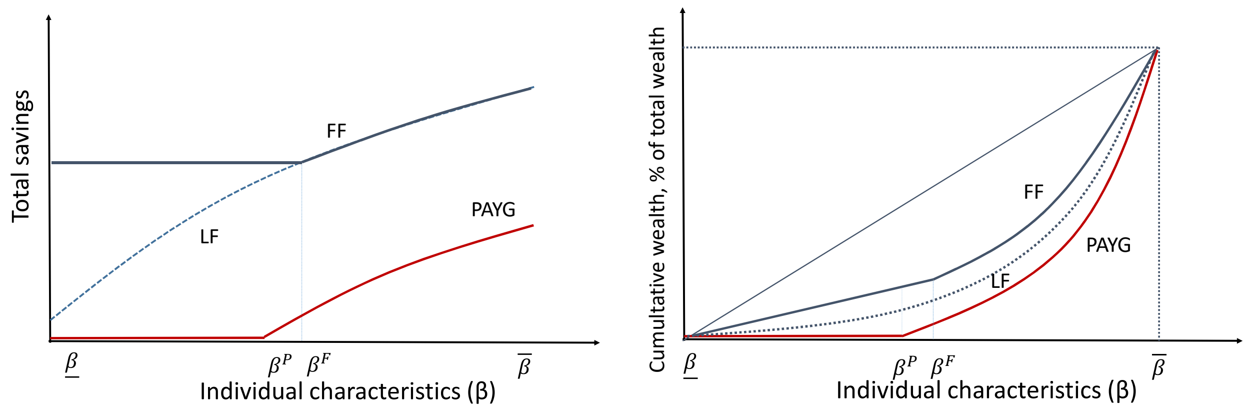
Compared to laissez faire: those with $\beta_i>\beta_F(\tau)$ accrue voluntary retirement wealth, $Rs_{i,F}=R\widetilde{s}(\beta_i)$, the same as in laissez faire, those with $\beta_i\le\beta_F(\tau)$ accumulate mandated wealth equal to $R\tau$ as compared to $R\widetilde{s}(\beta_i)$. If $\tau>\widetilde{s}(\underline{\beta})$, i.e., the FF-scheme payout exceeds the voluntary retirement saving of the least patient agent, then agents with $\underline{\beta}\le\beta_i\le\beta_F(\tau)$ hold more wealth than under laissez faire. It is in this sense, the FF scheme decreases wealth inequality relative to laissez faire. High β agents save the same as before, whereas low β agents save more.

2.3 Comparison

To compare two systems assume a given contribution rate applying for both a FF scheme and a PAYG scheme. To that end, it is useful to know whether $\beta_F(\tau)<\beta_P(\tau)$. Since $u_{c^y}(w-\tau)-R\beta_F(\tau)u_{c^o}(R\tau)\equiv 0$ and $u_{c^y}(w-\tau)-R\beta_P(\tau)u_{c^o}(\tau)\equiv 0$ and $R>1$, it follows $\beta_F(\tau)>\beta_P(\tau)$. Savings and thus wealth levels can now be compared across the different regimes. Total pension savings and hence wealth for any type i is always at least as high under a FF scheme as in the absence of a pension scheme, and higher for those with lowest savings (patience). Total savings and hence wealth is unambiguously lower for all types - and for the lowest savers (patience) zero - under a PAYG scheme compared to laissez faire, and thus lower than under a FF scheme. Recall

Figure 1: Savings and Lorenz curves

(a) Total savings (b) Lorenz curve



wealth under laissez faire $\chi_i \equiv R\widetilde{s}(w_i, \beta_i)$, $R\widetilde{s}(\beta_i) > Rs_P(\beta_i, \tau)$ and wealth under two pension systems:

$$\chi_{i,P} = \left\{ \begin{array}{c} 0 \text{ for } \beta_i \leq \beta_P(\tau) \\ Rs_P(\beta_i, \tau) \text{ for } \beta_i > \beta_P(\tau) \end{array} \right| \cdots \chi_{i,F} = \left\{ \begin{array}{c} R\tau \text{ for } \beta_i \leq \beta_F(\tau) \\ R\widetilde{s}(\beta_i) \text{ for } \beta_i > \beta_F(\tau) \end{array} \right. .$$

We can divide the range of β into three intervals, where it holds:

$$\begin{aligned} \chi_{i,F} &> \chi_i > \chi_{i,P} = 0 \text{ for } \beta_i \leq \beta_P(\tau) \\ \chi_{i,F} &> \chi_i > \chi_{i,P} \ > 0 \text{ for } \beta_P(\tau) < \beta_i \leq \beta_F(\tau) \\ \chi_{i,F} &= \chi_i > \chi_{i,P} \ > 0 \text{ for } \beta_i > \beta_F\,(\tau) \end{aligned}$$

Total savings levels depending on individual characteristics and the Lorenz curve are illustrated in Figure 1. The pension system affects both the level of wealth and its distribution. Under PAYG, the wealth share for low β-types decreases compared to the laissez faire and FF case because their savings are driven to zero. However, for high β-types the comparison is ambiguous: they have lower savings due to crowding out, but total savings are also lower (both the denominator and nominator in the ratio change.) It cannot thus generally be stated whether the Lorenz curves can cross for high levels of β. However, two facts can be generally established. A FF scheme implies:

1. a higher wealth level than laissez faire and PAYG,
2. a changed distribution of wealth increasing the wealth share of "low savers" (here: low patience) compared to laissez faire and PAYG.[13]

Whether or not a shift from PAYG to FF leads to a more equal wealth distribution is an empirical issue and depends on further assumptions. We explore the factual implications of a transition from

[13]Compared to laissez faire, the wealth share also unambigously decreases for "high savers" (here: high patience).

mostly PAYG to a hybrid pension system with a more dominant role of FF scheme by examining the Danish data in Section 4. We document that the shift in the pension system was accompanied by a considerable decrease in wealth inequality among Danish households. The structural model in Section 5 confirms that the pension reform played a dominant role in generating the obseved pattern in wealth inequality.

3 The Danish pension system

Over the last decades, the Danish pension system has undergone a major change from a mainly defined-benefit (DB) PAYG scheme towards a hybrid scheme with an important role for defined contributions (DC) funded occupational pensions (see Andersen, 2021).[14]

Before 1993, the DB PAYG scheme was dominant in Denmark. Unlike the U.S. social security system, the DB scheme in Denmark is tax financed and offers an universal entitlement – a flat-rate pension (common to all) and a means-tested supplement – to any resident reaching the statutory retirement age.[15] The system is relatively complicated due to specificities in the means testing (on a household basis) and additional supplements (e.g., for housing). The key features are, however, easily captured in Figure 2 which shows the relationship between public pensions (y-axis) and "private income" – income from work or pensions but not including public pension payouts (x-axis). A mandatory, funded DB scheme, ATP, is a supplement to the public pensions.

For a long time, supplementary occupational pension schemes only existed for particular groups, primarily in the public sector and for the highly educated. In the late 1980s, an agreement was reached to broaden the coverage of occupational pensions to almost the entire labor market. These DC occupational pensions arose out of a concern that while public pension amounts may forestall poverty among the retired, they may not be enough to generate decent living standards (pension adequacy) since private retirement saving levels were too low.

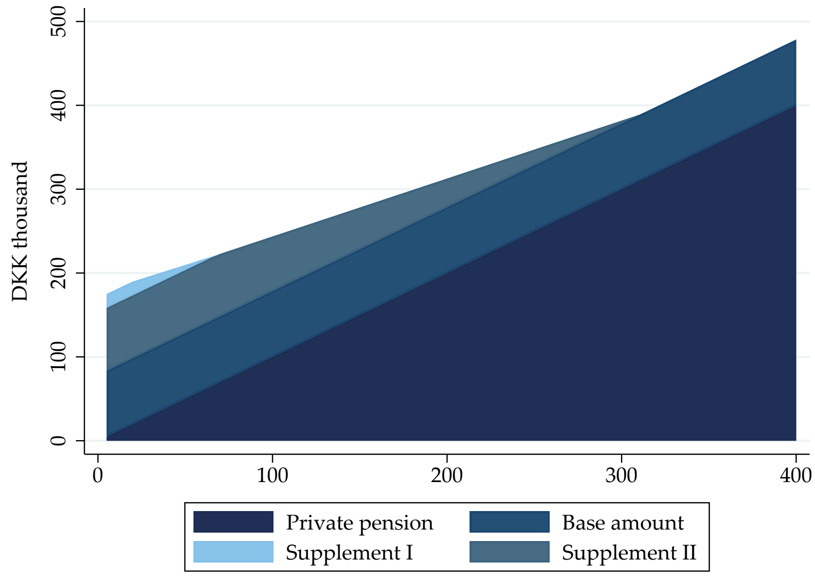
Figure 3 (a) illustrates how contribution rates to DC pension schemes have evolved since the early 1990s for large groups of blue and white-collar workers in the private labor market. By way of comparison, the figure also shows contribution rates for lawyers and economists (DJØF), and nursery teachers (public sector). Evidently, the transition saw a substantial increase in contribution rates over 1993-2009. By now, participation in DC schemes are quasi-mandatory with about 85% of the Danish working population being covered.[16] In this respect, the Danish system differs from

[14]This set-up differs from the model in Section 2 where the PAYG pillar is fully replaced by the FF pillar. However, the insights apply to the Danish case. In the structural model in Section 5 we allow for a two-pillar system with co-existing PAYG and FF elements. Voluntary retirement savings, which are tax-incentivized in Denmark, are sometimes called the third pillar.

[15]There are two supplements: i) Supplement I - pensionstillæg - (introduced in 1994) is means-tested against any type of income, and ii) Supplement II – ældrecheken - (introduced 2003) is means-tested against pension income only. Both the base pension and the supplements differ between singles and couples. The base pension has an earning-income threshold affecting those working post the statutory retirement age, but there is also a possibility of postponing the base pension on actuarial terms to strengthen incentives to postpone retirement.

[16]Technically, these schemes are considered 'voluntary' because they result from collective agreements, but participation

Figure 2: Structure of public pensions, 2020



Note: The figure is a stylized illustration of the public pension system in Denmark 2020. Public pensions are represented on the y-axis and private income - income from work or pensions but not including public pensions - on the x-axis. Supplement I: pensionstillæg; Supplement II: ældrecheken. Other supplements to the old (including housing) are disregarded. Pensions are taxable income. 100.000 DKK is approx. 13.000 euro.

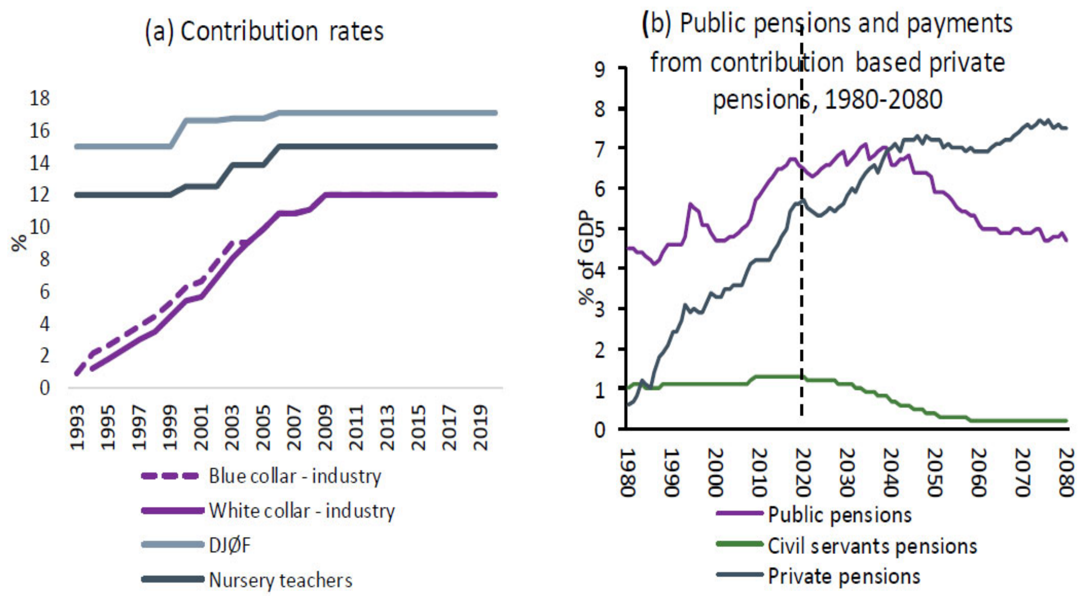
DC schemes in the U.S., like IRAs, which are voluntary.

Overall, partly due to the expansion of FF schemes, Denmark has seen a steep increase in the aggregate household savings rate from near 5% in the 1980s to 18% in 2019 (with a temporary drop during the financial crisis). Consequently, substantial pension funds have been accumulated. Pension assets have been rising from around 50% of GDP in the 1980s to currently more than 200% of GDP, the highest among OECD countries (OECD, 2020). This significant change is mirrored in a shift in the current account – from systematic deficits to surpluses – transforming Denmark from a net-debt position nation (vis-a-vis other countries) into a net-wealth position one (the Net International Investment Position in 2020 was nearly 65% of GDP).

When matured, the Danish system will be a hybrid with public PAYG-pensions mainly serving distributional objectives, and the occupational FF-pensions, insurance and replacement objectives, in accordance with the recommendations by the World Bank (1994). However, it takes several decades for the occupational funded pension scheme to mature in the sense that contributions at the politically desired level apply to an entire work career and pensions benefits for the entire pension period are received based on such contributions. The increasingly dominant presence of FF pensions is seen clearly from Figure 3 (b). The figure shows pension payments from contribution-

is mandatory for the individual worker. In addition, where bargained pensions do not apply, there are often occupational pension arrangements at the firm level. This creates a so-called residual group with no coverage which includes the self-employed. Also, individuals with a marginal attachment to the labor market have low accrued pension entitlements via occupational pensions.

Figure 3: Contribution rates, occupational labour market pensions, and total pension expenditures for public sector and contribution based pension funds.



Note: Contribution rates apply to DI/CO collective agreements. Since 2009 contribution rates have been identical for blue and white collar workers. Expenditures: data 2015 onwards are projected expenditures. Civil servants pensions are gradually phased out and of minor interest for this analysis. Data source: Finansministeriet (2017a).

based pension funds as a share of GDP; the increasing trend until about 2045 reflects the maturation of the scheme. Public expenditures are falling due to individuals having accumulated larger private pensions and therefore receiving less in public pensions due to means testing. This is despite an increasing old-age dependency ratio in par with the average OECD development.[17] Pensions from FF schemes are projected to be larger than the PAYG pensions after 2045.

The Danish system delivers replacement rates among the highest in the OECD and EU, see OECD (2019) and European Commission (2018a,b). The funded DC pensions are, by construction, financially viable. Important for the public part, criteria for fiscal sustainability are satisfied, see Economic Council (2019) and Finansministeriet (2019). Overall, the system is strong on various outcome measures (poverty alleviation, replacements rates) and financially viable, which is also reflected in Denmark consistently being ranked at the top in the Melbourne Mercer Global Pension Index.[18]

[17] In this context, increases in the statutory retirement ages are also relevant. Recent reforms have increased the statutory retirement age in steps from 60 to 64 years for early retirement (2023) and the public pension from 65 years to 67 years (2022). The second element in the reforms is an indexation of the early retirement age and pension age to the development in life expectancy at the age of 60 in order to target the expected pension period to 14.5 years (17.5 including early retirement) in the long run (currently about 18.5/23.5 years).

[18] The index is based on sub-indices for adequacy, sustainability and integrity, and includes in total 40 indicators, see https://info.mercer.com/rs/521-DEV-513/images/MMGPI%202019%20Full%20Report.pdf.

4 Facts on wealth and its distribution in Denmark

This section provides new stylized facts of the wealth distribution based on Danish data from *Statistics Denmark*. Several datasets are merged to obtain an annual longitudinal panel with information on wealth, earnings, household composition, etc., for all Danish households. The databases and variables used in our analysis are described in Appendix A. We work with the data from 1986 to 2017 and pay particular attention to 1992 and 2017. 1992 is a benchmark, pre-transition year and is chosen because from 1993, contribution rates for private-sector employees started to increase, see Figure 3 (a). 2017 is the last year in the sample, and contribution rates have reached their permanent level this year. From 2014 on, *Statistics Denmark* provides a new database on households' assets and liabilities. This database allows for a straightforward construction of various net and gross wealth measures, including person-specific pension assets and durables for the period 2014-2017. Before 2014, wealth was subject to slightly other definitions coming from a different dataset (see Appendix A). Note individual pension asset data are available only from 2014, and pension assets for previous years are imputed as described in later parts of this section. See Appendix A for data sources and definitions.

Throughout, net wealth is defined to include real and financial assets (encompassing pension assets net of taxes but not consumer durables such as cars) minus liabilities. This definition follows a vast international literature as described in Appendix A.2 that discusses wealth measures used in other studies. Wealth is considered from the households' perspective, as savings decisions are often made at the household, and not the individual level. The sample is restricted to individuals above twenty years of age, excluding those above that age who live with their parents. We aggregate household wealth by considering assets owned by each household member and create a normalized (equal-split) household measure. The aggregate household wealth is divided between adult members of the household. Moreover, we attribute specific household head characteristics, such as age, to the entire household. The household head is defined as the man in mixed-sex households. We keep the original household head definition provided by *Statistics Denmark* in the case of same-sex households.[19] The data reported in this section excludes self-employed people because they do not pay into the mandatory pension scheme. Therefore, their savings are not comparable to the rest of the population. It also excludes households with more than two adult members due to the difficulty of attributing wealth items correctly in the case of multi-person households. The overall conclusions are the same when considering all households. Appendix A.3.2 presents wealth measures for this category to facilitate comparison with other papers in the wealth inequality literature, including the recent paper by Jakobsen et al. (2020).

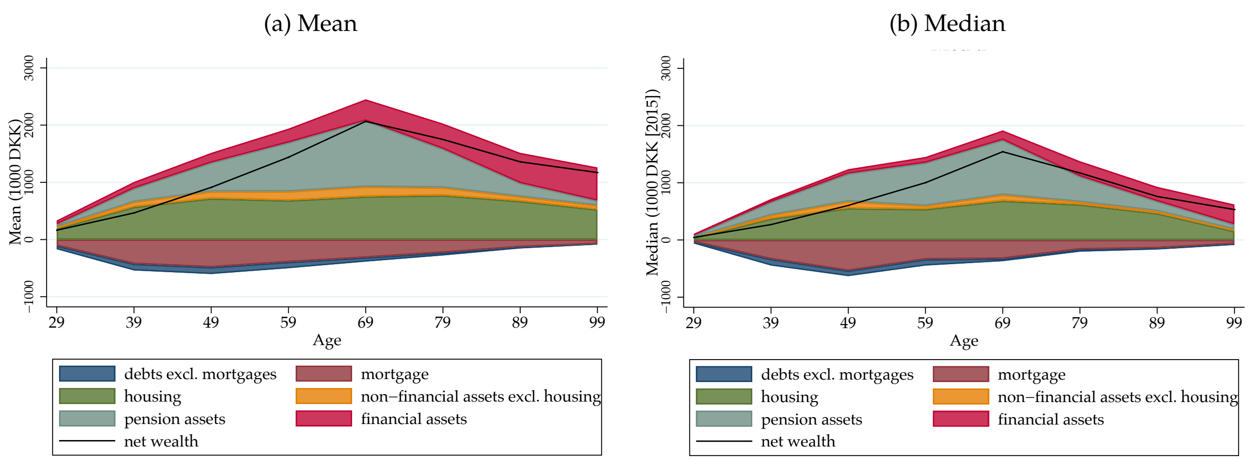
[19]Alternatively, we could define the household head as the primary earner. This alternative definition resulted in similar wealth distributions at the household level as in the current paper.

4.1 The role of pension assets in the asset portfolio of Danish households

We start by presenting some basic facts using the most recent and reliable data on wealth holdings of Danish households. Figure 4 presents cross-sectional wealth holdings of Danish households in 2017. Here and in the following graphs, numbers are presented in constant 2015 DKK. Panel (a) visualises the asset and debt holdings of the mean and panel (b) of the median household.[20] For the purpose of this graph, the Danish households are grouped into ten-year age bins, where the first bin covers households with a head between 20 and 29 years old, etc., and we restrict the sample to households whose head is below 100 years old. The life-cycle pattern of asset and debt holdings across age groups among Danish households resembles the one known from other countries (see Cowell et al., 2012, for a comparative study).

Young households hold relatively little net wealth, given that the acquisition of assets is paralleled by an increase in debts until their 40s. Most debt is housing-related (mortgages). Assets and net wealth peak for households between 60-69 years of age. Households tend to hold on to their wealth possessions after they retire. They do so also with housing, even though to a lesser extent. Pension assets peak for households between ages 60-69, after which most households retire, and begin to consume their pension wealth. Even households in the highest age-bin hold a considerable amount of assets, which may reveal a bequest motive. A comparison between the mean and median values indicates that the distribution is right-skewed, as the median household in the age group 90-99 holds relatively less net wealth than the mean household at the end of the lifetime. It is also visible that for the median household in 2017, private pension assets play a dominant role in their asset portfolio, particularly near retirement age.

Figure 4: Cross-sectional wealth holdings of Danish households, 2017



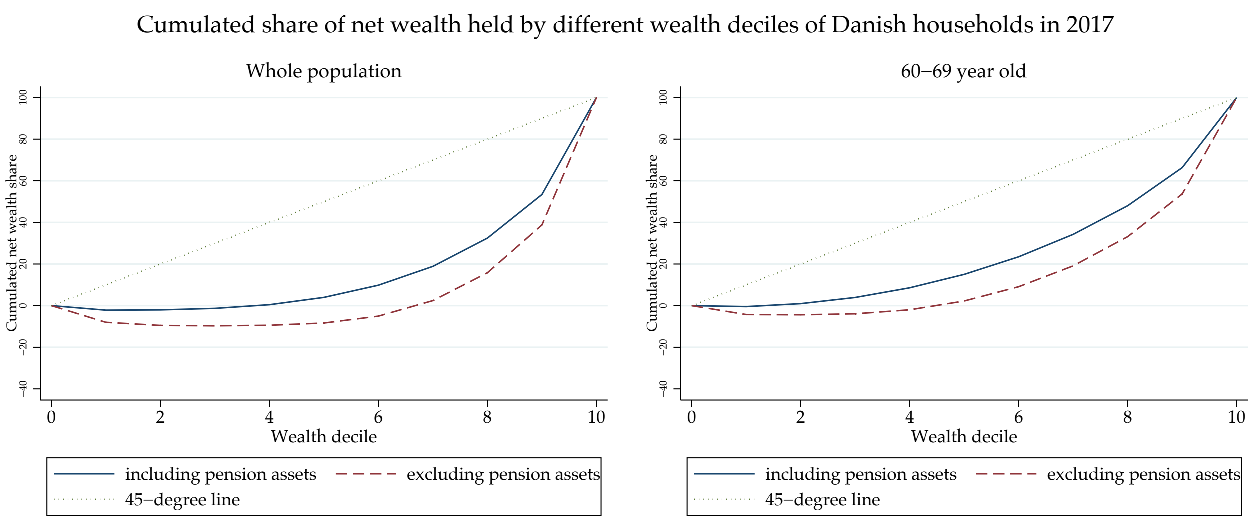
Note: Wealth is presented at the normalized (equal-split) household level. Self-employed and multi-person households are excluded.

[20]Due to the data restrictions from Denmark Statistics, the 'median' graph is based on the average asset and debt holdings of 100 households in a given age group with net wealth closest to the median value.

The role of pension assets in the asset portfolio may be visualised with Lorenz curves for wealth in 2017 presented in Figure 5. The left side of the graph presents accumulated net wealth shares held by each wealth decile including (solid line) and excluding (dashed line) pension assets for the entire population. Analogous Lorenz curves are presented for the group of 60-69 year olds on the right hand side of the graph. We focus on that group for several reasons. The Gini coefficient across all age groups displays wealth inequality mainly because age groups are in different phases of the life-cycle. This effect is eliminated by considering specific age-groups, and 60-69 group is relevant since people start to retire in their 60s in Denmark. Moreover, private pension assets peak for households in this age group.

If we do not include pension assets in the calculation, accumulated net wealth is negative for the first six (all age groups) and four (age group 60-69) wealth deciles. When private pension assets are included, both curves approach the 45-degree line, indicating lower wealth inequality as compared to the measure without pension assets. The corresponding Gini coefficients are presented in Table 1.

Figure 5: Lorenz curve for net wealth, 2017

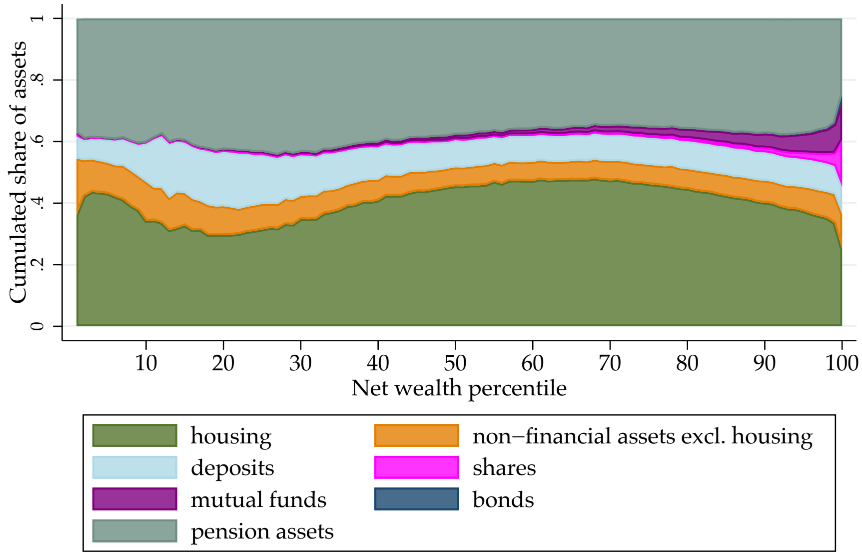


Note: Wealth corresponds to net wealth without consumer durables including pension assets net of taxes and is presented at the normalized (equal-split) household level. Self-employed and multi-person households are excluded.

Figure 5 thus shows how important pension assets are for the wealth distribution in Denmark, but also differences across age groups. Some of this is because they were subject to different pension policies over the last decades.

Apart from age, one can also look at asset holdings from the perspective of net wealth deciles. The portfolios of poorer and richer households are expected to differ considerably. This is confirmed by Figure 6 presenting different asset holdings of households ranked by wealth percentile. Private pension assets dominate the portfolios of the lowest-ranked households, even more prominent than housing in some cases. The role of private pension assets declines with the increase in net wealth decile and is the lowest for the most wealthy Danish households. Top wealth deciles have higher voluntary savings reflected in holdings of stocks. Housing is one of the two (aside from private pensions) most important asset categories for all households, but plays a relatively minor role for the top percentiles.

Figure 6: Cumulated mean share of different wealth categories held by Danish households in 2017



Note: Wealth corresponds to net wealth without consumer durables including pension assets net of taxes and is presented at the normalized (equal-split) household level. Self-employed and multi-person households are excluded.

4.2 Long-run trends in Danish wealth inequality

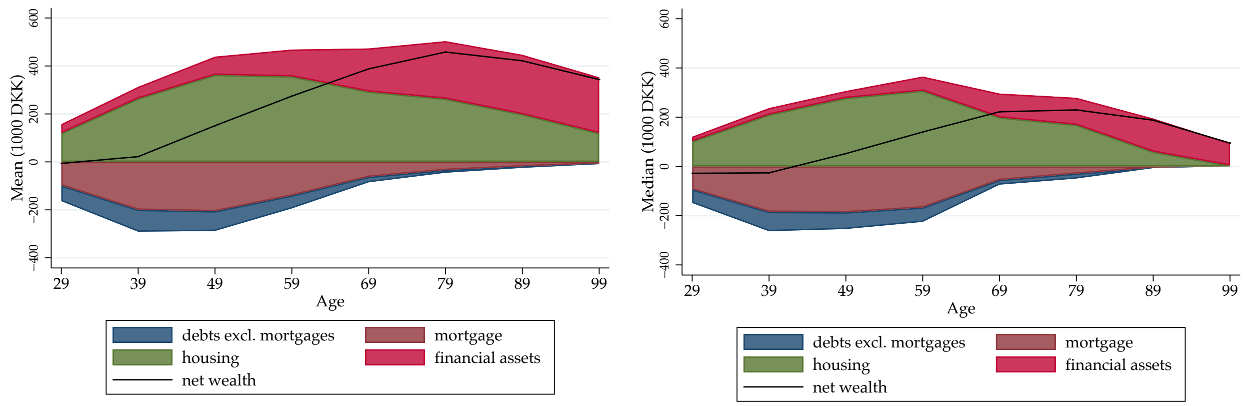
Comparing 2017 with 1992 data is not straightforward, as new wealth statistics is only available from 2014 on. In particular, individual-specific pension data is not available before 2014, and needs to be imputed. The cross-sectional distribution of assets and debts of the mean (panel a) the median (panel b) household in different age groups in 1992 as present in our raw dataset (i.e., without imputation) is presented in Figure 7. The raw data is presented because our private-pension imputation, while matching the aggregates quite well (in terms of Gini coefficients and wealth shares going to different wealth deciles), is not sufficiently precise at the individual level and could give a wrong impression about the historical distribution of these assets across the cohorts.

Comparing the scale of Figure 7 with that of Figure 4, it is clear assets and net wealth of Danish households grew considerably between 1992 and 2017, even ignoring private pension assets. In 1992, housing was the most important asset for most Danish households. Towards the end of life, financial assets accounted for the majority of net wealth. Also, households near retirement had much lower debt than younger households in 1992.[21]

[21]Note that mortgage debt, defined as such in historical data is unreliable, and hence, we assume mortgage debt constitutes 77% of all debt in each year of our sample. The choice of 77% is supported by the micro-evidence from more recent years indicating that mortgages account for 79% of household debt between 2014-2017 with a slightly increasing trend. Aggregate data for families presented on the website of *Statistics Denmark* shows that 76% of household debt

Figure 7: Cross-sectional wealth holdings of Danish households, 1992

(a) Mean (b) Median



Note: Wealth is presented at the normalized (equal-split) household level. Self-employed and multi-person households are excluded.

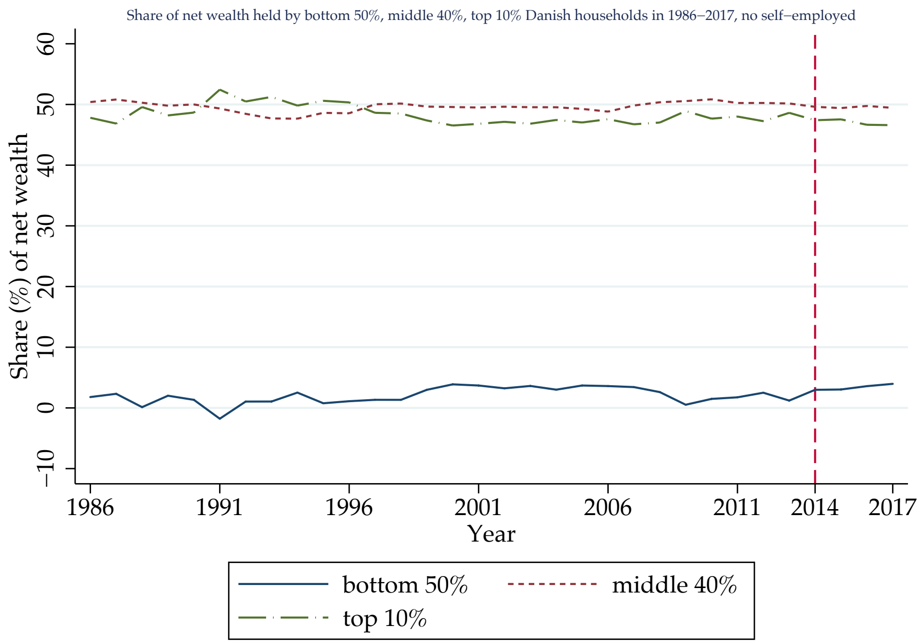
To look at long-run trends in net wealth, we impute pension assets for those years in the sample where we lack this data. In our imputation, we slightly modify the method used in Jakobsen et al. (2020) – aggregate private pension assets are attributed to individual agents based on their share of income and contributions to private pensions. Details on the imputation are available in Appendix A.1. We compare shares of net wealth (including private pension assets) held by different deciles between 1986-2017. It is also possible to compare our imputation with the new data in years 2014-2017. Figure 8 illustrates the evolution of the share of net wealth held by the bottom 50%, middle 40% (6th to 9th decile) and top 10% of Danish households over the past 30 years. The dashed vertical line in 2014 separates our imputed measure from the most recent data using the new wealth dataset. The bottom 50% of the Danish households hold net wealth near zero or even negative in some periods. In the last years, the share of wealth held by these households is increasing. Top 10% and middle 40% households hold comparable shares of net wealth, with the share held by the top 10% slightly decreasing over time.[22] Figure A.3 in Appendix A.3 shows the overlap of our imputed series with the data in 2014-2017. Our imputation matches the aggregate wealth shares held by specific wealth deciles quite well.

Our analytical model suggests a PAYG to FF transition increases the wealth of "low savers". Figure 9 presents the evolution of net wealth shares held by the first five wealth deciles of households in 1986-2017. Evidently, the increase in the wealth share of the bottom 50% – Figure 8 – is mostly driven by the lowest three wealth deciles, in particular the lowest one. In this sense, our

2014-2017 was in mortgages. Lastly, Òscar et al. (2017) presents historical data for the average share of mortgage loans to total non-financial private sector lending during 1987-2017. They find the number moved from near 66% to 69%. It is not evident there is a trend in this share for Denmark, and hence, going forward, we assume the share of mortgages in all households' debt was fixed over the considered time period.

[22]Note this graph shows a slightly different relation between the shares held by top 10% and middle 40% than the corresponding graph in Jakobsen et al. (2020). This is because in our sample we excluded households that have (or had in the past) self-employed members. Self-employed constitute a large share of the top 10%, and once we include them, the decrease in Danish wealth inequality becomes even more evident, see Figure A.10 in Appendix A.3.2.

Figure 8: Share of net wealth held by Danish households in 1986-2017



Note: Wealth corresponds to net wealth without consumer durables including pension assets net of taxes and is presented at the normalized (equal-split) household level. We remove self-employed households and households with more than two adult members in the sample.

model predictions find support in the data. Our conclusions hold once we include self-employed households in the sample.

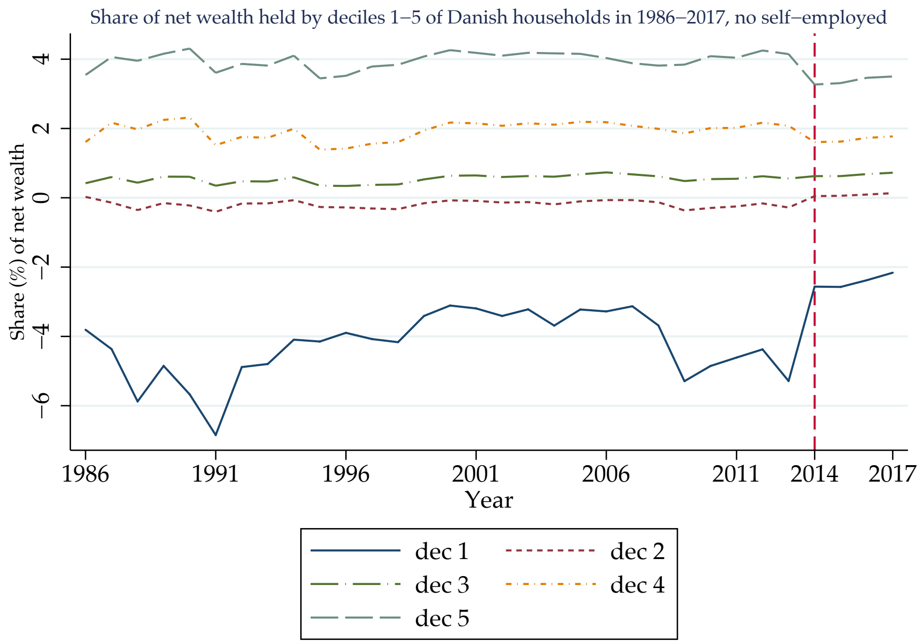
Lastly, even though our paper does not aim to understand the wealth held by most wealthy, the evolution of the top 1 and 0.1 % shares over time is shown in Appendix A.3.1 in Figure A.4. These shares, too, have decreased substantially compared to the early 1990s. Our imputation matches the top wealth shares well.[23]

Following the trends in the shares of wealth held by different groups of Danish households, it can be concluded that despite being high, the Danish wealth inequality has been decreasing in the recent years. This pattern is different from that in many other countries (see Figure A.15 in Appendix A.3).

This development over 1992-2017 is even more evident, when considering the Gini coefficients for 1992 and 2017 for our main sample excluding self-employed households, documented in Table 1. The Gini coefficient for our sample in 1992 was 0.75 (including pension assets) and 0.69 in 2017. It is evident from the table that pension assets play a big role in accounting for wealth inequality. Apart from Gini measures for the whole population, Table 1 shows also corresponding numbers for households where the household head is in the age group 60-69. For the latter, the inclusion of private pension assets into the wealth calculation considerably changes the Gini coefficient in

[23]Figure A.5 in Appendix A.3 shows the overlap between the imputed and combined series in years 2014-2017. Our conclusions hold once we include self-employed households in the sample.

Figure 9: Share of net wealth held by the lowest wealth deciles of Danish households in 1986-2017



Note: Wealth corresponds to net wealth without consumer durables including pension assets net of taxes and is presented at the normalized (equal-split) household level. We remove self-employed households and households with more than two adult members in the sample.

2017, which is consistent with the Lorenz curve presented in Figure 5. Over time, wealth inequality as measured by the Gini coefficient with pension assets included declined more for the retirement group compared to the whole population (-18% compared to -8% once we include all age groups).[24]

Table 1: Gini coefficients in our main sample

	All households			**Head 60-69 years old**		
	Pension assets Excluded	Pension assets Included	% Δ	Pension assets Excluded	Pension assets Included	% Δ
1992	1.04	0.75	-28%	0.66	0.63	-5%
2017	0.91	0.69	-24%	0.71	0.51	-28%
% Δ 1992-2017	-13%	-8%		+8%	-18%	

Note: Wealth corresponds to net wealth without consumer durables including pension assets net of taxes and is presented at the normalized (equal-split) household level. We remove self-employed households and households with more than two adult members in the sample.

In sum, the developments are in accordance with our theoretical predictions. A large role for FF pension should both increase overall saving and wealth accumulation, as well as wealth shares held by the poorest part of the population. The transition towards a more funded pension scheme

[24]Note that the 19% decline from 0.86 to 0.7 mentioned in the introduction refers to all Danish households, including self-employed, see Table A1.

was accompanied by a notable decrease in Danish wealth inequality. Equipped with these new stylized facts about wealth inequality and asset accumulation of Danish households over the life-cycle, we proceed with the presentation of a life-cycle model which offers a lens with which to view these massive changes.

5 A quantitative life-cycle model of the Danish economy

We formalize the transition in the Danish pension system in an otherwise standard life-cycle model wherein overlapping generations of long-lived agents live through work and retirement phases and save optimally over their life-cycle. Inequality between agents within a cohort is generated in two ways: via heterogeneous time preferences and transitory and permanent shocks to labor income. The model features a small open economy with an uncertain lifespan and demographic aging. All factor prices are assumed exogenous capturing the small, open economy status of the Danish economy.

To quantify the effect of the Danish pension reform on the wealth distribution and life-cycle savings, the model is calibrated to the Danish economy in 1992, the year prior to the pension reform. Annual simulations are done between two steady states: 1970 and 2100 (by which time, the pension system has fully matured).

5.1 Model set-up

Demographics

Time is discrete and denoted by $t = 1, 2, ..., \infty$. At each date t, agents are born at age $n = N^b$ and start work immediately. They may live to a maximum age of N^d and survival until this point is stochastic: $\delta_{n,t}$ is the probability with which an agent of age n survives until age $n + 1$. Agents retire at a fixed age of N^r.

Preferences

Agents have expected lifetime utility[25]

$$(1) \qquad \Omega_{0,i} = \mathbb{E}\left[\sum_{n=N^b}^{N^d} \beta_i^{n-N^b} \tilde{\delta}_n \frac{C_{i,n,t}^{1-\vartheta}}{1-\vartheta}\right],$$

where $C_{i,n,t}$ is consumption of agent i at age n at calendar date t, ϑ is the CRRA parameter, and $\tilde{\delta}_n$ is the cumulative survival probability between periods N^b and n, $\tilde{\delta}_n = \prod_{m=N^b}^{n} \delta_m$, which is 0 at age $n = N^d$.

[25]In Section 5.4 we consider a model extension with bequest motive.

Time preferences are idiosyncratic (but fixed over an agent's lifetime), with β_i denoting the individual-specific discount factor. Heterogeneity in β_i not only represents heterogeneity in patience, but is also a catch-all for other factors that differ across people, e.g. differences in investment opportunities, tax schedules, risk aversion or income growth expectations. We model heterogeneity in time preferences by assuming a uniform distribution for $\beta_i \in \left[\underline{\beta}, \bar{\beta}\right]$, from which agents draw at the beginning of their lives.

Labor income process

All labor income is generated exogenously. As is common in the life-cycle literature (going back to Zeldes, 1989 and Carroll, 1992, for example), we write labor income (after taxes and contributions) for agent i of age n at time t as

$$(2) \qquad Y_{i,n,t} = P_{i,n,t}\epsilon_{i,n,t} \qquad \text{if} \quad n < N^r,$$

with $P_{i,n,t}$, a permanent income component, defined as

$$(3) \qquad P_{i,n,t} = G_{n,t}P_{i,n-1,t-1}\eta_{i,n,t}.$$

$G_{n,t}$ is an age- and time-specific component of income, $\epsilon_{i,n,t}$ is an idiosyncratic, transitory income shock with $\ln \epsilon_{i,n,t} \sim \mathcal{N}(0, \sigma_\epsilon^2)$, and $\eta_{i,n,t}$ is a permanent income shock with $\ln \eta_{i,n,t} \sim \mathcal{N}(0, \sigma_\eta^2)$, uncorrelated with $\epsilon_{i,n,t}$. In this way, the labor income process generates both precautionary and life-cycle saving consumption smoothing saving motives.

Pension system

As described in Section 3, the PAYG pillar consists of a flat-rate scheme, providing a basic amount to everybody, and two means-tested supplements, but without loss of generalty it can be collapsed into one. Denoting FF pension annuities by $\tilde{Y}^{FF}_{i,n,t}$, PAYG income is

$$(4) \qquad \tilde{Y}^{PG}_{i,n,t} = \theta_{0,t} + \max\left\{0, \theta_{1,t} - \max\left(\kappa\left(\tilde{Y}^{FF}_{i,n,t}\right), 0\right)\right\} \quad \text{if} \quad n \geq N^r,$$

where $\theta_{0,t}$ and $\theta_{1,t}$ are DKK values of the flat-rate scheme and the maximum amount to be obtained under the means-tested scheme, respectively. $\kappa(\tilde{Y}^{FF}_{i,n,t})$ is a positive function of the labor market pension income introducing a negative relationship between private and public pension payouts. We abstract from public pension contributions. The Danish pension system is tax-financed, so the public pension scheme itself does not have to adhere to a balanced budget. Defining $Y_{i,n,t}$ as after-tax income, changes in the financing costs of the PAYG pension system along the transition path will show up as a change in the time-varying element, $G_{n,t}$, which we estimate.

Agents contribute a fraction $\pi_{i,n,t}$ of their labor income into the mandatory FF pension scheme, so that $(1 - \pi_{i,n,t})Y_{i,n,t}$ is disposable income net of taxes and contributions. We specify $\pi_{i,n,t}$ as

$\pi_{i,n,t} = \bar{\pi}_t + \pi(Y_{i,n,t})$ where $\bar{\pi}_t$ is a positive trend, the essence of the pension reform, and $\pi(Y_{i,n,t}) > 0$ captures the empirical observation that contribution rates are higher for occupations with higher salaries. Pension assets are administered by pension funds and cannot be accessed pre-retirement. Contributions are invested in the financial market and earn an exogenous gross market rate of return R_t. It follows, just before retiring, agents hold end-of-period pension assets

$$LW_{i,N^r-1,t} = \sum_{n=N^b}^{N^r-1} (\pi_{i,n,t-N^r-n+1} Y_{i,n,t-N^r-n+1}) R_{t-N^r-n+1}^{N^r-n}$$

FF pensions are paid out as annuities.[26] With T_t being the expected number of remaining years of life, the annuity paid out at the beginning of period t is

$$\text{(5)} \qquad \tilde{Y}^{FF}_{i,n,t} = \frac{LW_{i,N^r,t-N^r-n} \prod_{s=1}^{T_t} \mathbb{E}_t \left[R_{t+s}\right]}{\sum_{s=1}^{T_t} \mathbb{E}_t \left[R_{t+s}^{s-1}\right]} \quad \forall n \geq N^r.$$

The present value of the annuity equals the current value of all payments from the annuity between retirement and the expected age of death, discounted at rate $\mathbb{E}_t\left[R_{t+s}\right]$ and weighted by the probability of surviving until the specific age. Pension wealth of retirees is decreased by the value of the annuity each year until it turns zero at age $N^r + T_t$. Note the risk diversification of individual mortality implied by such life-annuitization schemes. The FF pension wealth of agents dying prematurely gets implicitly added to the accounts of the survivors within the same cohort ensuring payment as long as the saver is alive.

Private saving

In addition to mandatory pension savings, agents can decide to accumulate private savings. They will do so for precautionary reasons and to smooth consumption over their life-cycle. We denote end-of-period accumulated voluntary savings by $A_{i,n,t}$. Households invest into a single, safe asset, which yields a return of R_t, the same as the mandatory pension savings. Although empirically relevant, we do not explicitly model housing. Housing is an illiquid asset, and most housing in Denmark is owner-occupied indicating it is held not just for investment reasons but also for consumption purposes. This is why it is often excluded in models of life-cycle savings decisions (see e.g., De Nardi and Fella, 2017). Additionally, because borrowing takes place mainly through mortgages, we abstract from it by introducing a zero-borrowing constraint, $A_{i,n,t} \geq 0$.

The budget constraint of a working-age individual therefore reads

$$\text{(6)} \qquad C_{i,n,t} + A_{i,n,t} \leq A_{i,n-1,t-1} R_t + (1 - \pi_t) Y_{i,n,t} \qquad \text{if} \quad n < N^r.$$

[26]In reality, a smaller part of the pensions is paid out as periodic installments and lump sum payments, but we abstract from these to keep the model simple. As we are interested mainly in the wealth of the near-retirees, the payout mode is of little consequence to our results.

Retired agents receive an annuity out of their funded pension wealth. The budget constraint is

$$(7) \quad C_{i,n,t} + A_{i,n,t} \leq A_{i,n-1,t-1}R_t + \tilde{Y}^{PG}_{i,n,t} + \tilde{Y}^{FF}_{i,n,t} \qquad \text{if} \quad n \geq N^r.$$

Agents choose their voluntary private asset holdings $A_{i,n,t}$ such as to maximize lifetime utility eq.(1) subject to eq.s (6) and (7). The solution to the optimization problem can be expressed by the Euler equation

$$(8) \quad C^{-\vartheta}_{i,n,t} \geq \beta_i \delta_n R\, \mathbb{E}\left[C^{-\vartheta}_{i,n+1,t+1}\right]$$

which holds with equality when the borrowing constraint is non-binding, i.e., $A_{i,n,t} > 0$.

5.2 Calibration and simulation

The phasing-in of the fully funded pensions is modeled as a transition from an initial, pre-transition steady state (1970) to a new, post-transition steady state (2100). This allows us to capture time trends in labor income in the 1970s and 1980s, important for achieving a good model fit. The long transition period reflects that the mandated pension system will take full effect only when all living individuals have contributed at the permanent level of contribution rates throughout their entire life cycle.

The model is solved using endogenous gridpoint method (Carroll, 2006). We simulate the transition between the two steady states by performing annual simulations (see Appendix B.1 for details). Agents are assumed to have perfect foresight and know the entire future path of pension contribution rates. To generate a distribution, the model is simulated for 5,000 independent draws of income shocks per age, year and preference type. The model is calibrated to match peak life-cycle FF pension and non-pension wealth and the Gini coefficient in 1992, the last pre-reform year.

Labor income process

In the estimation of the labor income process for Danish households, we apply the methodology of Cocco et al. (2005), described in detail in Appendix A.4 and use registry data from 1987-2017. We regress the logarithm of equal-split after-tax labor income on age fixed effects and a set of demographic control variables, and fit a fifth-order polynomial as

$$(9) \quad \ln(G_n) = 4.635 - 0.774age + 0.472\frac{age^2}{10} + 0.130\frac{age^3}{100} + 0.018\frac{age^4}{1000} - 0.090\frac{age^5}{1000000}.$$

A time trend is included to capture aggregate income growth. Aggregate income in our benchmark sample grows by 36.4% between 1987 and 2017. The error structure of the labor income process is determined by a variance decomposition (Carroll and Samwick, 1997) and implies $\sigma^2_\epsilon = 0.063$ and $\sigma^2_\eta = 0.010$.

FF pensions

The pension contribution rate is determined as

$$\pi_{i,n,t} = \min \left(\max \left\{0, \bar{\pi}_t + \pi(Y_{i,n,t})\right\}, 0.18\right) \tag{10}$$

We limit the contribution rates to take a value between 0 and 0.18 to reflect the reality in Denmark. For $\bar{\pi}_t$, we use average contribution rates for the individuals in our benchmark sample. Between 1992 and 2009, the contribution rate in the data increases from 5.2% to 10.9%. We calibrate $\pi(Y_{i,n,t})$ to match peak FF pension assets in 1992, roughly 132 (thousands, 2015 DKK). This yields

$$\pi(Y_{i,n,t}) = -0.112 + 0.00035 Y_{i,n,t}$$

Note, aggregate income growth implies average pension contributions increase by much more than what is implied by the change in $\bar{\pi}_t$. Prior to the pension reform, pension contributions are zero for a sizable share of individuals.

PAYG pensions

The parameters θ_0, θ_1 and $\kappa(\tilde{Y}^{FF}_{i,n,t})$ are calibrated to match PAYG replacement rates over time from Finansministeriet (2017b). The Danish Ministry of Finance documents a hump shape for replacement rates between 1960 and the 2000s, peaking around 1995, and projects a decline from 41% in 2016 to 34 % in 2070. We approximate this shape by fitting a third-order polynomial for θ_1. Matching these numbers, we get

$$\theta_0 = 83.92 \qquad \theta_1 = 66.68 - \frac{3}{21875}s^3 + \frac{199}{4375}s^2 - \frac{359}{175}s \qquad \kappa = 0.309\left(\tilde{Y}^{FF}_{i,n,t} - 0.14\right)$$

where s indexes the transition year, being 1 in 1971 and 49 in 2019. All numbers are in thousands, 2015 DKK.

Time preferences

The uniform distribution of β_i is approximated by an equi-distant grid following Carroll et al. (2017). Our grid is defined as

$$[b-3d \quad b-2d \quad b-d \quad b \quad b+d \quad b+2d \quad b+3d]$$

We assume an equal probability mass at each grid point and calibrate b and d jointly to match two data moments for our benchmark sample in 1992: the peak of non-pension life-cycle assets, which is 590.3 (thousands, 2015 DKK), and the Gini coefficient, which is 0.626 for the 60-69 year olds. The resulting values are $b = 0.952$; $d = 0.0125$. This results in a discount factor $\beta_i \in [0.9145, 0.9895]$.

The dispersion is slightly larger than in other macroeconomic papers (e.g., Krusell and Smith, 1998, Carroll et al., 2017 and Hendricks, 2007) because of two reasons. First, we consider a life-cycle framework wherein a larger difference in patience is typically needed to generate wealth inequality compared to models with infinitely-lived agents. Second, our agents are ex-ante identical in their labor income trajectories. While the income-shocks they face in their lives will render the agents different ex-post, this mechanism generates less income inequality than models where agents start with innately different levels of permanent income. For example, Carroll et al., 2017 consider three educational groups with different income trajectories. For the data on Denmark, a specification with homogeneous initial permanent income is more suitable as it results in a good match for the income Gini coefficient. The behavioral literature tends to find quite large differences in discount factors, closer to our values – see e.g., Harrison et al. (2002), Andersen et al. (2014), and Epper et al. (2020) for estimates for Denmark.

Demographics

Agents become economically active are born at age 20 (N^b) and live to a maximum of age 100 (N^d). For the survival probabilities $\delta_{n,t}$ and for the remaining life expectancy at retirement, T_t, we use numbers from the UN Population Prospects (UNPOP). The population of Denmark has been enjoying increases in life expectancy over the last decades. Remaining life expectancy at age 15 was 60 in 1970 and increased to 67 by 2020. The population size by age corresponds to the number of individuals in our benchmark sample from the registry data.

The retirement age, N^r, is set to 65. This has been the statutory retirement age between 2004-2018. Prior to that, it was 67, and from 2019, it is gradually increasing again. Given that the actual retirement age tends to be below the official one, 65 matches the data well.

Other parameters

The coefficient of relative risk aversion ϑ is set to 0.65 based on an estimate by Andersen et al. (2014) for Denmark. This relatively low number may be justified by people expecting to rely more on home production post retirement. The return on assets R is 1.037, which corresponds to the real return on retirement savings plans during 2001-2019 (OECD, 2020) subtracting income tax and tax on returns.

5.3 Quantifying the effects of pension reform on savings and wealth

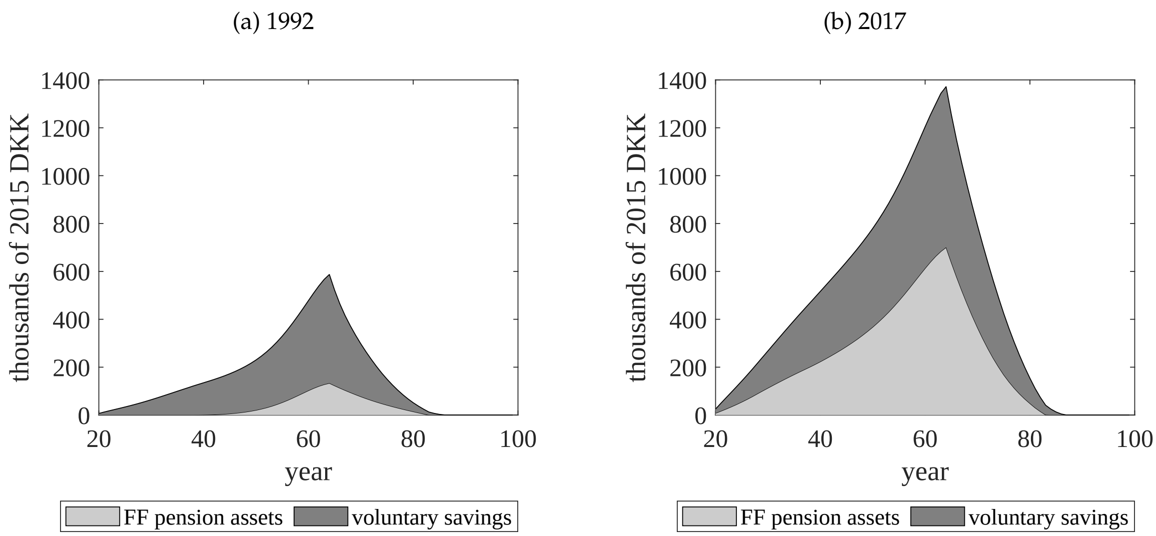
In the following, we start by showcasing the model predictions for individual pension and non-pension wealth before proceeding to discuss the wealth distribution. The primary focus is the years 1992 and 2017, for which we can compare the model predictions with the data. Steady state results can be found in Appendix B.2. In order to determine how much of the change in wealth inequality can be attributed to the pension reform – rather than, for example, aggregate income growth –

we carry out several counterfactual analyses. Finally, we discuss the implications of the pension reform for consumption inequality.

Individual asset profiles

Figure 10 presents cross-sectional averages of asset holdings along the age distribution for 1992 and 2017 for a representative household (where income shocks are set at their expected values and averaged across β-types).

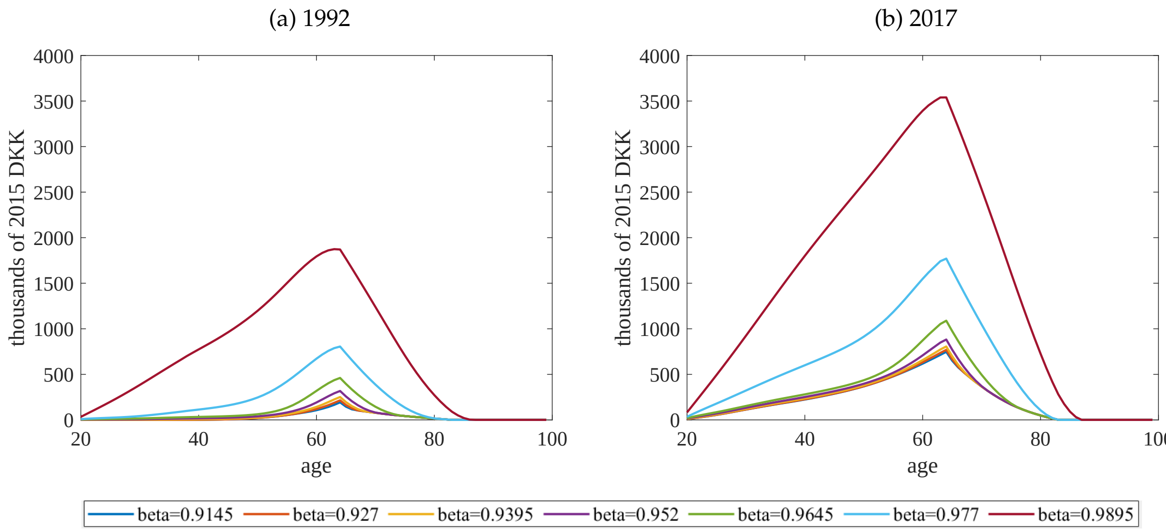
Figure 10: Cross-sectional asset holdings



The peaks of both pension and non-pension assets in the 1992 cross-section were targeted; therefore, the model matches the data exactly. Comparing it to 2017, FF pension assets increased by a factor of 5 to 6 (from 132 to 701 tsd. DKK). The increase is slightly lower than in the data shown in Figure 4. This is because the definition of pension assets in the data includes voluntary top-up of the mandatory contributions which show up as non-pension assets in the model. Non-pension assets peak at 617 tsd. DKK, roughly in line with the data. They constitute almost half of total pension wealth compared to just below one fourth in 1992. This last observation highlights the important role the pension reform plays for individual asset accumulation.

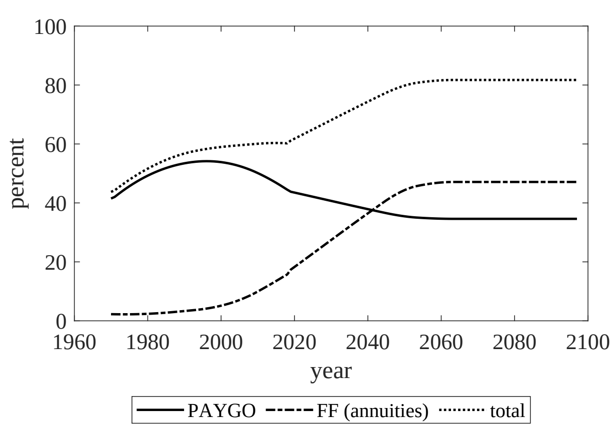
Figure 11 shows total asset holdings by discount factor. Not surprisingly, there is large variation across β-types. However, the dispersion decreases over time. In 1992, pre-retirement wealth is almost 15-times higher for the most patient agents compared to the most impatient ones. In 2017, they only hold about five times more wealth. This finding aligns with the results from the toy model: when shifting from a mainly PAYGO to a a hybrid system, overall savings in the economy increase and the wealth share of impatient agents becomes larger.

Figure 11: Cross-sectional asset holdings, by β type



The increasing importance of funded pension income during retirement can be seen in Figure 12 showing PAYG and FF pension income as a share of the average labor income in the last five working periods. Note from 2019 onwards, as the model is transitioning to the new steady state, the fitted polynomial for PAYG is kept fixed. This is the reason for the discontinuity in the graph. The PAYG replacement rate is hump-shaped, whereas the FF replacement rate increases continuously over time. In line with the predictions shown in Figure 3 (b), FF pension income becomes the most important retirement income from around 2040 onwards and stabilizes around 2060.

Figure 12: Pension replacement rates and FF pension wealth



Wealth distribution

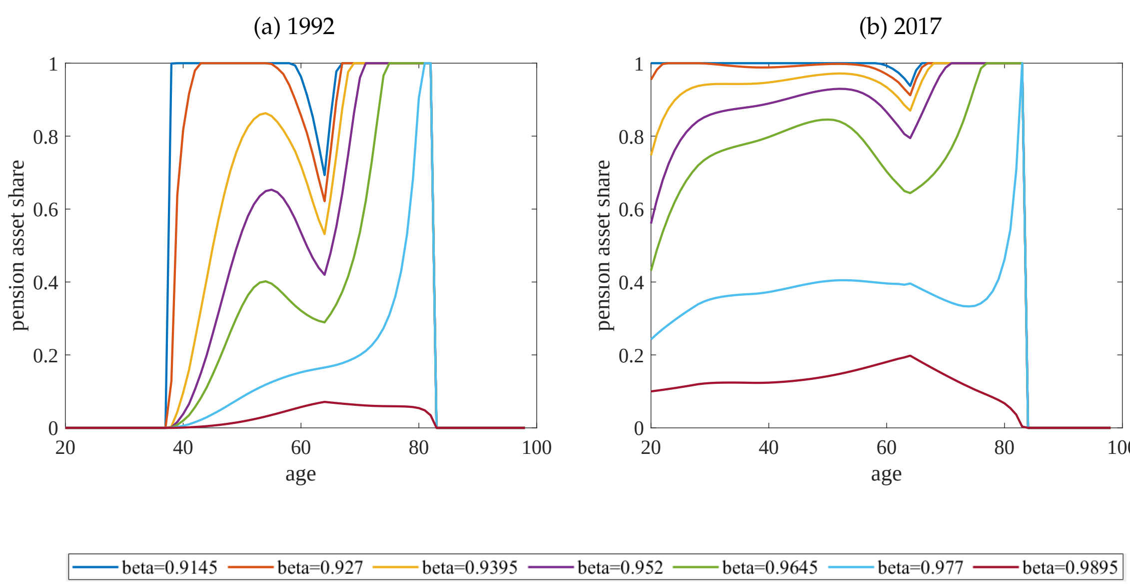
Turning to the distribution of wealth, it is important to take into account that individuals differ by age, income, and discount factor. Table 2 shows wealth inequality in 1992 and 2017. The model is calibrated to match the Gini coefficient for the 60-69 year olds in 1992 perfectly. The model also does well on the Gini coefficient for the entire population, which was not targeted. Turning to the Gini coefficients in 2017, the model predicts a decline of almost ten percentage points, close to the data decline. The remaining decrease of the Gini coefficient in the data may, e.g., be due to short-term macroeconomic fluctuations in the Danish economy, which our model does not consider.

Table 2: Gini coefficients - 1992 and 2017

	1992		**2017**	
	60-69yrs	all	60-69yrs	all
model	0.626	0.773	0.534	0.642
data	0.626	0.750	0.511	0.691

The model slightly overpredicts the decrease in the Gini coefficient for the total population. One reason could be the changing income distribution and an increase in the variance of income shocks. This has been documented for the U.S. (Heathcote et al., 2010), and also seems to be the case in Denmark: the Gini coefficient for labor income in the data increased from 0.23 in 1992 to 0.28 in 2017, whereas in the model there is a slight decline due to the increase in aggregate income.

Figure 13: Share of pension assets in total assets, by β type



Why did the pension reform lead to lower wealth inequality? The mechanism at work in the structural model captures the essence of the analytical model of Section 2: Poor and impatient agents are mandated to save (through the pension system) a larger share of their labor income than they would like to. Since they are not allowed to borrow with pensions as collateral, they cannot offset the high mandatory saving, and consequently, hit the zero voluntary saving corner. The structural model generalizes the analytical model by allowing for precautionary saving. This form of saving turns out to be small and does not play a significant quantitative role. Figure 13 shows the share of pension assets in total saving by discount factor type. Again, realizations of income shocks are set to equal their expected value.

Before comparing the profiles across β-types, be reminded that pension assets are zero after age T_t ($T_{1992} = 83, T_{2017} = 84$), the expected age of death. Pension assets are also zero in 1992 for young agents because their income is below the threshold (see eq.10). This does not mean that young agents do not pay into the pension system; it is just that a person experiencing average income shocks will not pay into the system below a certain age.

Next, consider how the discount factor affects saving behavior. Both in 1992 and 2017, the most impatient agents do not hold any significant amount of assets beyond the FF assets for most of their lives. They are at the zero savings corner, and they would (if possible) like to offset the mandatory pension savings by borrowing. This group is very impatient, wishing to consume more at young ages. Forcing them to save through the pension system means taking away young-age consumption opportunities against the preferences of low β-types.[27] A few years before retirement, this type starts saving a small amount and runs down the assets quickly upon retirement. Even though this group is hitting the borrowing constraint both in 1992 and 2017, the constraint is more binding in 2017.

In contrast, the group with the highest discount factor always chooses to save a much larger amount than their mandatory pension contributions. As they are very patient, they have a strong desire to consume when old, putting aside a large share of their labor income during working age. The most significant difference between 1992 and 2017 is observable for the groups in the middle of the β-distribution – they are pushed to the zero savings corner by the pension reform. This effect primarily concerns those with $\beta \in [0.927, 0.952]$. The different saving behavior of these groups plays a significant role in how pension reform affects wealth inequality.

Counterfactual analyses

As shown in the previous section, the pension reform massively affects the saving behavior of the low-β types and changes the distribution of wealth through this channel. But there are additional forces at work in the model: Because of aggregate income growth, all agents want to save more in absolute terms in 2017 than in 1992, which could lower inequality. The change in PAYG replacement rates should also play a role in voluntary savings behavior. To disentangle the effects

[27] Andersen and Bhattacharya (2021) discuss the welfare implications of mandating the young to save for retirement.

Table 3: Gini coefficient: counterfactual vs. full model

		60-69 yrs	all
(a)	full model	100%	100%
(b)	no trend in $\bar{\pi}_t$	43.5%	43.4%
(c)	$\pi(Y_{i,n,t})$ fix	87.7%	95.7%
(d)	no aggregate income growth	81.2%	60.4%
(e)	no change in PAYG rate	97.8%	102.2%

of these developments and highlight the crucial role played by the pension reform, we carry out counterfactual analyses in which we mute each channel one by one.

Table 3 compares changes in the Gini coefficient between 1992 and 2017 in each counterfactual scenario to the change in the base case considered above. Each number divides the percentage change in the Gini coefficient of the counterfactual case by the percentage change in the Gini coefficient in the base case. A higher ratio thus implies that this channel is less important because, in the absence of this channel, the model still produces similar results as the base case. In contrast, a low number means that when we shut down a channel, the decline in the Gini coefficient is much less pronounced, so the muted channel plays a vital role in the change in inequality.

In (b) we keep $\bar{\pi}_t$ at its value of 1992. This mutes the trend growth of contribution rates over time, i.e., it assumes the pension reform had not taken place. However, this does not necessarily imply that individual pension contribution rates stay the same as in 1992. Because of changes in aggregate income, the income-dependent element of the pension contributions in $\pi(Y_{i,n,t})$ still increases, which means many agents will pay higher contributions. This additional pension channel is taken into account in scenarios (c), where we make the pension contributions income-independent, but hold fixed the growth in $\bar{\pi}_t$. In (d) we take out the growth in aggregate income. In (e) we fix the PAYG replacement rate at its 1992 value.

Shutting down the pension reform leads to much lower inequality decreases than the baseline model. More than half the decline in the Gini coefficient can be attributed directly to the pension reform. In contrast, when muting the income-dependent part of FF pensions,the reduction in the Gini coefficient is almost the same as in the full model, so this element of the pension system by itself has barely any effect on inequality.

Aggregate income growth also makes the wealth distribution more equal, but its effect is weaker than that of the pension reform. In particular, for the group of 60 to 69-year-olds, inequality still increases considerably in the absence of aggregate income growth. Fixing PAYG replacement rates barely leads to any reduction in inequality. On the one hand, agents have less desire to save because the pension system is more generous. On the other hand, the means-tested element is muted.

Consumption inequality

The preceding analysis focuses on the effects of a fully funded, mandated pension system on wealth accumulation and distribution. However, as noted before, wealth does not map directly into consumption (nor wealth inequality into consumption inequality) in a PAYG regime because part of the cash-flow comes from government transfers that have no correspondent in accummulated assets. Table 4 reports changes in the disparity in consumption between 1992 and 2027 at retirement and for all retirees. For the former group, consumption inequality increases while decreasing for the latter group. The latter finding reflects that the mandated pension system aims to “protect” old-age consumption, particularly by increasing consumption possibilities for those who otherwise would have low consumption due to insufficient savings.

Table 4: Gini coefficients for consumption

	at retirement (age 65)	ages 65-100
1992	0.080	0.196
2017	0.089	0.184

From a distributional perspective, it is often argued that a FF scheme induces more inequality than a PAYG scheme, since the latter by design has redistributive features, and the former projects the distribution of earned income into a distribution of pension payouts that are contingent on individuals’ labor market history. The present analysis argues the interpretation may be incomplete since transition to FF schemes may render the wealth distribution more equal. Moreover, as the comparison of Gini coefficients for consumption makes clear, depending on the age group considered, consumption inequality may decrease as well. This by no means suggest such such a transition comes at a zero cost. There may be groups in the society, presumably the young, for which such a transition may imply higher consumption inequality. In the case of our model, for the working age population, the consumption inequality actually slightly decreases between 1992 and 2017, as it does for the whole population. It is beyond this paper to analyze the welfare rationale for the changes in the Danish pension system and relate it to the theoretical literature pointing to various reasons why individuals “undersave” and thus the need for intervention in pension savings. Andersen and Bhattacharya (2021), for example, consider the welfare case for mandated pensions in a model with present-biased preferences. Welfare implications of the transition in the pension system are an interesting question for future research to explore, particularly which groups have benefited the most from the changes in the pension system.

5.4 Model extension: bequests

As shown in Figures 4 and 7, individuals in Denmark tend to die holding a considerable amount of assets. This suggests a bequest motive. We present here a model extension including a “warm glow" of leaving a bequest in the utility function (see e.g., De Nardi, 2004). Then, expected lifetime utility becomes

$$\text{(11)} \quad \Omega_{0,i} = \mathbb{E}\left[\sum_{n=N^b}^{N^d} \beta_i^{n-N^b} \tilde{\delta}_{n-1} \left(\delta_n \frac{C_{i,n,t}^{1-\vartheta}}{1-\vartheta} + (1-\delta_n)\mu^{\vartheta} \frac{X_{i,n,t}^{1-\vartheta}}{1-\vartheta}\right)\right],$$

$\tilde{\delta}_{n-1}$ is the cumulative survival probability between periods N^d and $n-1$, $X_{i,n,t}$ is cash-on-hand, the right-hand side in eq.s (6) and (7), and μ measures the strength of the bequest motive. In the absence of explicit intergenerational links, we assume bequests – both intentional and accidental – get redistributed equally among all living households. Inheritances are unexpected.

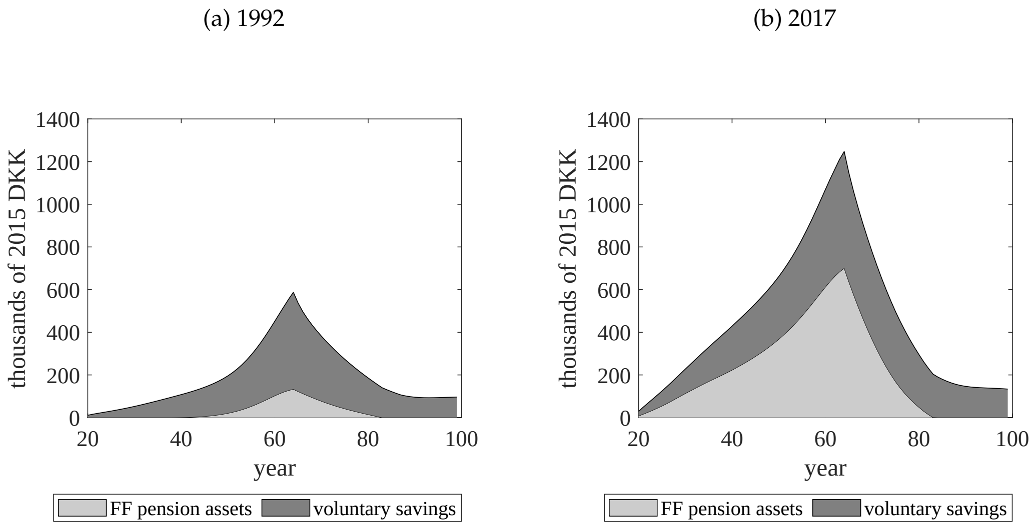
Introducing a bequest motive requires re-calibrating b and d as agents with a given discount factor now wish to hold a larger amount of assets. The parameters b, d and μ are calibrated jointly to match peak life-cycle asset holdings and the Gini coefficient in 1992 as before, as well as assets held at the end of life in 1992. We target a value slightly lower than the value presented in Figure 7 (a) since this number is driven by wealthy households. Richer individuals have a higher life expectancy than poorer individuals in the real world, whereas in the model, all agents have the same survival probability. Therefore, we target the value of the median household instead, which is around 100,000 DKK (Figure 7 (b)). The resulting values are $\mu = 1.05$, $b = 0.919$, $d = 0.0217$, which imply $\beta \in [0.8539, 0.9841]$.

Figure 14 shows the cross-sectional asset holdings, whereas Table 5 presents the Gini coefficients. The values are not very different from the model without a bequest motive. The pension reform still leads to a substantial reduction in wealth inequality among the 60-69 year-olds as well as in the general population. The model with bequests matches the Gini coefficient of the entire population slightly better, owing to the fact that now the old choose to hold a considerable amount of wealth.

Table 5: Gini coefficients - bequests

	1992		**2017**	
	60-69yrs	all	60-69yrs	all
bequest motive	0.626	0.740	0.541	0.624
data	0.626	0.750	0.511	0.691

Figure 14: Cross-sectional asset holdings, with bequest motive



6 Conclusion

The design of the pension system has been shown to have a quantitatively significant effect on wealth accumulation and its distribution. Exploiting Danish data and a transition from a largely PAYG-pension system to a hybrid with a large weight on FF pensions, we document how this transition, in accordance with theory, implies not only increased wealth accumulation but also a more equal wealth distribution.

It should be emphasized that the preceding analysis is positive because we do not attach value judgments to whether the post-reform functional distribution of income (ownership of capital) is desirable in any sense. This is not to suggest our topic is devoid of normative content. The rising inequality concerns those who correlate wealth with augmented political power and fret about the impact on democratic institutions, more so if the wealth distribution is widely perceived to be unfair.[28] Others see wealth inequality as an important – again, possibly unfair – channel by which economic advantage, opportunities, and affluence are transmitted across generations.[29] To

[28]The worry surrounding worsening ownership of capital is for some its effect on politics. For others, there is jubilance over the emergence of a 'public shareholder class' and how 'even blue-collar workers now often have sufficient personal savings to justify investment in equity securities [through pension funds]...no longer do labor and capital constitute clearly distinct interest groups in society' (Ireland, 2005). Drucker (1976) was among the first to notice the potential for pension programs to reshape the ownership of capital. He commented on how in the late 60s and early 70s, American financial institutions, notably retirement funds were diversifying away from mostly "safe" investments, such as treasuries and bonds, and investing in public company stock. To Drucker, this meant the future "owners" of American companies would not be old-style capitalists but workers in the public and private sector (through their fiduciaries, trustees and managers of pension funds).

[29]As Kopczuk (2015) frames its "the extent to which the well-off are going to rely on work versus rely on the returns to their wealth in the future is clearly important for assessing the extent to which a society will view itself as in some way a meritocracy."

neoclassical economists like Kuznets and Solow, the issue of who owns capital is not important: along a balanced growth path, both owners of capital and labor would benefit from growth to the same degree (Piketty, 2014). Data from the 1990s onward are at odds with this "balanced growth" view: if anything, labor's share of output is falling which raises the imperative to know who owns capital.[30] After all, as the percentage of value-added in output going to owners of capital is rising, FF schemes, by allowing workers to become capital owners, may help pass on some of the benefits to the workers. Our work demonstrates how pension reform has democratized the ownership of capital thereby helping to bring down wealth inequality in Denmark. Whatever the underlying motivation, governments are concerned about the rise in wealth inequality, and intervene in ways to curb it. Many such attempts suggest the direction of the taxation of wealth. We do not enter that discussion. We merely offer the view that pension reform, already underway in many high wealth-inequality nations, may reduce wealth inequality by itself.

[30]Between 1991 and 2014, the labor share declined in 29 of the largest 50 economies overturning one of the enduring Kaldor stylized facts (Karabarbounis and Neiman, 2014). There is mounting evidence that the labor income share has decreased (increased) for low (high)-skilled workers (Dao et al., 2017).

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APPENDIX

A Data appendix

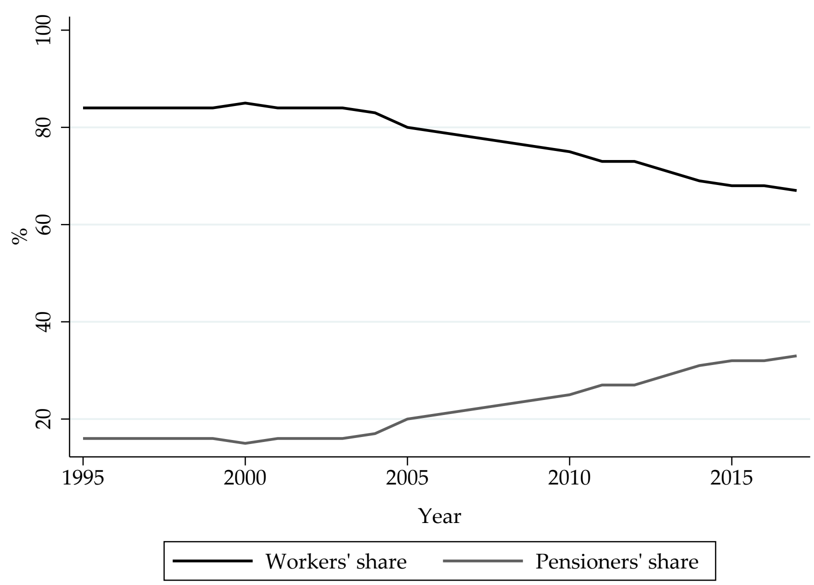
A.1 Datasets and variables used in the analysis

Wealth data

While the income registry includes information on end-of-year assets and liabilities of Danish individuals prior to 2014, following third-party reporting, it has certain shortcomings. Most importantly, it does not provide information on pension wealth.

We construct imputed pension wealth for 1986-2013 slightly modifying the approach used in Jakobsen et al. (2020). Jakobsen et al. (2020) assume that aggregate pension assets are distributed among pensioners and wage earners according to shares that are fixed over time and correspond to the distribution of pension assets in 2012. They later attribute the pension wealth to each individual according to their share in income. We modify this imputation in two ways. First, we do not assume that the share of pension assets belonging to pensioners is fixed in 30 years. The nature of the Danish reform leads to an increase in pension assets held by pensioners over time, from 16 % in 1995 to 33% in 2017 (data from Denmarks Statistik), see Figure A.1. We extrapolate this trend back to 1992, before which the shares are assumed to be fixed.

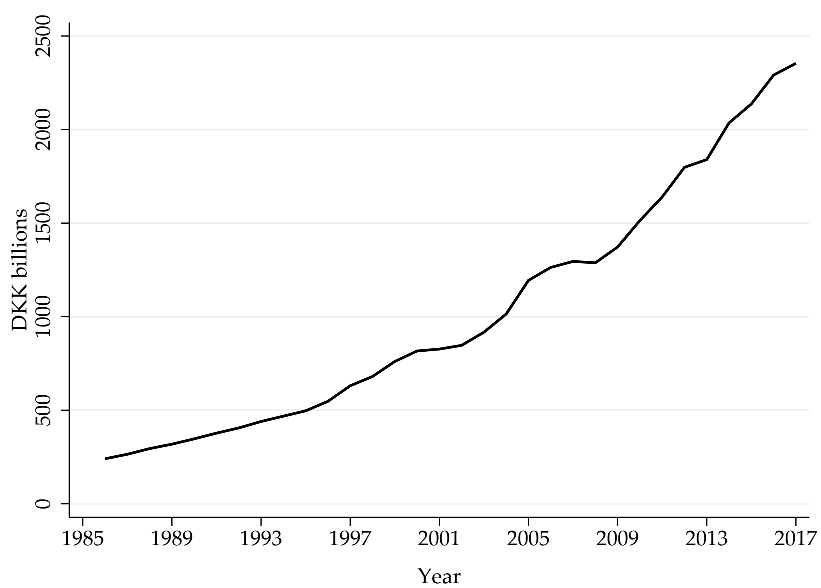
Figure A.1: Share of pension assets belonging to workers and pensioners in Denmark, 1995-2017



Data: Provided by Statistics Denmark.

Second, if we assume that the pension assets are distributed in the population based on the income distribution as in Jakobsen et al. (2020), the resulting distribution of pension assets is far too equal, as it follows the relatively more equal income distribution (the Gini coefficient of net income was 0.23 in 1992). As a consequence, in order to calculate the share of pension assets going to each individual, we take an average based on their income share and pension contribution share. This imputation results in a slightly more unequal distribution of pension assets in the population and matches the data better than the one based only on the income distribution. We impute pension assets at the individual level. Throughout the years, we assume that 4% of pension assets belong to self-employed, and the share belonging to pensioners is increasing over time according to the data from Figure A.1. Each year, the aggregate pension wealth after taxes (provided to us by Denmark Statistics, see Figure A.2) is distributed to workers, pensioners and self-employed proportionately to their incomes from a given type of activity (we only take into account positive incomes, setting the negative accounts to 0 in order to avoid attributing negative pension wealth to households) and proportionately to their share in pension contributions in a given year (we take the average of two). We then add the imputed pension wealth to the reported net wealth for these years and aggregate individuals into households to compute a measure of net wealth at the equal-split household level. To provide comparability of wealth measures across different time periods, we treat net wealth without consumer durables with pension assets net of taxes as our benchmark wealth measure. For a discussion on different wealth measures used in the literature, see Appendix A.2.

Figure A.2: Aggregate pension assets after taxes in Denmark, 1986-2017



Data: Provided by Statistics Denmark.

Note, the figures reported in the main text merge imputed wealth measures with the new wealth data from 2014.[31] The differences bewteen the lines may be attributed to different definitions of similar variables in the 'old' and 'new' datasets and to our pension imputation.

The list below specifies which variables are used for the construction of our socio-economic measures:

- Before 2014: We use the IND database merged with the net pension assets provided to us by Denmark Statistics to impute wealth. For net wealth without consumer durables, we use the variable FORM before and including 1997, and the variable FORMREST_NY05 after 1997. While these variables have a slightly different definition, in the overlap year 1997, they have a correlation of 99%. Pension assets are imputed, as described above. Pension assets are redistributed proportionately to workers wage incomes (LOENMV in 1986, and LOENMV_13 afterwards), pensioners pension incomes (QPENSIALT-FOLKEFORTID_13-QEFTLON before and in 2013, PRIVAT_PENSION_13 after 2013), self-employed incomes (NETOVSKUD_13; for 1986, constructed as ERHVERDSINDK_GL-LOENMV) and pension contribution rates being the sum of QARBPEN and QPRIPEN. Note that pension contributions are available only from 1995. Before that date, we assume the individual contribution share to be equal to their share in 1995.

- After 2014: We use the FORMGELD database to construct a measure of net wealth without durables with pension assets net of taxes. Note that we do not take into account the asset category defined as D in the wealth database of Statistics Denmark, which refers to deposits under the business scheme. The share of assets under this category is very low for most of the households. As such, when we talk about the share of particular asset category in total assets, we mean total assets excluding deposits under the business scheme.

Income data

Our main source of income data is the IND database provided by Statistics Denmark. We construct our net income measure from this database:

- Before 1991: Net income = DISPON_13 - FORMUEIND_BRUTTO + KAPINDKP - LEJEV_EGEN_BOLIG - QTILPENS. We take the net disposable income that includes certain types of income that we want to exclude and deduct from it interest income and gains and losses on securities, add the taxes paid on capital income back, deduct the rental value of own accommodation and ATP pensions. Data on other types of private pensions is only available from 1991 and hence, we use a slightly enhanced definition starting in 1991.

[31]We can compare the numbers for years 2014-2017, when we have both the new wealth data and the imputation based on the old dataset. We present corresponding graphs in Appendix A.3, where the imputed lines correspond to our imputed data, while the combined lines are those shown in the main text, created from combining the imputed data until 2013 with the new wealth data from 2014 onward.

- After 1991: Net income = DISPON_13 - FORMUEIND_BRUTTO + KAPINDKP - LEJEV_EGEN_BOLIG - PRIVAT_PENSION_13*0.6. As before 1991, we disregard capital income and taxes paid on it, remove the rental value of own accomodation and net value of private pensions (including ATP).

Other data

- CPI - we use the Consumer Price Index indexed at 100 in 2015 (PRISD112).
- self-employed dummy: we use the variables BESKTST and BESKST13 to indentify self-employed according to the description in IND (the dummy takes value of 1 if one of these variables takes the value of 1).
- socio-economic characteristics: we use the BEF database to obtain individual's household identifier, civil status, position in the household, sex and age. From the FAM database, we obtain the number of children living in the household. This includes children below the age of 25.

A.1.1 Household unit

We consider two categories of households: All and Benchmark, which we later use for calibration of the model and present in the main paper. The category All includes all households in our dataset and we present wealth measures for this category in Appendix A.3 to be comparable with other papers in the wealth inequality literature, including the recent paper by Jakobsen et al. (2020). In the category Benchmark, we exclude households in which more than two adult members live given that it is difficult to appropriately attribute wealth in such cases (think of intergenerational households). Moreover, we exclude all households which at any point in time had any self-employed member due to the fact that self-employed are not required to pay into the mandatory pension scheme. In addition, self-employed usually face a distinct income process and accumulate wealth differently from the rest of population (see e.g. evidence by De Nardi and Fella, 2017 for the U.S.). After these exclusions, our Benchmark category includes approx. 80 % of all households included in All (the exact share depends on the year).

A.2 Wealth definitions

In our study, we focus on net wealth without consumer durables including pension assets net of taxes as the benchmark wealth definition. We present the data at the normalized household level, dividing household wealth equally between adult members of the household (equal-split). Our choice of wealth measure is motivated by the vast literature on wealth inequality.

Roine and Waldenström (2015) discuss different concepts of wealth and the wealth owner, noting that when wealth tax-based data are used, such as in the Nordic countries, the most common unit of reference is a household.[32] The most common measure of wealth is marketable wealth, consisting of all nonhuman real and financial assets minus debt. Assets may include pension and social security wealth (historically often excluded, but arguably important, since interacting with incentives to save privately), consumer durables (often excluded due to issues with historical data) and foreign wealth holdings (often unavailable due to data limitations and tax evasion). Roine and Waldenström (2009) present their wealth inequality estimates by plotting the share of wealth owned by certain fractions of population (for Denmark, percentiles 90-95, 95-99, 99-100, and the top 0.1 percentile). The marketable wealth is used as a measure of wealth. Similarly, Davies et al. (2017) use marketable wealth to conduct cross-country comparisons, but the study includes private pension wealth, not taking into account merely the public pensions. For the U.S., analysis at the household level is standard, and the wealth measure used is net worth including the value of most defined contribution plans, but excluding the implied values of defined benefit plans and social security (Cagetti and De Nardi, 2008). This approach is also followed more recently by Saez and Zucman (2016) and Zucman (2019) that include all pension wealth in their definition of net wealth, with the exception of social security and unfunded defined benefit pensions. Due to the data availability, Saez and Zucman (2016) and Zucman (2019) do not consider consumer durables. In order to be comparable with these papers, we use a similar wealth measure.

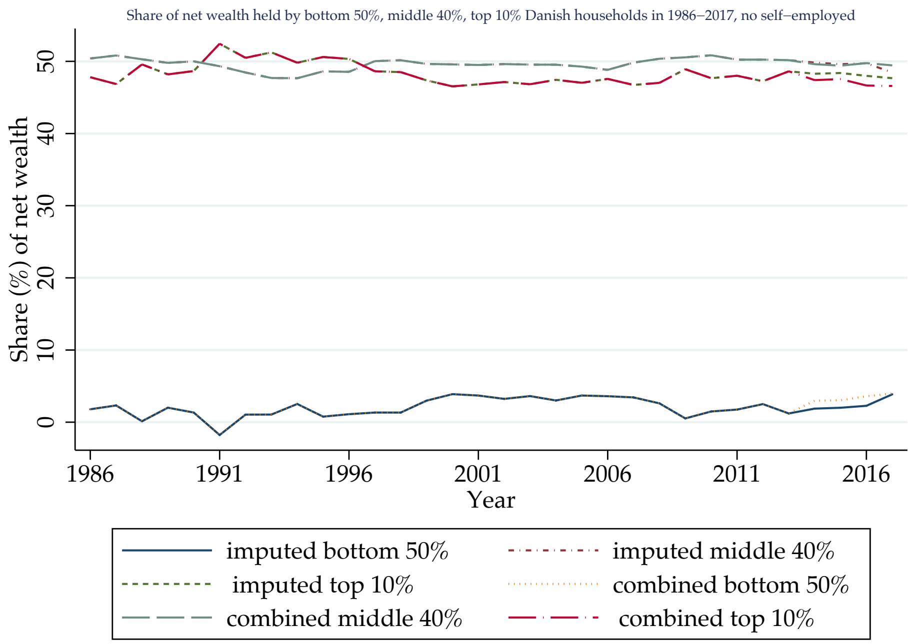
Notably, Jakobsen et al. (2020) provide the most recent analysis of Danish household wealth with the use of administrative wealth registry provided by the Danish Statistical Agency since 1980. The paper focuses on years 1980-2012. Jakobsen et al. (2020) use a measure of wealth including all non-financial and financial assets belonging to Danish residents minus debt. Similarly to the international literature discussed above, their analysis does not take into account the present value of future government transfers, consumer durables and valuables. In order to take into account private pension assets (not needed in the main part of their analysis), they impute them from aggregate data, as we do in our paper (see the discussion in A.1).

[32]Using individual as opposed to household as the wealth holding unit may give raise to a more unequal wealth distribution, even though this has not been shown not to be the case for Sweden, see Roine and Waldenström (2009) and for the U.S., Kopczuk and Saez (2004).

A.3 Additional graphs and tables

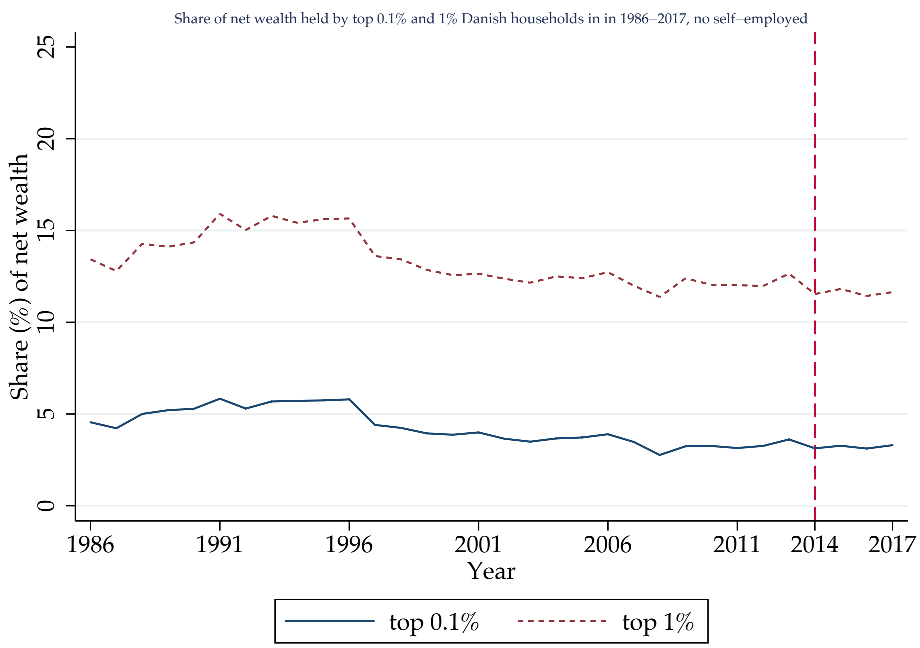
A.3.1 Additional graphs for the main sample excluding self-employed households

Figure A.3: Share of net wealth held by bottom 50%, middle 40% and top 10% Danish households in 1986-2017 - comparison of imputed and combined measures for the main sample, excluding self-employed



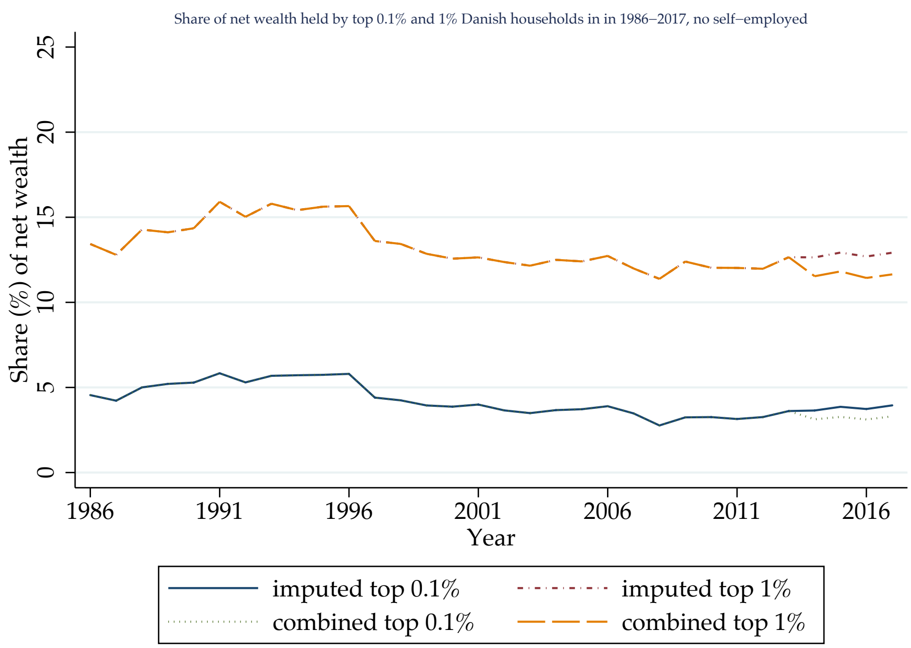
Note: Wealth corresponds to net wealth without consumer durables including pension assets net of taxes and is presented at the normalized (equal-split) household level. We remove self-employed households and households with more than two adult members in the sample. The imputed series shows our imputation for years 1986-2017. The combined series is constituted of the imputed series in years 1986-2013 and the new wealth data in years 2014-2017 and corresponds to the series presented in the main paper.

Figure A.4: Share of net wealth held by top 1% and 0.1% Danish households in 1986-2017



Note: Wealth corresponds to net wealth without consumer durables including pension assets net of taxes and is presented at the normalized (equal-split) household level. We remove self-employed households and households with more than two adult members in the sample.

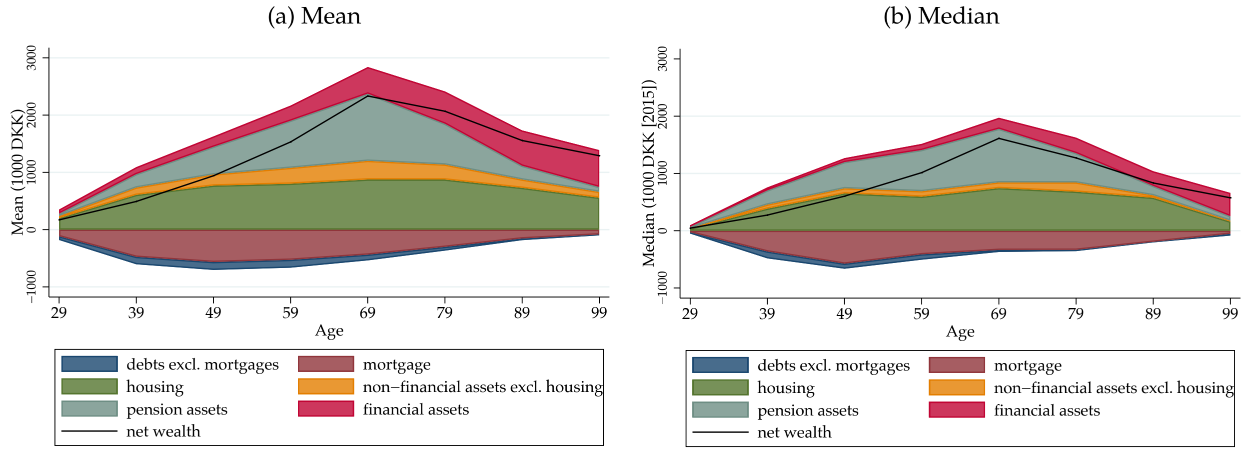
Figure A.5: Share of net wealth held by top 1% and 0.1% Danish households in 1986-2017



Note: Wealth corresponds to net wealth without consumer durables including pension assets net of taxes and is presented at the normalized (equal-split) household level. We remove self-employed households and households with more than two adult members in the sample. The imputed series shows our imputation for years 1986-2017. The combined series is constituted of the imputed series in years 1986-2013 and the new wealth data in years 2014-2017 and corresponds to the series presented in the main paper.

A.3.2 Additional graphs for the extended sample including all households

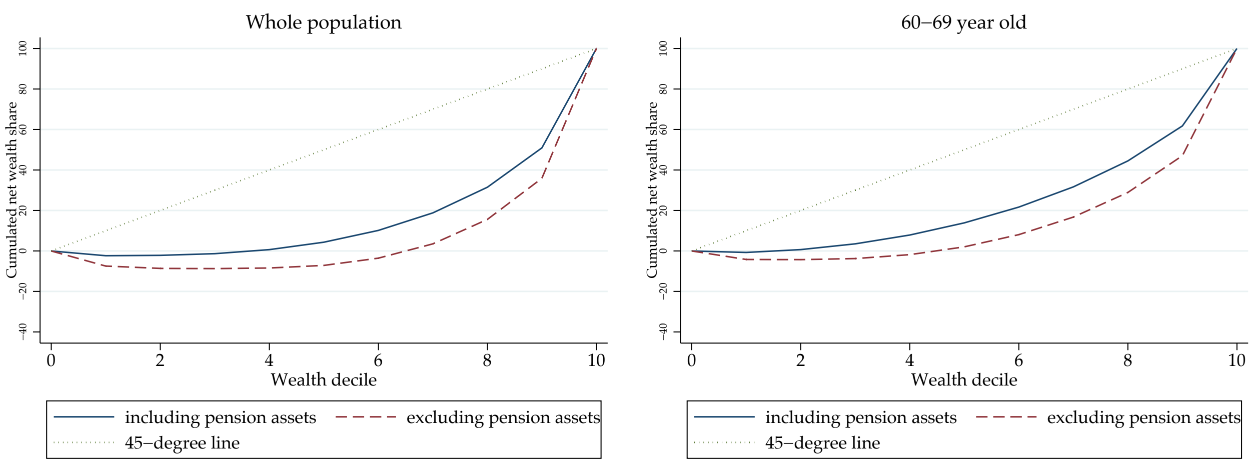
Figure A.6: Cross-sectional wealth holdings of Danish households, including self-employed, 2017



Note: Wealth is presented at the normalized (equal-split) household level. In this graph, we consider all households present in our original dataset.

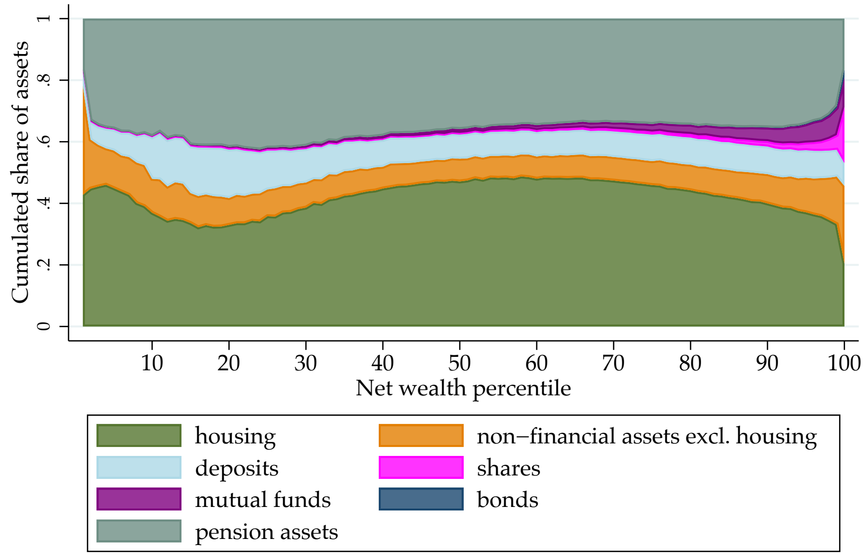
Figure A.7: Lorenz curve for net wealth, sample including self-employed, 2017

Cumulated share of net wealth held by different wealth deciles of Danish households in 2017



Note: Wealth corresponds to net wealth without consumer durables including pension assets net of taxes and is presented at the normalized (equal-split) household level. In this graph, we consider all households present in our original dataset.

Figure A.8: Cumulated mean share of different wealth categories held by Danish households, including self-employed, in 2017



Note: Wealth corresponds to net wealth without consumer durables including pension assets net of taxes and is presented at the normalized (equal-split) household level. In this graph, we consider all households present in our original dataset.

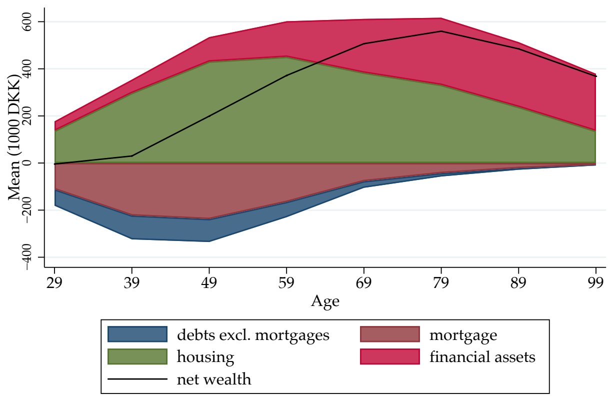
Table A1: Gini coefficients for the whole population, including self-employed households

	All households			**Head 60-69 years old**		
	Pension assets Excluded	Pension assets Included	% Δ	Pension assets Excluded	Pension assets Included	% Δ
1992	1.17	0.86	-26%	0.69	0.66	-5%
2017	0.91	0.7	-23%	0.75	0.55	-27%
% Δ 1992-2017	-22%	-19%		+8%	-17%	

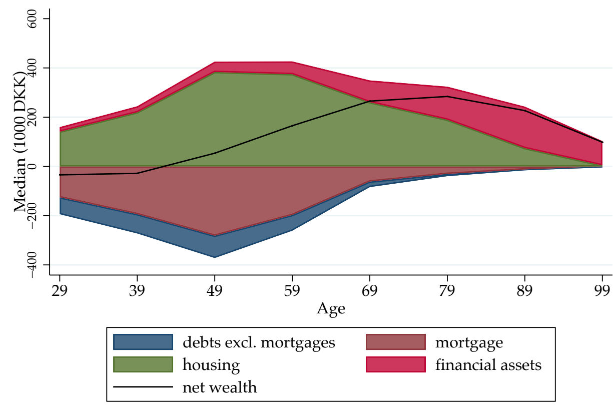
Note: Wealth corresponds to net wealth without consumer durables including pension assets net of taxes and is presented at the normalized (equal-split) household level. In this table, we consider all households present in our original dataset.

Figure A.9: Cross-sectional wealth holdings of Danish households including self-employed, 1992

(a) Mean

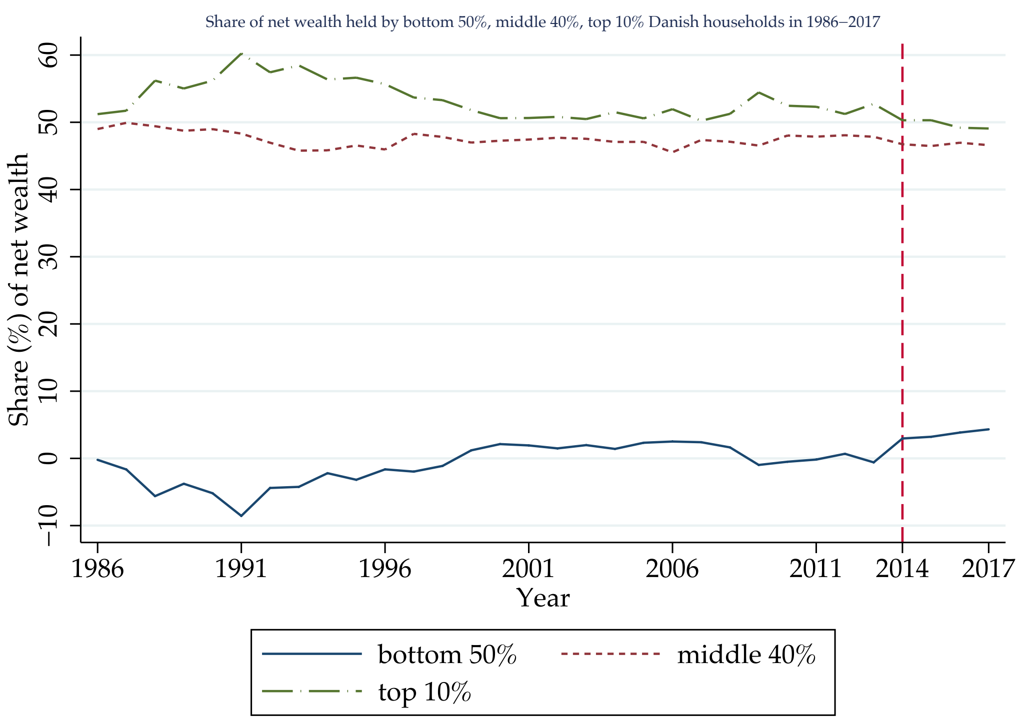


(b) Median



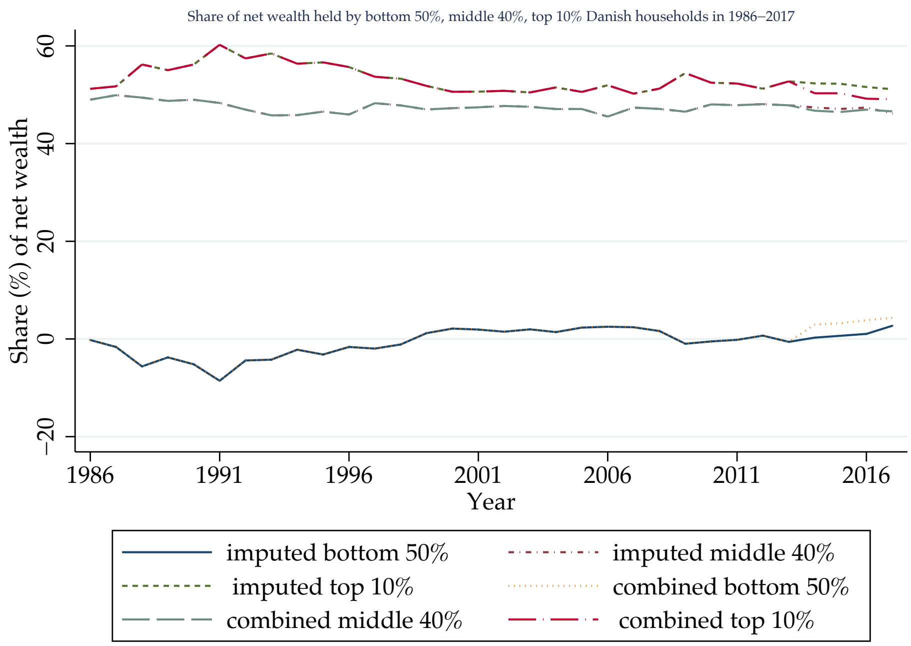
Note: Wealth is presented at the normalized (equal-split) household level. In this graph, we consider all households present in our original dataset.

Figure A.10: Share of net wealth held by Danish households in 1986-2017, including self-employed



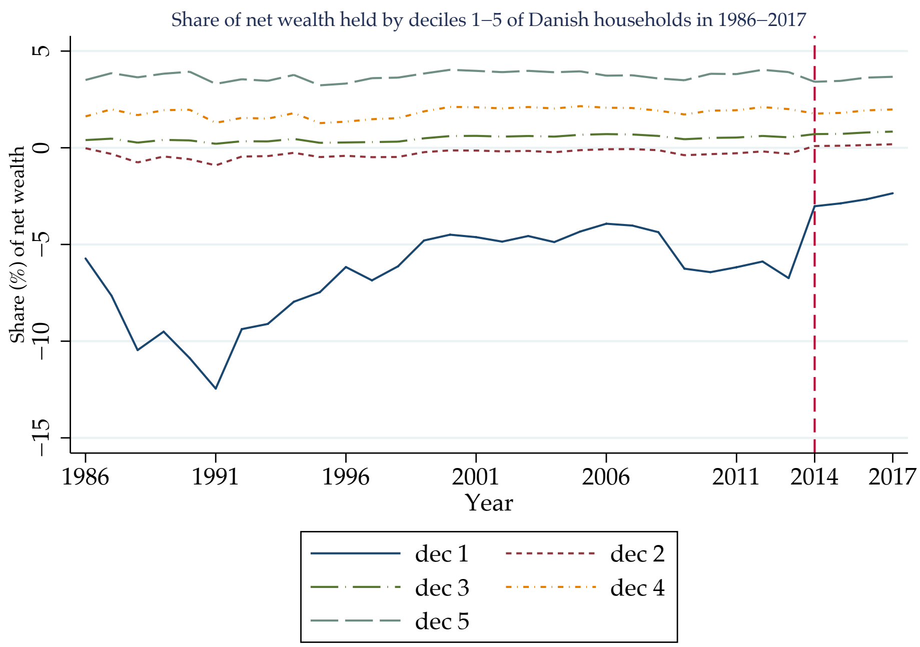
Note: Wealth corresponds to net wealth without consumer durables including pension assets net of taxes and is presented at the normalized (equal-split) household level. In this graph, we consider all households present in our original dataset.

Figure A.11: Share of net wealth held by bottom 50%, middle 40% and top 10% Danish households in 1986-2017 - comparison of imputed and combined measures for all households, including self-employed



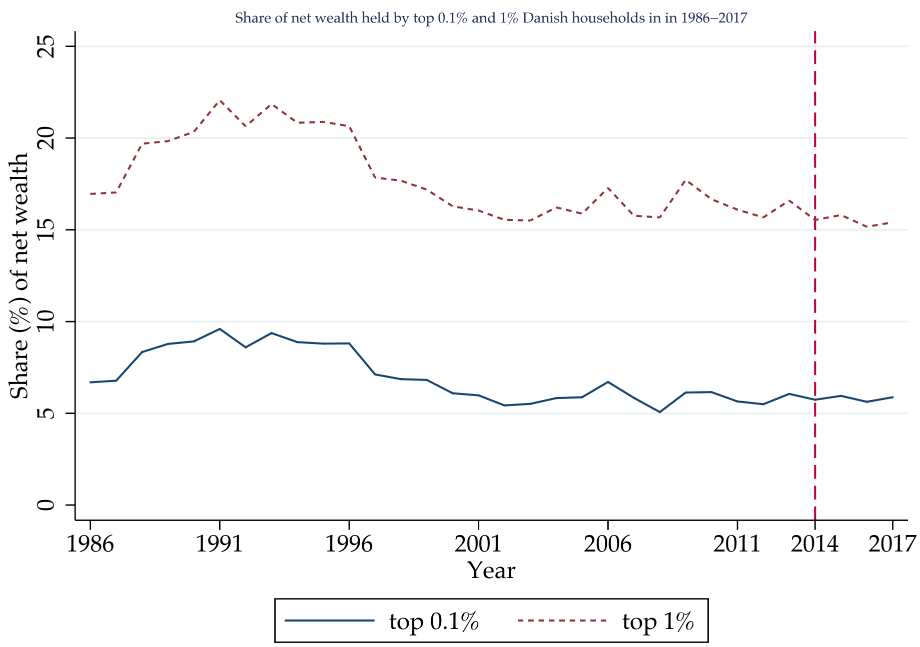
Note: Wealth corresponds to net wealth without consumer durables including pension assets net of taxes and is presented at the normalized (equal-split) household level. In this graph, we consider all households present in our original dataset. The imputed series shows our imputation for years 1986-2017. The combined series is constituted of the imputed series in years 1986-2013 and the new wealth data in years 2014-2017.

Figure A.12: Share of net wealth held by the lowest wealth deciles of Danish households in 1986-2017 for all households, including self-employed



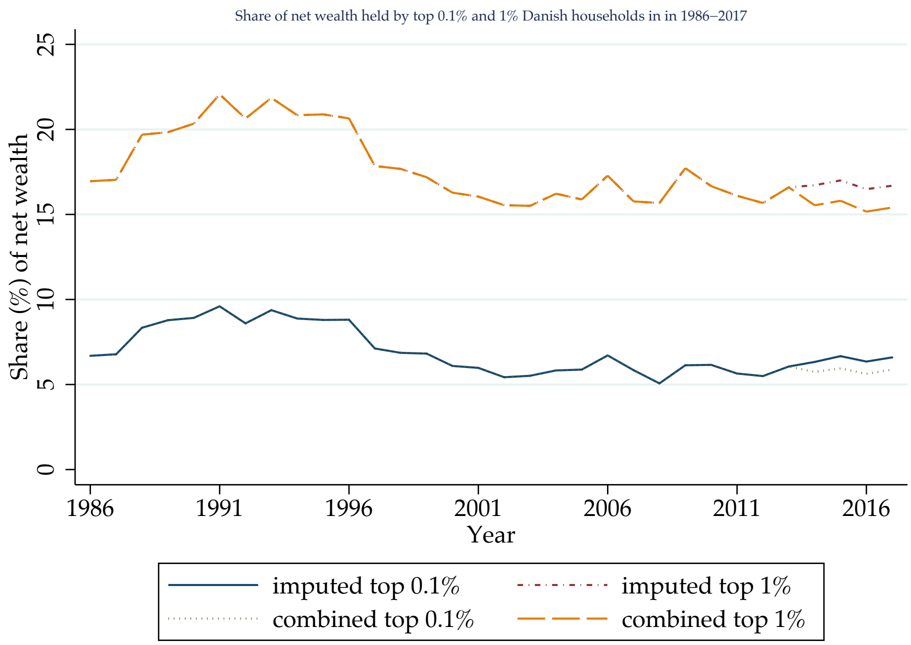
Note: Wealth corresponds to net wealth without consumer durables including pension assets net of taxes and is presented at the normalized (equal-split) household level. In this graph, we consider all households. present in our original dataset.

Figure A.13: Share of net wealth held by top 1% and 0.1% Danish households in 1986-2017 for all households, including self-employed



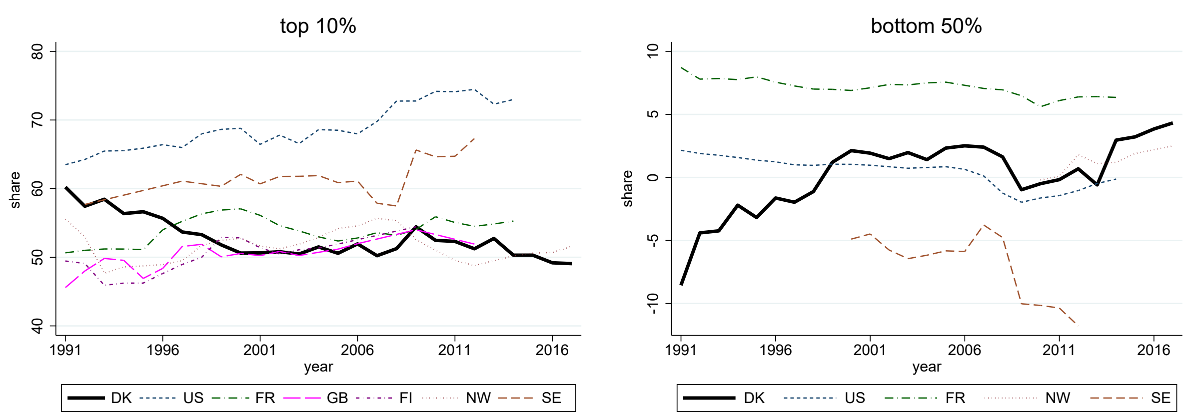
Note: Wealth corresponds to net wealth without consumer durables including pension assets net of taxes and is presented at the normalized (equal-split) household level. In this graph, we consider all households present in our original dataset.

Figure A.14: Share of net wealth held by top 1% and 0.1% Danish households in 1986-2017 - comparison of imputed and combined measures for all households, including self-employed



Note: Wealth corresponds to net wealth without consumer durables including pension assets net of taxes and is presented at the normalized (equal-split) household level. In this graph, we consider all households present in our original dataset. The imputed series shows our imputation for years 1986-2017. The combined series is constituted of the imputed series in years 1986-2013 and the new wealth data in years 2014-2017.

Figure A.15: Wealth concentration - international comparison



Note: DK stands for Denmark, US: United States, FR: France, GB: Great Britain, FI: Finland, NW: Norway, SE: Sweden. Source: World Inequality Database, Roine and Waldenström (2009) (Swedish data up to 2009), Lundberg and Waldenström (2018) (Swedish data from 2000 on) and own calculations based on Danish (DK) registry data (all households, see Figure A.11).

A.4 Estimation of labor income process

We use data from 1987-2017.[33] Details on the data sources and the definition of variables used in the estimation are provided in Appendix A.1. For the purpose of estimation, we exclude households with self-employed members. This group does not have mandatory contributions to labor market pensions, and additionally faces an income process that is systematically different from employees. We also drop all households for which our broad measure of net income is 0. We focus on households where the head is between 20 and 65 years old and drop households where the head is not in the labor force. This leaves us for 41 mio. observations over 30 years that we use in the estimation. The head is defined as the man in mixed-sex couples. In case of the same-sex couples, we kept the original definition of household head available in Statistics Denmark data.

We follow Cocco et al. (2005) and use a broad measure of labor income, including labor income plus benefits (unemployment, parental etc.), other transfers, as well as social security. We estimate

$$\ln(Y_{i,n,t}) = \alpha_n + \gamma_i + X'_{i,n,t}\beta + \varepsilon_{i,n,t}$$

where $Y_{i,n,t}$ is the labor and transfer income of the household head and spouse divided by two, where applicable (we use the equal-split measure to be consistent with the wealth measure). α_n is an age fixed effect, the main outcome variable of interest. Age is the age of the household head γ_i is a household fixed effect. $X_{i,n,t}$ is a vector that contains other characteristics of the household. We take into account whether the household is a couple (partnership or married), the number of children living in the household, the home ownership status of the household, the sex of the household head.

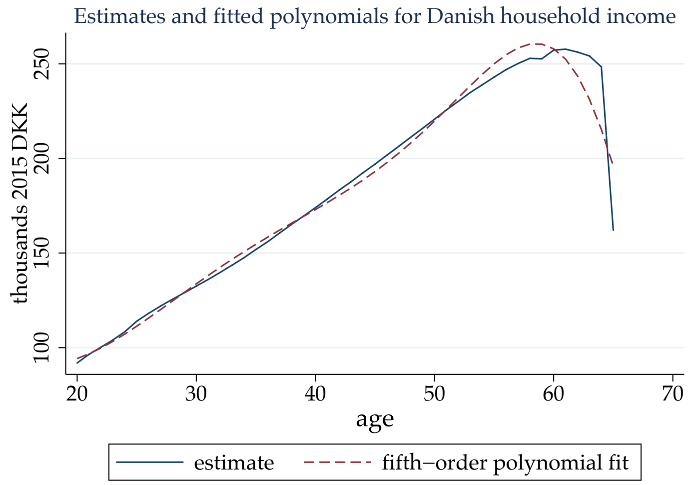
The main outcome variable of interest is the age fixed effect (G_n), where we fit a fifth-order polynomial to the estimated age fixed effects of working-age individuals, assuming that individuals retire at age 65. The fitted polynomial is presented in main text eq.(9). Figure A.16 shows both the estimate and the polynomial.

In the simulation, we use a trend obtained from HP-filtering with smoothing parameter 1600.

To estimate σ_ϵ^2 and σ_η^2, we follow the variance decomposition of Carroll and Samwick (1997). We calculate for each household and year the first difference in the deviation of the actual from the predicted labor income. The variance of this term is a combination of σ_ϵ^2 and σ_η^2, which can be estimated.

[33] We could theoretically estimate the income process on the pre-reform data, i.e. before 1992. However, this would leave us with only six years of data, which would make the estimates of variances of shocks unreliable.

Figure A.16: Labor income process, fifth-order polynomial



B Model appendix

B.1 Solution algorithm

Our model does not have an analytical solution and therefore needs to be solved numerically. The solution to the model proceeds by backward induction starting from the last period of life (age 100).

We use the endogenous gridpoint method (EGM) introduced by Carroll (2006). This implies computing policy rules on an exogenous grid for end-of-period wealth. We find optimal consumption at each age by iterating backwards on the Euler equation, eq.(8). Then, we can derive an endogenous grid for cash-on-hand as exogenous end-of-period assets plus consumption at each age. We apply linear interpolation to infer the points in between the gripdoints.

We solve the model for a discrete set of 50 admissible values for the exogenous end-of-period wealth on an exponential grid. The continuous distribution for shocks to labor income are approximated using Gaussian quadrature method following Tauchen (1986).

B.2 Steady state results

Figure B.1 displays labor and pension income in the pre-reform and in the post-reform steady state. A few things stand out: first, labor income increases quite substantially between the two steady states at all points in the life-cycle, owing to aggregate income growth. Second, the graph shows the effect of the pension reform: PAYG income is a much lower share of retirement income in the initial steady state than in the new steady state.

The pension reform together with the increase in aggregate income have a big impact on life-cycle savings. Figure B.2 shows mandatory and voluntary savings in the two steady states. The peak of total life-cycle asset holdings increases from 637 to 2,839 (thousand 2015 DKK). Pension

Figure B.1: Life-cycle labor and retirement income

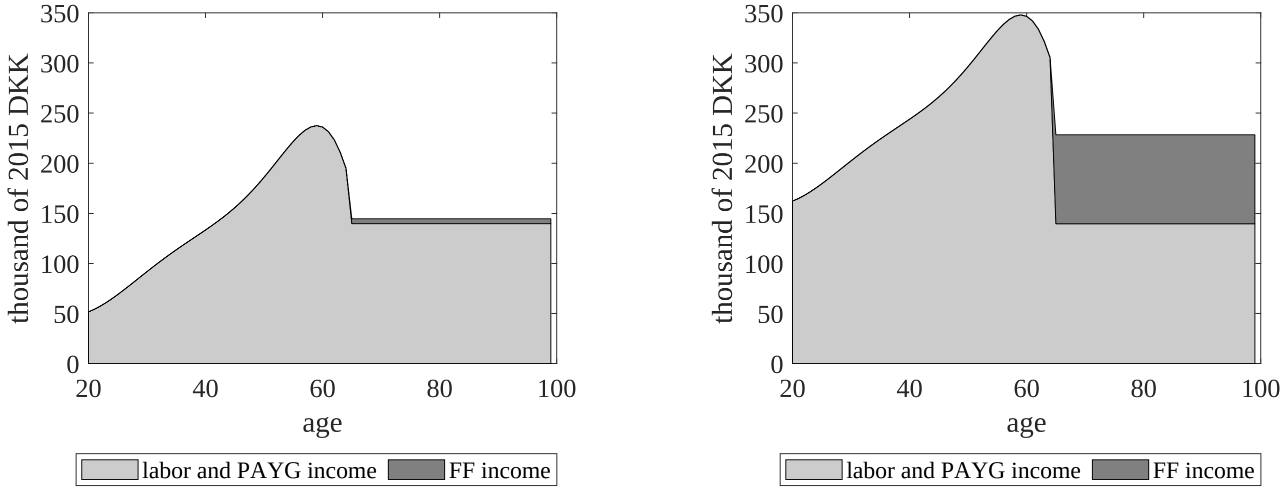
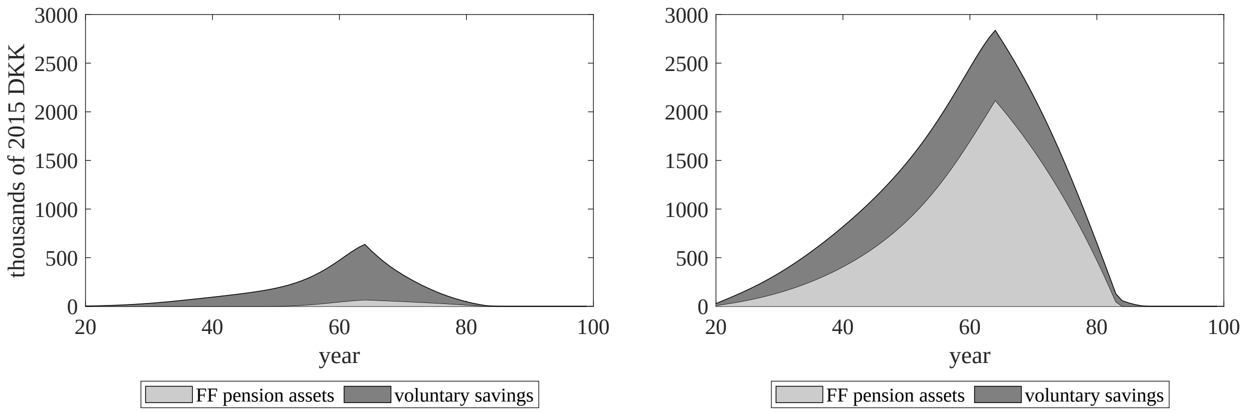


Figure B.2: Steady state asset holdings

(a) Steady state 1 (b) Steady state 2



assets make for 9% of total life-cycle savings in steady state 1, but 66% of life-cycle savings in steady state 2. This underlines how massive the impact of the pension reform is on savings when taking the long-run perspective.

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