Technical Appendix For Online Publication
“Involuntary Unemployment and the Business Cycle”

by

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A. Workers and Household

The economy consists of a continuum of households. In turn, each household consists of a continuum of workers. Workers have no access to credit or insurance markets other than through their arrangements with the household. In part, we view the household construct as a stand-in for the market and non-market arrangements that actual workers use to insure against idiosyncratic labor market experiences. In part, we are following Andolfatto (1996) and Merz (1995), in using the household construct as a technical device to prevent the appearance of difficult-to-model wealth dispersion among workers. Households have sufficiently many members, i.e. workers, that there is no idiosyncratic household-level labor market uncertainty.

A.1. Preferences and Search Technology

A worker can either work, or not. At the start of the period, each worker draws a privately observed idiosyncratic shock, l, from a stochastic process with support on the unit interval, $[0,1]$. We assume the stochastic process for l exhibits dependence over time, but that its cross sectional distribution is constant across dates and uniform. A workers's realized value of l determines its utility cost of working:

$$\varsigma\left(1+\sigma_L\right) l^{\sigma_L}. \tag{A.1}$$

The parameters, ς and $\sigma_L \geq 0$ are common to all workers. In (A.1) we have structured the utility cost of employment so that σ_L affects its variance in the cross section and not its mean.[46]

After drawing l, a worker decides whether or not to participate in the labor force. The

[46] To see this, note:

$$\int_0^1 \left(1+\sigma_L\right) l^{\sigma_L} dl = 1,\ \int_0^1 \left[\left(1+\sigma_L\right) l^{\sigma_L} - 1\right]^2 dl = \frac{\sigma_L^2}{1+2\sigma_L}.$$

probability that a worker which participates in the labor market actually finds work is $p(e_{l,t};\tilde{\eta}_t)$, where $e_{l,t} \geq 0$ is a privately observed level of effort expended by the worker. We find it convenient to adopt the following piecewise linear functional form for $p(e_{l,t};\tilde{\eta}_t)$. Let

$$\tilde{p}(e_{l,t};\tilde{\eta}_t) = \eta_t + ae_{l,t} \tag{A.2}$$

where $a > 0$. The sign of a implies that the marginal product of effort is non-negative. Further,

$$\tilde{\eta}_t = \eta + \mathcal{M}(\bar{m}_t/\bar{m}_{t-1}) \tag{A.3}$$

where $\eta < 0$. We discuss the negative sign on η below. The function $\mathcal{M}(\bar{m}_t/\bar{m}_{t-1})$ reflects the impact of aggregate economic conditions – in particular the change of the aggregate labor force $\bar{m}_t/\bar{m}_{t-1}$ – on the worker's probability to find work. We will discuss details about the function $\mathcal{M}$ in subsection B.6 and estimate its key parameter in the empirical model.

We assume:

$$p(e_{l,t};\tilde{\eta}_t) = \begin{cases} 1 & \tilde{p}(e_{l,t};\tilde{\eta}_t) > 1 \\ \tilde{p}(e_{l,t};\tilde{\eta}_t) & 0 \leq \tilde{p}(e_{l,t};\tilde{\eta}_t) \leq 1 \\ 0 & \tilde{p}(e_{l,t};\tilde{\eta}_t) < 0 \end{cases}. \tag{A.4}$$

We adopt this simple representation in order to preserve analytic tractability.

A worker whose work aversion is l and which participates in the labor market and exerts effort e_l enjoys the following utility:

$$p(e_{l,t};\tilde{\eta}_t) \overbrace{\left[\ln(c_t^w - bC_{t-1}) - \varsigma(1+\sigma_L) l^{\sigma_L} - \frac{1}{2}e_{l,t}^2\right]}^{\text{ex post utility of worker that joins labor force and finds a job}} \tag{A.5}$$
$$+ (1 - p(e_{l,t};\tilde{\eta}_t)) \overbrace{\left[\ln(c_t^{nw} - bC_{t-1}) - \frac{1}{2}e_{l,t}^2\right]}^{\text{ex post utility of worker that joins labor force and fails to find a job}}.$$

Here, $e_{l,t}^2/2$ is the utility cost associated with effort. In (A.5), c_t^w and c_t^{nw} denote the consumption of employed and non-employed workers, respectively. These are outside the control of a worker and are determined in equilibrium given the arrangements which we describe below. In addition, $\tilde{\eta}_t$ is also outside the control of a worker. Our notation reflects that in our environment, a worker's consumption can only be dependent on its current employment status as this is the only worker characteristic that is publicly observed. For example, we do not allow worker consumption allocations to depend upon the history of worker reports of l. We make the latter assumption to preserve tractability. It would be interesting to investigate whether the results are sensitive to our assumption about the absence of history.[47] The term

[47] We suspect that if the history of past reports were publicly known, then the difference between discounted utility when household types and labor effort are public or private would narrow (see, e.g., Atkeson and Lucas (1995)).

bC_{t-1} reflects habit persistence in consumption at the household level which the worker takes as given. We assume that $0 \leq b < 1$.

In case the worker chooses non-participation in the labor market, its utility is simply:

$$\ln\left(c_t^{nw} - bC_{t-1}\right). \tag{A.6}$$

A non-participating worker does not experience any disutility from work or from exerting effort to find a job.

We now characterize the effort and labor force participation decisions of the worker. Because workers' work aversion type and effort choice are private information, their effort and labor force decisions are privately optimal conditional on c_t^{nw} and c_t^{w}. In particular, the worker decides its level of effort and labor force participation by comparing the magnitude of (A.6) with the maximized value of (A.5). In the case of indifference, we assume the worker chooses non-participation.

A.2. Characterizing Worker Behavior

As described above, the worker takes the replacement ratio $r_t \equiv c_t^{nw}/c_t^{w} < 1$ as given. The workers's utility of participating in the labor market, minus the utility, $\ln\left(c_t^{nw} - bC_{t-1}\right)$, of non-participation is given by:

$$\max_{e_{l,t}\geq 0} f\left(e_{l,t}\right),\ \ f\left(e_{l,t}\right) \equiv p\left(e_{l,t};\tilde{\eta}_t\right)\left[\ln\left(\frac{c_t^{w} - bC_{t-1}}{c_t^{nw} - bC_{t-1}}\right) - \varsigma\left(1+\sigma_L\right)l^{\sigma_L}\right] - \frac{1}{2}e_{l,t}^2.$$

Denote

$$\tilde{r}_t = \frac{c_t^{nw} - bC_{t-1}}{c_t^{w} - bC_{t-1}},$$

and note the distinction between this expression and the replacement ratio, r_t. In either case, the household treats this variable as given. Then, the difference in utility can be expressed as follows:

$$\max_{e_{l,t}\geq 0} f\left(e_{l,t}\right),\ \ f\left(e_{l,t}\right) \equiv p\left(e_{l,t};\tilde{\eta}_t\right)\left[\ln\left(1/\tilde{r}_t\right) - \varsigma\left(1+\sigma_L\right)l^{\sigma_L}\right] - \frac{1}{2}e_{l,t}^2. \tag{A.7}$$

We suppose that if more than one value of $e_{l,t}$ solves (A.7), then the worker chooses the smaller of the two. The worker chooses non-participation if the maximized value of (A.7) is smaller than, or equal to, zero. It chooses to participate in the labor force if the maximized value of f in (A.7) is strictly positive.

A.2.1. Optimal Effort

It is convenient to consider a version of (A.7) in which the sign restriction on $e_{l,t} \geq 0$ is ignored and $p(e_{l,t};\tilde{\eta}_t)$ in (A.7) is replaced with the linear function, $\tilde{p}(e_{l,t};\tilde{\eta}_t)$ (see (A.2)):[48]

$$\max_{e_{l,t}} \tilde{f}(e_{l,t};\tilde{\eta}_t,\tilde{r}_t),\ \tilde{f}(e_{l,t};\tilde{\eta}_t,\tilde{r}_t) \equiv \tilde{p}(e_{l,t};\tilde{\eta}_t)\left[\ln(1/\tilde{r}_t) - \varsigma(1+\sigma_L) l^{\sigma_L}\right] - \frac{1}{2}e_{l,t}^2. \tag{A.8}$$

The function, $\tilde{f}$, is quadratic with negative second derivative, and so the unique value of $e_{l,t}$ that solves the above problem is the one that sets the derivative of $\tilde{f}$ to zero:

$$\tilde{e}_{l,t} = a\left[\ln(1/\tilde{r}_t) - \varsigma(1+\sigma_L) l^{\sigma_L}\right]. \tag{A.9}$$

Substituting this expression into (A.8), we obtain:

$$\tilde{f}(\tilde{e}_{l,t};\tilde{\eta}_t) \equiv \frac{\tilde{e}_{l,t}}{2}\left[\frac{2}{a}\tilde{\eta}_t + \tilde{e}_{l,t}\right], \tag{A.10}$$

where $\tilde{e}_{l,t}$ is the particular function of l given in (A.9). We want to express $\tilde{e}_{l,t}$ as a function of l. Doing so results in the following restriction:

$$a\ln(1/\tilde{r}_t) > -\frac{2}{a}\tilde{\eta}_t > a\left[\ln(1/\tilde{r}_t) - \varsigma(1+\sigma_L)\right]. \tag{A.11}$$

The object on the left of (A.11) is $\tilde{e}_{0,t}$.

Further, keep in mind that $0 < \tilde{r}_t < 1$ so that $\tilde{e}_{0,t} > 0$ by equation (A.9). The first inequality ensures that $\frac{2}{a}\tilde{\eta}_t + \tilde{e}_{l,t} > 0$, so that $l = 0$ workers choose to participate in the labor force, i.e. the square bracket in (A.10) is positive. Inserting $\tilde{e}_{0,t}$ into the last inequality and re-arranging yields $a\ln(1/\tilde{r}_t) > -\frac{2}{a}\tilde{\eta}_t$ which is the condition that says that $l = 0$ workers exert positive effort and choose to participate in the labor force.

The second inequality in (A.11) ensures that the object in square brackets in (A.10) is negative for $l = 1$ so that households with the greatest aversion to work choose not to participate in the labor force.

A.2.2. Optimal Participation

By continuity and monotonicity of $\tilde{e}_{l,t}$, there exists a unique $0 < l < 1$ such that the object in square brackets in (A.10) is zero. That value of l is the labor force participation rate, which we denote by m_t and which solves:

$$a\left[\ln(1/\tilde{r}_t) - \varsigma(1+\sigma_L) m_t^{\sigma_L}\right] = -\frac{2}{a}\tilde{\eta}_t, \tag{A.12}$$

[48] Considering the unconstrained case first will be helpful to understand more easily the constrained case, i.e. $e_{l,t} \geq 0$ and $0 \leq p(e_{l,t};\tilde{\eta}_t) \leq 1$ which we characterize below.

or,

$$m_t = \left[\frac{\ln(1/\tilde{r}_t) + \frac{2}{a^2}\tilde{\eta}_t}{\varsigma(1+\sigma_L)} \right]^{\frac{1}{\sigma_L}}. \tag{A.13}$$

Note that for all $l \geq m_t$ such that $\tilde{e}_{l,t} \geq 0$, $\tilde{f}(\tilde{e}_{l,t}; \tilde{\eta}_t, \tilde{r}_t) \leq 0$ and for all $l < m_t$, $\tilde{f}(\tilde{e}_{l,t}; \tilde{\eta}_t, \tilde{r}_t) > 0$. We summarize these findings in the form of a proposition:

Proposition A.1. *Suppose that (A.11) is satisfied and the l^{th} worker's objective is described in (A.8), with $\tilde{r}_t$ taken as given by the worker. Let m_t be as defined in (A.13). Then, $0 < m_t < 1$, workers with $1 \geq l \geq m_t$ choose non-participation and workers with $l < m_t$ and $\tilde{e}_{l,t} \geq 0$ choose participation. For those that choose participation, their effort level is given by (A.9).*

The previous proposition was derived under the counterfactual assumption that the workers's objective is (A.8). We use the results based on (A.8) to understand the relevant case of (A.7). One can show that there is a largest value of l, denoted $\mathring{l}_t$, such that for all $l \leq \mathring{l}_t$, the constraint, $p(e_{l,t}; \tilde{\eta}_t) \leq 1$ is binding. In other words, there is a share of workers $\mathring{l}_t$ that has $p\left(e_{\mathring{l},t}; \tilde{\eta}_t\right) = 1$. The cutoff, $\mathring{l}_t$, solves:

$$p\left(e_{\mathring{l},t}; \tilde{\eta}_t\right) = \tilde{\eta}_t + a^2 \left[\ln(1/\tilde{r}_t) - \varsigma(1+\sigma_L)\mathring{l}_t^{\sigma_L}\right] = 1,$$

or after making use of (A.12) to substitute out $\ln(1/\tilde{r}_t)$:

$$p\left(e_{\mathring{l},t}; \tilde{\eta}_t\right) = \tilde{\eta}_t + a^2 \left[\varsigma(1+\sigma_L)\left(m_t^{\sigma_L} - \mathring{l}_t^{\sigma_L}\right) - \frac{2}{a^2}\tilde{\eta}_t\right] = 1,$$

or

$$\mathring{l}_t = \left[m_t^{\sigma_L} - \frac{1+\tilde{\eta}_t}{\varsigma(1+\sigma_L)a^2}\right]^{\frac{1}{\sigma_L}}. \tag{A.14}$$

A.3. Household Utility Function

Utility of the household is given by:

$$\int_0^{m_t} \left(p(e_{l,t}; \tilde{\eta}_t)\left[\ln(c_t^w - bC_{t-1}) - \varsigma(1+\sigma_L)l^{\sigma_L}\right] + (1 - p(e_{l,t}; \tilde{\eta}_t))\ln(c_t^{nw} - bC_{t-1}) - \frac{1}{2}e_{l,t}^2 \right) dl$$
$$+ (1-m_t)\ln(c_t^{nw} - bC_{t-1})$$

We wish to express this as a function of C_t and h_t (recalling that the household takes C_{t-1} and $\tilde{\eta}_t$ as given) only using the results in the previous section.

Below we will need the restriction that the marginal worker, $l = m_t$, chooses effort according to (A.9). That is, we require that for the marginal worker,

$$\tilde{p}(e_{m,t}; \tilde{\eta}_t) = \tilde{\eta}_t + ae_{m,t} \leq 1$$

Note that by (A.9)

$$e_{m,t} = a\left[\ln\left(1/\tilde{r}_t\right) - \varsigma\left(1+\sigma_L\right)m^{\sigma_L}\right].$$

Further, the indifference condition for the marginal worker is given by (A.12)

$$a\left[\ln\left(1/\tilde{r}_t\right) - \varsigma\left(1+\sigma_L\right)m_t^{\sigma_L}\right] = -\frac{2}{a}\tilde{\eta}_t,$$

Combining the last three equations gives:

$$\tilde{p}\left(e_{m,t};\tilde{\eta}_t\right) = -\tilde{\eta}_t$$

Thus, we adopt the restriction, $-\tilde{\eta}_t \leq 1$. It is also convenient to have $\tilde{p}\left(e_{m,t};\tilde{\eta}_t\right) \geq 0$. Thus,

$$0 \leq -\tilde{\eta}_t \leq 1. \tag{A.15}$$

Simplifying the expression above for the household utility function,

$$\int_0^{m_t}\left(p\left(e_{l,t};\tilde{\eta}_t\right)\left[\ln\left(1/\tilde{r}_t\right) - \varsigma\left(1+\sigma_L\right)l^{\sigma_L}\right] - \frac{1}{2}e_{l,t}^2\right)dl + \ln\left(c_t^{nw} - bC_{t-1}\right)$$

Rewriting the incentive constraint, (A.13), in a more convenient form:

$$\ln\left(1/\tilde{r}_t\right) = \varsigma\left(1+\sigma_L\right)m_t^{\sigma_L} - \frac{2}{a^2}\tilde{\eta}_t. \tag{A.16}$$

It is also useful to have an expression for $c_t^{nw} - bC_{t-1}$. The household resource constraint is given by:

$$c_t^w h_t + \left(1-h_t\right)c_t^{nw} = C_t, \tag{A.17}$$

so that

$$c_t^w = \frac{C_t}{h_t + \left(1-h_t\right)r_t},\quad c_t^{nw} = \frac{r_t C_t}{h_t + \left(1-h_t\right)r_t}. \tag{A.18}$$

Using these results, household utility can be written as follows:

$$B\left(m_t;\tilde{\eta}_t\right) + \ln\left(\frac{r_t C_t}{h_t + \left(1-h_t\right)r_t} - bC_{t-1}\right), \tag{A.19}$$

where

$$B\left(m_t;\tilde{\eta}_t\right) \equiv \int_0^{m_t}\left(p\left(e_{l,t};\tilde{\eta}_t\right)\left[\varsigma\left(1+\sigma_L\right)\left(m_t^{\sigma_L} - l^{\sigma_L}\right) - \frac{2}{a^2}\tilde{\eta}_t\right] - \frac{1}{2}e_{l,t}^2\right)dl \tag{A.20}$$

is a term capturing the disutility of work and costly job search.

A.4. Expressing $B(m_t;\tilde{\eta}_t)$

We seek to provide an expression for $B(m_t;\tilde{\eta}_t)$ where the integral is evaluated. Suppose that $p(e_{l,t};\tilde{\eta}_t) \leq 1$ is binding for a measure of $m_t > l \geq 0$, that is, that (A.14) holds. In particular, we require that $e_{m,t}$ in (A.9) lies inside the admissible probability region. We permit $e_{l,t}$ in (A.9) to lie above the admissible probability region for $l < m_t$.

Under our supposition, there exists an $\mathring{l} \geq 0$ that solves (A.14). Then, (A.20) can be written

$$B\left(m_t, \mathring{l}_t;\tilde{\eta}_t\right) = B_1\left(m_t, \mathring{l}_t;\tilde{\eta}_t\right) + B_2\left(m_t, \mathring{l}_t;\tilde{\eta}_t\right),$$

where

$$B_1\left(m_t, \mathring{l}_t;\tilde{\eta}_t\right) \equiv \int_{\mathring{l}_t}^{m_t}\left\{p(e_{l,t};\tilde{\eta}_t)\left[\varsigma(1+\sigma_L)(m_t^{\sigma_L} - l^{\sigma_L}) - \frac{2}{a^2}\tilde{\eta}_t\right] - \frac{1}{2}e_{l,t}^2\right\}dl \quad \text{(A.21)}$$

$$B_2\left(m_t, \mathring{l}_t;\tilde{\eta}_t\right) \equiv \int_0^{\mathring{l}_t}\left\{\varsigma(1+\sigma_L)(m_t^{\sigma_L} - l^{\sigma_L}) - \frac{2}{a^2}\tilde{\eta}_t - \frac{1}{2}e_{l,t}^2\right\}dl. \quad \text{(A.22)}$$

We desire expressions for $e_{l,t}$. Note that for $l \geq \mathring{l}_t$, the optimal effort equation (A.9) together with the incentive constraint (A.16) yields:

$$e_{l,t} = a\left[\varsigma(1+\sigma_L)(m_t^{\sigma_L} - l^{\sigma_L}) - \frac{2}{a^2}\tilde{\eta}_t\right].$$

Note that for $l \leq \mathring{l}_t$,

$$\tilde{p}(e_{l,t};\tilde{\eta}_t) = \tilde{\eta}_t + ae_{l,t} = 1$$

Solving for $e_{l,t}$ yields:

$$e_{l,t} = \frac{1-\tilde{\eta}_t}{a}$$

Summarizing the previous results for optimal effort:

$$e_{l,t} = \begin{cases} \frac{1-\tilde{\eta}_t}{a} & l \leq \mathring{l}_t \\ a\left[\varsigma(1+\sigma_L)(m^{\sigma_L} - l^{\sigma_L}) - \frac{2}{a^2}\tilde{\eta}_t\right] & l \geq \mathring{l}_t \end{cases}. \quad \text{(A.23)}$$

Note that the $e_{l,t}$ function defined in (A.23) is continuous. That is,

$$a\left[\varsigma(1+\sigma_L)\left(m^{\sigma_L} - \mathring{l}^{\sigma_L}\right) - \frac{2}{a^2}\tilde{\eta}_t\right] = \frac{1-\tilde{\eta}_t}{a}$$

for $\mathring{l}_t$ given in (A.14).

We now develop an expression for $B_1\left(m_t, \mathring{l}_t;\tilde{\eta}_t\right)$ in (A.21). Substituting for $p(e_{l,t};\tilde{\eta}_t)$

and optimal effort, the integrand is:

$$\begin{aligned}
& \left[\tilde{\eta}_t + a^2\left[\varsigma\left(1+\sigma_L\right)\left(m^{\sigma_L} - l^{\sigma_L}\right) - \frac{2}{a^2}\tilde{\eta}_t\right]\right] \\
& \times\left[\varsigma\left(1+\sigma_L\right)\left(m_t^{\sigma_L} - l^{\sigma_L}\right) - \frac{2}{a^2}\tilde{\eta}_t\right] \\
& -\frac{1}{2}a^2\left[\varsigma\left(1+\sigma_L\right)\left(m^{\sigma_L} - l^{\sigma_L}\right) - \frac{2}{a^2}\tilde{\eta}_t\right]^2 \\
= \; & \tilde{\eta}_t\left[\varsigma\left(1+\sigma_L\right)\left(m_t^{\sigma_L} - l^{\sigma_L}\right) - \frac{2}{a^2}\tilde{\eta}_t\right] \\
& +\frac{1}{2}a^2\left[\varsigma\left(1+\sigma_L\right)\left(m^{\sigma_L} - l^{\sigma_L}\right) - \frac{2}{a^2}\tilde{\eta}_t\right]^2
\end{aligned}$$

Then,

$$\begin{aligned}
= \; & \tilde{\eta}_t\left[\varsigma\left(1+\sigma_L\right)\left(m_t^{\sigma_L} - l^{\sigma_L}\right) - \frac{2}{a^2}\tilde{\eta}_t\right] + \frac{1}{2}a^2\left[\varsigma\left(1+\sigma_L\right)\left(m_t^{\sigma_L} - l^{\sigma_L}\right) - \frac{2}{a^2}\tilde{\eta}_t\right]^2 \\
= \; & \tilde{\eta}_t\varsigma\left(1+\sigma_L\right)\left(m_t^{\sigma_L} - l^{\sigma_L}\right) - \frac{2}{a^2}\tilde{\eta}_t^2 \\
& +\frac{1}{2}a^2\varsigma^2\left(1+\sigma_L\right)^2\left(m_t^{\sigma_L} - l^{\sigma_L}\right)^2 - 2\tilde{\eta}_t\varsigma\left(1+\sigma_L\right)\left(m_t^{\sigma_L} - l^{\sigma_L}\right) + \frac{1}{2}a^2\left(\frac{2}{a^2}\tilde{\eta}_t\right)^2 \\
= \; & -\varsigma\tilde{\eta}_t\left(1+\sigma_L\right)\left(m_t^{\sigma_L} - l^{\sigma_L}\right) + \frac{1}{2}a^2\varsigma^2\left(1+\sigma_L\right)^2\left(m_t^{2\sigma_L} - 2m_t^{\sigma_L}l^{\sigma_L} + l^{2\sigma_L}\right)
\end{aligned}$$

We must integrate the previous expression over $l = \mathring{l}_t$ to m_t. For this, the following results are useful:

$$\begin{aligned}
\int_{\mathring{l}_t}^{m_t}\left(m_t^{\sigma_L} - l^{\sigma_L}\right)dl &= m_t^{\sigma_L}l|_{\mathring{l}_t}^{m_t} - \frac{l^{\sigma_L+1}}{\sigma_L+1}|_{\mathring{l}_t}^{m_t} \\
&= \left(m_t - \mathring{l}_t\right)m_t^{\sigma_L} - \frac{m_t^{\sigma_L+1} - \mathring{l}_t^{\sigma_L+1}}{\sigma_L+1} \\
\int_{\mathring{l}_t}^{m_t}\left(m_t^{2\sigma_L} - 2m_t^{\sigma_L}l^{\sigma_L} + l^{2\sigma_L}\right)dl &= m_t^{2\sigma_L}\left(m_t - \mathring{l}_t\right) - 2m_t^{\sigma_L}\frac{m_t^{\sigma_L+1} - \mathring{l}_t^{\sigma_L+1}}{\sigma_L+1} \\
&\quad + \frac{m_t^{2\sigma_L+1} - \mathring{l}_t^{2\sigma_L+1}}{2\sigma_L+1}.
\end{aligned}$$

Then,

$$B_1\left(m_t,\mathring{l}_t;\tilde{\eta}_t\right) \equiv \int_{\mathring{l}_t}^{m_t}\left[-\varsigma\tilde{\eta}_t(1+\sigma_L)\left(m_t^{\sigma_L}-l^{\sigma_L}\right)+\frac{1}{2}a^2\varsigma^2(1+\sigma_L)^2\left(m_t^{2\sigma_L}-2m_t^{\sigma_L}l^{\sigma_L}+l^{2\sigma_L}\right)\right]dl$$

$$= -\varsigma\tilde{\eta}_t(1+\sigma_L)\left[\left(m_t-\mathring{l}_t\right)m_t^{\sigma_L}-\frac{m_t^{\sigma_L+1}-\mathring{l}_t^{\sigma_L+1}}{\sigma_L+1}\right]$$

$$+\frac{1}{2}a^2\varsigma^2(1+\sigma_L)^2\left[m_t^{2\sigma_L}\left(m_t-\mathring{l}_t\right)-2m_t^{\sigma_L}\frac{m_t^{\sigma_L+1}-\mathring{l}_t^{\sigma_L+1}}{\sigma_L+1}+\frac{m_t^{2\sigma_L+1}-\mathring{l}_t^{2\sigma_L+1}}{2\sigma_L+1}\right]$$

$$= -\varsigma\tilde{\eta}_t\left[\sigma_L m_t^{\sigma_L+1}-(\sigma_L+1)\mathring{l}_t m_t^{\sigma_L}+\mathring{l}_t^{\sigma_L+1}\right]$$

$$+\frac{1}{2}a^2\varsigma^2(1+\sigma_L)^2\left[m_t^{2\sigma_L+1}\left(\frac{\sigma_L-1}{\sigma_L+1}+\frac{1}{2\sigma_L+1}\right)-m_t^{2\sigma_L}\mathring{l}_t+\frac{2m_t^{\sigma_L}\mathring{l}_t^{\sigma_L+1}}{\sigma_L+1}-\frac{\mathring{l}_t^{2\sigma_L+1}}{2\sigma_L+1}\right]$$

or, after further simplification, we have:

$$B_1\left(m_t,\mathring{l}_t;\tilde{\eta}_t\right)=-\varsigma\tilde{\eta}_t\left[\sigma_L m_t^{\sigma_L+1}-(1+\sigma_L)\mathring{l}_t m_t^{\sigma_L}+\mathring{l}_t^{\sigma_L+1}\right] \quad \text{(A.24)}$$

$$+\frac{1}{2}a^2\varsigma^2(1+\sigma_L)^2\left[\frac{2\sigma_L^2 m_t^{2\sigma_L+1}}{(\sigma_L+1)(2\sigma_L+1)}-m_t^{2\sigma_L}\mathring{l}_t+\frac{2m_t^{\sigma_L}\mathring{l}_t^{\sigma_L+1}}{\sigma_L+1}-\frac{\mathring{l}_t^{2\sigma_L+1}}{2\sigma_L+1}\right].$$

This completes our discussion of $B_1\left(m_t,\mathring{l}_t;\tilde{\eta}_t\right)$ in (A.21).

Next, we evaluate $B_2\left(m_t,\mathring{l}_t;\tilde{\eta}_t\right)$ in (A.22):

$$B_2\left(m_t,\mathring{l}_t;\tilde{\eta}_t\right) \equiv \int_0^{\mathring{l}_t}\left\{\varsigma(1+\sigma_L)\left(m_t^{\sigma_L}-l^{\sigma_L}\right)-\frac{2}{a^2}\tilde{\eta}_t-\frac{1}{2}e_{l,t}^2\right\}dl$$

$$= \int_0^{\mathring{l}_t}\left\{\varsigma(1+\sigma_L)\left(m_t^{\sigma_L}-l^{\sigma_L}\right)-\frac{2}{a^2}\tilde{\eta}_t-\frac{1}{2}\left[\frac{1-\tilde{\eta}_t}{a}\right]^2\right\}dl$$

by (A.23). Then,

$$B_2\left(m_t,\mathring{l}_t;\tilde{\eta}_t\right)=\varsigma(1+\sigma_L)\left(m_t^{\sigma_L}\mathring{l}_t-\frac{\mathring{l}_t^{\sigma_L+1}}{1+\sigma_L}\right)-\frac{2\tilde{\eta}_t}{a^2}\mathring{l}_t-\frac{1}{2}\left[\frac{1-\tilde{\eta}_t}{a}\right]^2\mathring{l}_t \quad \text{(A.25)}$$

We conclude that, after adding (A.24) and (A.25),

$$B\left(m_t,\mathring{l}_t;\tilde{\eta}_t\right) = B_1\left(m_t,\mathring{l}_t;\tilde{\eta}_t\right)+B_2\left(m_t,\mathring{l}_t;\tilde{\eta}_t\right)$$

$$= -\varsigma\tilde{\eta}_t\sigma_L m_t^{\sigma_L+1}-\varsigma\tilde{\eta}_t\left(\mathring{l}_t^{\sigma_L+1}-(1+\sigma_L)\mathring{l}_t m_t^{\sigma_L}\right)$$

$$+\frac{1}{2}a^2\varsigma^2(1+\sigma_L)^2\frac{2\sigma_L^2 m_t^{2\sigma_L+1}}{(\sigma_L+1)(2\sigma_L+1)}$$

$$+\frac{1}{2}a^2\varsigma^2(1+\sigma_L)^2\left(-\mathring{l}_t m_t^{2\sigma_L}+\frac{\mathring{l}_t^{\sigma_L+1}}{\sigma_L+1}2m_t^{\sigma_L}-\frac{\mathring{l}_t^{2\sigma_L+1}}{2\sigma_L+1}\right)$$

$$+\varsigma(1+\sigma_L)\mathring{l}_t\left[m_t^{\sigma_L}-\frac{\mathring{l}_t^{\sigma_L}}{1+\sigma_L}-\frac{2\tilde{\eta}_t}{a^2}-\frac{1}{2}\left[\frac{1-\tilde{\eta}_t}{a}\right]^2\right]$$

or,

$$B\left(m_t,\mathring{l}_t;\tilde{\eta}_t\right) = -\varsigma\tilde{\eta}_t\sigma_L m_t^{\sigma_L+1} + \frac{a^2\varsigma^2\left(1+\sigma_L\right)\sigma_L^2}{2\sigma_L+1}m_t^{2\sigma_L+1} + \mathring{l}_t\left[\begin{array}{c}\frac{1}{2}a^2\varsigma^2\left(1+\sigma_L\right)^2\left(-m_t^{2\sigma_L}+\frac{\mathring{l}_t^{\sigma_L}}{\sigma_L+1}2m_t^{\sigma_L}-\frac{\mathring{l}_t^{2\sigma_L}}{2\sigma_L+1}\right)\\ +\varsigma\left(1+\sigma_L\right)\left[m_t^{\sigma_L}-\frac{\mathring{l}_t^{\sigma_L}}{1+\sigma_L}-\frac{2\tilde{\eta}_t}{a^2}-\frac{1}{2}\left(\frac{1-\tilde{\eta}_t}{a}\right)^2\right]-\varsigma\tilde{\eta}_t\left(\mathring{l}_t^{\sigma_L}-\left(1+\sigma_L\right)m_t^{\sigma_L}\right)\end{array}\right] \tag{A.26}$$

We seek to simplify $B\left(m_t,\mathring{l}_t;\tilde{\eta}_t\right)$. The following expressions for (A.14) will be useful:

$$\mathring{l}_t^{\sigma_L} = m_t^{\sigma_L} - \frac{1+\tilde{\eta}_t}{\varsigma\left(1+\sigma_L\right)a^2}$$

Or

$$\mathring{l}_t = \left[m_t^{\sigma_L} - \frac{1+\tilde{\eta}_t}{\varsigma\left(1+\sigma_L\right)a^2}\right]^{\frac{1}{\sigma_L}}$$

Or

$$\begin{aligned}\mathring{l}_t^{2\sigma_L} &= \left[m_t^{\sigma_L} - \frac{1+\tilde{\eta}_t}{\varsigma\left(1+\sigma_L\right)a^2}\right]^2\\ &= m_t^{2\sigma_L} - 2m_t^{\sigma_L}\frac{1+\tilde{\eta}_t}{\varsigma\left(1+\sigma_L\right)a^2} + \left(\frac{1+\tilde{\eta}_t}{\varsigma\left(1+\sigma_L\right)a^2}\right)^2\end{aligned}$$

Substituting for $\mathring{l}_t^{\sigma_L}$ and $\mathring{l}_t^{2\sigma_L}$, equation (A.26) can be rewritten as follows:

$$B\left(m_t,\mathring{l}_t;\tilde{\eta}_t\right) = -\varsigma\sigma_L\tilde{\eta}_t m_t^{\sigma_L+1} + \frac{a^2\varsigma^2\left(1+\sigma_L\right)\sigma_L^2}{2\sigma_L+1}m_t^{2\sigma_L+1} + \mathring{l}_t\left[\begin{array}{c}\frac{2\varsigma\sigma_L^2}{2\sigma_L+1}\left(1+\tilde{\eta}_t\right)m_t^{\sigma_L} - a^2\varsigma^2\sigma_L^2\frac{\sigma_L+1}{2\sigma_L+1}m_t^{2\sigma_L}\\ -\frac{1}{2a^2}\frac{\left(\varsigma-3\sigma_L+4\varsigma\sigma_L+5\varsigma\sigma_L^2+2\varsigma\sigma_L^3-1\right)}{2\sigma_L^2+3\sigma_L+1}\left(1+\tilde{\eta}_t\right)^2\end{array}\right]$$

Or:

$$\begin{aligned}B\left(m_t,\mathring{l}_t;\tilde{\eta}_t\right) &= \alpha_1\tilde{\eta}_t m_t^{\sigma_L+1} + \alpha_2 m_t^{2\sigma_L+1} + \alpha_3\left(1+\tilde{\eta}_t\right)m_t^{\sigma_L}\mathring{l}_t - \alpha_2 m_t^{2\sigma_L}\mathring{l}_t + \alpha_4\left(1+\tilde{\eta}_t\right)^2\mathring{l}_t\\ &= \left(\alpha_1\tilde{\eta}_t m_t + \alpha_2\left(m_t-\mathring{l}_t\right)m_t^{\sigma_L} + \alpha_3\left(1+\tilde{\eta}_t\right)\mathring{l}_t\right)m_t^{\sigma_L} + \alpha_4\left(1+\tilde{\eta}_t\right)^2\mathring{l}_t\end{aligned} \tag{A.27}$$

where

$$\begin{aligned}\alpha_1 &= -\varsigma\sigma_L\\ \alpha_2 &= a^2\varsigma^2\sigma_L^2\frac{\left(1+\sigma_L\right)}{2\sigma_L+1}\\ \alpha_3 &= \frac{2\varsigma\sigma_L^2}{2\sigma_L+1}\\ \alpha_4 &= -\frac{1}{2a^2}\frac{\left(\varsigma-3\sigma_L+4\varsigma\sigma_L+5\varsigma\sigma_L^2+2\varsigma\sigma_L^3-1\right)}{2\sigma_L^2+3\sigma_L+1}\end{aligned}$$

and

$$\mathring{l}_t = \left[m_t^{\sigma_L} - \frac{1+\tilde{\eta}_t}{\varsigma\left(1+\sigma_L\right)a^2}\right]^{\frac{1}{\sigma_L}}$$

and

$$\tilde{\eta}_t = \eta + \omega_1\left(\bar{u}_t - \omega_2\bar{u}_{t-1}\right)$$

A.5. Expressing $\ln\left(\frac{r_t C_t}{h_t+(1-h_t)r_t} - bC_{t-1}\right)$

We now simplify the ln term in (A.19). To do so, we first establish a relationship between the replacement ratio,

$$r_t = c_t^{nw}/c_t^w$$

and

$$\tilde{r}_t = \frac{c_t^{nw} - bC_{t-1}}{c_t^w - bC_{t-1}}.$$

The latter equation can be written as:

$$r_t c_t^w - bC_{t-1} = \tilde{r}_t\left(c_t^w - bC_{t-1}\right)$$

Recall that the budget constraint of the household is:

$$c_t^w = \frac{C_t}{h_t + \left(1-h_t\right)r_t}$$

Substituting out c_t^w in the previous equation:

$$r_t\frac{C_t}{h_t + \left(1-h_t\right)r_t} - bC_{t-1} = \tilde{r}_t\left(\frac{C_t}{h_t + \left(1-h_t\right)r_t} - bC_{t-1}\right)$$

Solving for r_t:

$$r_t = \frac{\left(C_t - h_t bC_{t-1}\right)\tilde{r}_t + h_t bC_{t-1}}{C_t - \left(1-h_t\right)bC_{t-1} + \left(1-h_t\right)bC_{t-1}\tilde{r}_t}. \qquad \text{(A.28)}$$

So, substituting into the ln term in (A.19):

$$\begin{aligned}
\ln\left(\frac{C_t}{\frac{h_t}{r_t} + 1 - h_t} - bC_{t-1}\right) &= \ln\left(\frac{C_t}{\frac{h_t}{\frac{(C_t - h_t bC_{t-1})\tilde{r}_t + h_t bC_{t-1}}{C_t - (1-h_t)bC_{t-1} + (1-h_t)bC_{t-1}\tilde{r}_t}} + 1 - h_t} - bC_{t-1}\right) \\
&= \ln\left(\frac{C_t}{\frac{C_t(h_t + \tilde{r}_t - h_t\tilde{r}_t)}{C_t\tilde{r}_t + bh_tC_{t-1} - bh_t\tilde{r}_tC_{t-1}}} - bC_{t-1}\right) \\
&= \ln\left(\tilde{r}_t\frac{C_t - bC_{t-1}}{h_t + \tilde{r}_t - h_t\tilde{r}_t}\right) \\
&= \ln\left(C_t - bC_{t-1}\right) + \ln\frac{\tilde{r}_t}{h_t + \tilde{r}_t - h_t\tilde{r}_t} \\
&= \ln\left(C_t - bC_{t-1}\right) - \ln\left(h_t\left(\frac{1}{\tilde{r}_t} - 1\right) + 1\right)
\end{aligned}$$

A.6. Expressing Household Utility Function

Pulling together all terms (A.19), the indirect household utility function can be written as follows:

$$U(C_t, h_t, m_t; C_{t-1}, \tilde{\eta}_t; \tilde{r}_t) = \ln(C_t - bC_{t-1}) - \ln\left(h_t\left(\frac{1}{\tilde{r}_t} - 1\right) + 1\right) + B\left(m_t, \mathring{l}_t; \tilde{\eta}_t\right), \quad \text{(A.29)}$$

where $B\left(m_t, \mathring{l}_t; \tilde{\eta}_t\right)$ is defined in (A.27). It remains to provide expressions relating $\tilde{r}_t$ and m_t to h_t.

From (A.16),

$$\frac{1}{\tilde{r}_t} = e^{\varsigma(1+\sigma_L)m_t^{\sigma_L} - \frac{2}{a^2}\tilde{\eta}_t}. \quad \text{(A.30)}$$

We now have a representation of $\tilde{r}_t$ in terms of m_t. We still require a representation of m_t in terms of h_t.

A.7. h-m Relationship

We now derive the relationship between m_t and h_t :

$$\begin{aligned}
h_t &= \int_0^{m_t} p(e_{l,t}; \tilde{\eta}_t)\, dl = \int_0^{\mathring{l}_t} 1 dl + \int_{\mathring{l}_t}^{m_t} \tilde{p}(e_{l,t}; \tilde{\eta}_t)\, dl \\
&= \mathring{l}_t + \int_{\mathring{l}_t}^{m_t} \left(\tilde{\eta}_t + \overbrace{a^2\left[\varsigma(1+\sigma_L)(m^{\sigma_L} - l^{\sigma_L}) - \frac{2}{a^2}\tilde{\eta}_t\right]}^{=ae_{l,t},\text{ for } l \geq \mathring{l}_t} \right) dl \\
&= \mathring{l}_t + \tilde{\eta}_t\left(m_t - \mathring{l}_t\right) + a^2\varsigma(1+\sigma_L)\left[\left(m_t - \mathring{l}_t\right)m_t^{\sigma_L} - \frac{m_t^{\sigma_L+1} - \mathring{l}_t^{\sigma_L+1}}{\sigma_L + 1}\right] - 2\tilde{\eta}_t\left(m_t - \mathring{l}_t\right) \\
&= \mathring{l}_t - \tilde{\eta}_t\left(m_t - \mathring{l}_t\right) + a^2\varsigma(1+\sigma_L)\left(m_t - \mathring{l}_t\right)m_t^{\sigma_L} - a^2\varsigma\left(m_t^{\sigma_L+1} - \mathring{l}_t^{\sigma_L+1}\right) \\
&= -\tilde{\eta}_t m_t + a^2\varsigma\sigma_L m_t^{\sigma_L+1} + \mathring{l}_t\left[1 + \tilde{\eta}_t + a^2\varsigma(1+\sigma_L)\left(-m_t^{\sigma_L} + \frac{\mathring{l}_t^{\sigma_L}}{1+\sigma_L}\right)\right].
\end{aligned}$$

According to (A.14),

$$1 + \tilde{\eta}_t = a^2\varsigma(1+\sigma_L)\left(m_t^{\sigma_L} - \mathring{l}_t^{\sigma_L}\right)$$

Using this to substitute out for $1 + \tilde{\eta}_t$ in the previous expression and re-arranging yields:

$$h_t = -\tilde{\eta}_t m_t + a^2\varsigma\sigma_L\left(m_t^{\sigma_L+1} - \mathring{l}_t^{\sigma_L+1}\right) \quad \text{(A.31)}$$

where $\mathring{l}_t$ is given in (A.14).

A.8. Summary of Household Utility

We summarize the preceding results in the form of a proposition:

Proposition A.2. *Under assumption (A.15), the household utility function is given by (A.29), where* $B\left(m_t, \mathring{l}_t; \tilde{\eta}_t\right)$ *is given in (A.26),* $\mathring{l}_t$ *is given by (A.14),* $\tilde{r}_t$ *is given by (A.16),* m_t *is the function of* h_t *defined by the inverse of (A.31), and* $\tilde{\eta}_t$ *is given by (A.3). For convenience, we list these equations here:*

$$\begin{aligned} U(C_t, h_t, m_t, \mathring{l}_t; C_{t-1}, \tilde{\eta}_t, \tilde{r}_t) &= \ln\left(C_t - bC_{t-1}\right) - \ln\left(h_t\left(\frac{1}{\tilde{r}_t} - 1\right) + 1\right) + B\left(m_t, \mathring{l}_t; \tilde{\eta}_t\right) \qquad \text{(A.32)} \\ B\left(m_t, \mathring{l}_t; \tilde{\eta}_t\right) &= \alpha_1 \tilde{\eta}_t m_t^{\sigma_L+1} + \alpha_2 m_t^{2\sigma_L+1} + \alpha_3\left(1+\tilde{\eta}_t\right) m_t^{\sigma_L} \mathring{l}_t - \alpha_2 m_t^{2\sigma_L} \mathring{l}_t + \alpha_4\left(1+\tilde{\eta}_t\right)^2 \mathring{l}_t \\ \mathring{l}_t^{\sigma_L} &= m_t^{\sigma_L} - \frac{1+\tilde{\eta}_t}{\varsigma\left(1+\sigma_L\right) a^2} \\ \ln\left(1/\tilde{r}_t\right) &= \varsigma\left(1+\sigma_L\right) m_t^{\sigma_L} - \frac{2}{a^2}\tilde{\eta}_t \\ h_t &= -\tilde{\eta}_t m_t + a^2 \varsigma \sigma_L\left(m_t^{\sigma_L+1} - \mathring{l}_t^{\sigma_L+1}\right) \\ \tilde{\eta}_t &= \eta + \mathcal{M}(\bar{m}_t/\bar{m}_{t-1}) \end{aligned}$$

A notable feature of (A.29) is that consumption enters the household's utility function in the same way that it enters the individual worker's utility function. Moreover, consumption and employment are separable in utility.

Use the $h - m$ and $\mathring{l}_t$ relationships to obtain:

$$m_t^{\sigma_L} = \frac{\frac{h_t}{m_t} + \tilde{\eta}_t}{a^2 \varsigma \sigma_L} + \frac{\mathring{l}_t^{\sigma_L+1}}{m_t}. \qquad \text{(A.33)}$$

There is a unique value of m_t, $m_t \geq 0$, that satisfies (A.33). To see this, note that the left side of (A.33) begins at zero and increases without bound as m increases. The right side starts at plus infinity (thus, greater than the left size) with $m_t = 0$ and (assuming the behavior of $\mathring{l}_t$ does not disrupt this conclusion) declines monotonically to a finite number as m_t increases (thus, the right side is eventually below the left side). By continuity and monotonicity, there is a unique value of m_t that satisfies the equality in (A.33).

Then, substitute for $\mathring{l}_t$ to obtain the following h-m relationship:

$$\begin{aligned} h_t &= -\tilde{\eta}_t m_t + a^2 \varsigma \sigma_L\left(m_t^{\sigma_L+1} - \left[m_t^{\sigma_L} - \frac{1+\tilde{\eta}_t}{\varsigma\left(1+\sigma_L\right) a^2}\right]^{\frac{\sigma_L+1}{\sigma_L}}\right) \equiv Q\left(m_t; \tilde{\eta}_t\right) \\ m_t &= Q^{-1}\left(h_t; \tilde{\eta}_t\right), \end{aligned}$$

or

$$m_t = Q^{-1}\left(h_t; \tilde{\eta}_t\right),$$

where Q^{-1} is the inverse function of Q, defined by:

$$h_t = Q\left(Q^{-1}\left(h_t; \tilde{\eta}_t\right); \tilde{\eta}_t\right).$$

Using $m_t = Q^{-1}\left(h_t; \tilde{\eta}_t\right)$ and also substituting out $\mathring{l}_t$, we can write (A.32) as:

$$\begin{aligned}
u(C_t, h_t; C_{t-1}, \tilde{\eta}_t) &= \ln\left(C_t - bC_{t-1}\right) - z(h_t; \tilde{\eta}_t) \qquad \text{(A.34)} \\
z(h_t; \tilde{\eta}_t) &= \ln\left(h_t\left[e^{\varsigma(1+\sigma_L)\left[Q^{-1}(h_t;\tilde{\eta}_t)\right]^{\sigma_L} - \frac{2}{a^2}\tilde{\eta}_t} - 1\right] + 1\right) \\
&\quad -\alpha_1\tilde{\eta}_t\left[Q^{-1}\left(h_t; \tilde{\eta}_t\right)\right]^{\sigma_L+1} - \alpha_2\left[Q^{-1}\left(h_t; \tilde{\eta}_t\right)\right]^{2\sigma_L+1} \\
&\quad -\left[\alpha_3\left(1+\tilde{\eta}_t\right)\left[Q^{-1}\left(h_t; \tilde{\eta}_t\right)\right]^{\sigma_L} - \alpha_2\left[Q^{-1}\left(h_t; \tilde{\eta}_t\right)\right]^{2\sigma_L} + \alpha_4\left(1+\tilde{\eta}_t\right)^2\right] \times \\
&\quad \left[\left[Q^{-1}\left(h_t; \tilde{\eta}_t\right)\right]^{\sigma_L} - \frac{1+\tilde{\eta}_t}{\varsigma\left(1+\sigma_L\right)a^2}\right]^{\frac{1}{\sigma_L}} \\
\tilde{\eta}_t &= \eta + \mathcal{M}(\bar{m}_t/\bar{m}_{t-1})
\end{aligned}$$

A.9. Derivatives of Household Utility

We need derivatives of household utility to calculate various elasticities.

A.9.1. Labor Supply Elasticity

We now derive the elasticity of labor supply associated with the household utility function, (A.29). Let w denote the wage and the first order condition associated with the choice of h is:

$$u_c w + u_h = 0,$$

or,

$$w = \frac{-u_h}{u_c}.$$

We differentiate this and set $du_c = 0$, which implies $dc = 0$ in our case of separability. Totally differentiating the first order condition and imposing the above restriction,

$$u_c dw + u_{hh} dh = 0,$$

or,

$$\frac{\frac{dh}{h}}{\frac{dw}{w}} = \frac{u_h}{u_{hh}h}.$$

A.9.2. MATLAB Symbolic Differentiation

We now describe a procedure based on symbolic arithmetic in MATLAB for calculating u_{hh}, u_h and $u_{h\tilde{\eta}}$. We need those expressions in the log-linearized wage Phillips curve as well as for the steady state computations including the steady state labor supply elasticity.

Suppose an object, $f(x,y)$, has been defined as a function of the particular arguments, x and y. Suppose that there is another function, $g(z,f)$. The latter is actually a shorthand for $G(z,x,y) = g(z, f(x,y))$. Thus, if g is differentiated with respect to, say, x, then MATLAB delivers dG/dx :

$$\frac{dg}{dx} = G_x(z,x,y) = g_2(z, f(x,y))\, f_x(x,y).$$

Recall that

$$\begin{aligned} h &\equiv Q(m;\tilde{\eta}) \\ m &= Q^{-1}(h;\tilde{\eta}) \end{aligned}$$

where Q^{-1} is the inverse function of Q, defined by:

$$h = Q\left(\underbrace{Q^{-1}(h;\tilde{\eta})}_{m};\tilde{\eta}\right)$$

Note that by differentiating both sides of the latter equation with respect to h we obtain:

$$1 = Q_m Q_h^{-1}$$

Or:

$$Q_h^{-1} = \frac{1}{Q_m}$$

To get the second derivative of the inverse function, Q^{-1} with respect to h we differentiate the previous expression once more:

$$Q_{hh}^{-1} = -\frac{1}{Q_m^2} Q_{mm} Q_h^{-1}$$

Or

$$Q_{hh}^{-1} = -\frac{Q_{mm}}{Q_m^3}$$

The utility function we are interested in, u, is related to U as follows:

$$u(C_t, h_t; C_{t-1}, \tilde{\eta}_t) = U\left(C_t, h_t, Q_t^{-1}; C_{t-1}, \tilde{\eta}_t\right). \tag{A.35}$$

Or more compactly after dropping time subscripts and variables taken as exogenous by the household:

$$u(C,h) = U\left(C, h, Q^{-1}\right).$$

Notice that $\mathring{l}$ has been substituted out in the utility function resulting in the utility function being a function of C, h and m only.

We require the first and second derivatives of u with respect to h :

$$\begin{aligned} u_h(C,h) &= U_h\left(C,h,Q^{-1}\right) + U_m\left(C,h,Q^{-1}\right)Q_h^{-1} \\ &= U_h\left(C,h,Q^{-1}\right) + \frac{U_m\left(C,h,Q^{-1}\right)}{Q_m}. \end{aligned}$$

Or more compactly

$$u_h = U_h + \frac{U_m}{Q_m}.$$

The second derivative with respect to h is:

$$\begin{aligned} u_{hh}(C,h) &= U_{hh}\left(C,h,Q^{-1}\right) + U_{hm}\left(C,h,Q^{-1}\right)Q_h^{-1} + U_{mh}\left(C,h,Q^{-1}\right)Q_h^{-1} \\ &+ U_{mm}\left(C,h,Q^{-1}\right)\left(Q_h^{-1}\right)^2 + U_m\left(C,h,Q^{-1}\right)Q_{hh}^{-1}. \end{aligned}$$

After substituting,

$$\begin{aligned} u_{hh}(C,h) &= U_{hh}\left(C,h,Q^{-1}\right) + 2\frac{U_{hm}\left(C,h,Q^{-1}\right)}{Q_m} \\ &+ \frac{U_{mm}\left(C,h,Q^{-1}\right)}{\left(Q_m\right)^2} - \frac{U_m\left(C,h,Q^{-1}\right)Q_{mm}}{Q_m^3} \end{aligned}$$

Or more compactly

$$u_{hh} = U_{hh} + 2\frac{U_{hm}}{Q_m} + \frac{U_{mm}}{Q_m^2} - \frac{U_m Q_{mm}}{Q_m^3}.$$

Later on, we also require the cross-derivative of u_h with respect to $\tilde{\eta}$. Recall that:

$$u_h(C,h;\tilde{\eta}) = U_h\left(C,h,Q^{-1}(h;\tilde{\eta});\tilde{\eta}\right) + \frac{U_m\left(C,h,Q^{-1}(h;\tilde{\eta});\tilde{\eta}\right)}{Q_m\left(Q^{-1}(h;\tilde{\eta});\tilde{\eta}\right)}$$

Differentiating with respect to $\tilde{\eta}$ gives:

$$\begin{aligned} u_{h\tilde{\eta}}(C,h;\tilde{\eta}) &= U_{h\tilde{\eta}}\left(C,h,Q^{-1}(h;\tilde{\eta});\tilde{\eta}\right) + U_{hm}\left(C,h,Q^{-1}(h;\tilde{\eta});\tilde{\eta}\right)Q_{\tilde{\eta}}^{-1}(h;\tilde{\eta}) \\ &+ \frac{1}{Q_m\left(Q^{-1}(h;\tilde{\eta});\tilde{\eta}\right)}\left[U_{m\tilde{\eta}}\left(C,h,Q^{-1}(h;\tilde{\eta});\tilde{\eta}\right) + U_{mm}\left(C,h,Q^{-1}(h;\tilde{\eta});\tilde{\eta}\right)Q_{\tilde{\eta}}^{-1}(h;\tilde{\eta})\right] \\ &- \frac{U_m\left(C,h,Q^{-1}(h;\tilde{\eta});\tilde{\eta}\right)}{Q_m\left(Q^{-1}(h;\tilde{\eta});\tilde{\eta}\right)^2}\left[Q_{m\tilde{\eta}}\left(Q^{-1}(h;\tilde{\eta});\tilde{\eta}\right) + Q_{mm}Q_{\tilde{\eta}}^{-1}(h;\tilde{\eta})\right] \end{aligned}$$

We require an expression for $Q_{\tilde{\eta}}^{-1}(h;\tilde{\eta})$. Recall that

$$h = Q\left(\underbrace{Q^{-1}(h;\tilde{\eta})}_{m};\tilde{\eta}\right)$$

Differentiating with respect to $\tilde{\eta}$ yields:

$$0 = Q_{\tilde{\eta}} + Q_m Q_{\tilde{\eta}}^{-1}$$

Rewriting gives:

$$Q_{\tilde{\eta}}^{-1} = -\frac{Q_{\tilde{\eta}}}{Q_m}$$

Substituting into the expression for $u_{h\tilde{\eta}}(C, h; \tilde{\eta})$ yields:

$$\begin{aligned} u_{h\tilde{\eta}}(C, h; \tilde{\eta}) &= U_{h\tilde{\eta}}\left(C, h, Q^{-1}(h; \tilde{\eta}); \tilde{\eta}\right) - U_{hm}\left(C, h, Q^{-1}(h; \tilde{\eta}); \tilde{\eta}\right)\frac{Q_{\tilde{\eta}}}{Q_m} \\ &+ \frac{U_{m\tilde{\eta}}(C, h, Q^{-1}(h; \tilde{\eta}); \tilde{\eta}) - U_{mm}(C, h, Q^{-1}(h; \tilde{\eta}); \tilde{\eta})\frac{Q_{\tilde{\eta}}}{Q_m}}{Q_m(Q^{-1}(h; \tilde{\eta}); \tilde{\eta})} \\ &- \frac{U_m(C, h, Q^{-1}(h; \tilde{\eta}); \tilde{\eta})}{Q_m(Q^{-1}(h; \tilde{\eta}); \tilde{\eta})^2}\left[Q_{m\tilde{\eta}}\left(Q^{-1}(h; \tilde{\eta}); \tilde{\eta}\right) - Q_{mm}\frac{Q_{\tilde{\eta}}}{Q_m}\right] \end{aligned}$$

Or more compactly:

$$u_{h\tilde{\eta}} = U_{h\tilde{\eta}} - U_{hm}\frac{Q_{\tilde{\eta}}}{Q_m} + U_{m\tilde{\eta}}\frac{1}{Q_m} - U_{mm}\frac{Q_{\tilde{\eta}}}{Q_m^2} - U_m\frac{Q_{m\tilde{\eta}}}{Q_m^2} + U_m\frac{Q_{mm}Q_{\tilde{\eta}}}{Q_m^3}.$$

B. Integrating Unemployment into a Medium-Sized DSGE Model

We now incorporate our unemployment modelling in a version of the medium-sized DSGE model in CEE or Smets and Wouters (2003, 2007). Below, we describe how to introduce our model of involuntary unemployment into this model. Towards the end of the section we derive the standard model (EHL as interpreted by Galí (2011)) as a special case of our model.

B.1. Final and Intermediate Goods

A final good is produced by a competitive, representative firm using a continuum of inputs as follows:

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{1}{\lambda_f}} di\right]^{\lambda_f},\ 1 \leq \lambda_f < \infty. \tag{B.1}$$

The i^{th} intermediate good is produced by a monopolist with the following production function:

$$Y_{i,t} = (z_t H_{i,t})^{1-\alpha} K_{i,t}^{\alpha} - \phi_t, \tag{B.2}$$

where $K_{i,t}$ denotes capital services used for production by the i^{th} intermediate good producer. Also, $\ln z_t$ is a technology shock whose first difference has a positive mean. ϕ_t denotes a fixed production cost. The economy has two sources of growth: the positive drift in $\ln(z_t)$ and a positive drift in $\ln(\Psi_t)$, where Ψ_t is the state of an investment-specific technology shock discussed below. The object, z_t^+, in (B.2) is defined as follows:

$$z_t^+ = \Psi_t^{\frac{\alpha}{1-\alpha}} z_t.$$

Along a non-stochastic steady state growth path, Y_t/z_t^+ and $Y_{i,t}/z_t^+$ converge to constants. The two shocks, z_t and Ψ_t, are specified to be unit root processes in order to be consistent with the assumptions we use in our VAR analysis to identify the dynamic response of the economy to neutral and capital-embodied technology shocks. The two shocks have the following time series representations:

$$\ln \mu_{z,t} = \ln \mu_z + \sigma_{\mu_z} \varepsilon_{\mu_z,t}/100, \; E\left(\varepsilon_{\mu_z,t}\right)^2 = 1 \tag{B.3}$$

$$\ln \mu_{\Psi,t} = (1-\rho_{\mu_\Psi}) \ln \mu_\Psi + \rho_{\mu_\Psi} \ln \mu_{\Psi,t-1} + \sigma_{\mu_\Psi} \varepsilon_{\mu_\Psi,t}/100, \; E\left(\varepsilon_{\mu_\Psi,t}\right)^2 = 1. \tag{B.4}$$

where $\mu_{z,t} = \frac{z_t}{z_{t-1}}$ and $\mu_{\Psi,t} = \frac{\Psi_t}{\Psi_{t-1}}$. Our assumption that the level of neutral technology follows a random walk matches closely the finding in Smets and Wouters (2007) who estimate $\ln z_t$ to be highly autocorrelated. The direct empirical analysis of Prescott (1986) also supports the notion that $\ln z_t$ is a random walk.

In (B.2), $H_{i,t}$ denotes homogeneous labor services hired by the i^{th} intermediate good producer. Intermediate good firms must borrow the wage bill in advance of production, so that one unit of labor costs is given by $W_t R_t$ where R_t denotes the gross nominal rate of interest. Intermediate good firms are subject to Calvo price-setting frictions. With probability ξ_p the intermediate good firm cannot reoptimize its price, in which case it is assumed to set its price according to the following rule:

$$P_{i,t} = \bar{\pi} P_{i,t-1}, \tag{B.5}$$

where $\bar{\pi}$ is the steady state inflation rate. With probability $1 - \xi_p$ the intermediate good firm can reoptimize its price. Apart from the fixed cost, the i^{th} intermediate good producer's profits are:

$$E_t \sum_{j=0}^{\infty} \beta^j \upsilon_{t+j} \{P_{i,t+j} Y_{i,t+j} - s_{t+j} P_{t+j} Y_{i,t+j}\},$$

where s_t denotes the marginal cost of production, denominated in units of the homogeneous good. s_t is a function only of the costs of capital and labor, and is described in section B.11.1. In the firm's discounted profits, $\beta^j \upsilon_{t+j}$ is the multiplier on the households's nominal period $t+j$ budget constraint. The equilibrium conditions associated with this optimization problem are reported in section B.11.1.

We suppose that the homogeneous labor hired by intermediate good producers is itself 'produced' by competitive labor contractors. Labor contractors produce homogeneous labor by aggregating different types of specialized labor, $j \in (0,1)$, as follows:

$$H_t = \left[\int_0^1 (h_{t,j})^{\frac{1}{\lambda_w}} dj\right]^{\lambda_w}, \; 1 \leq \lambda_w < \infty. \tag{B.6}$$

Labor contractors take the wage rate of H_t and $h_{t,j}$ as given and equal to W_t and $W_{t,j}$, respectively. Profit maximization by labor contractors leads to the following first order necessary condition:

$$W_{j,t} = W_t \left(\frac{H_t}{h_{t,j}} \right)^{\frac{\lambda_w - 1}{\lambda_w}}. \tag{B.7}$$

Equation (B.7) is the demand curve for the j^{th} type of labor.

B.2. Worker and Household Preferences

We integrate the model of unemployment in the previous section into the Erceg, Henderson and Levin (2000) (EHL) model of sticky wages used in the standard DSGE model. Each type, $j \in [0, 1]$, of labor is assumed to be supplied by a particular household. The j^{th} household resembles the single representative household in the previous section, with one exception. The exception is that the unit measure of workers in the j^{th} household is only able to supply the j^{th} type of labor service. Each worker in the j^{th} household has the utility cost of working, (A.1), and the technology for job finding, (A.4). The preference and job finding technology parameters are the same across households.

Let $c_{j,t}^{nw}$ and $c_{j,t}^{w}$ denote the consumption levels allocated by the j^{th} household to non-employed and employed workers within the household. Although households all enjoy the same level of consumption, C_t, for reasons described momentarily each household experiences a different level of employment, $h_{j,t}$. Because employment across households is different, each type j household chooses a different way to balance the trade-off between the need for consumption insurance and the need to provide work incentives. For the j^{th} type of household with high $h_{j,t}$, the premium of consumption for employed workers to non-employed workers must be high. Accordingly, the incentive constraint is given by (A.16) which we repeat here for convenience:

$$\ln \left(\frac{c_{j,t}^{w} - bC_{t-1}}{c_{j,t}^{nw} - bC_{t-1}} \right) = \varsigma \left(1 + \sigma_L\right) m_{j,t}^{\sigma_L} - \frac{2}{a^2} \tilde{\eta}_t$$

where $m_{j,t}$ solves the analog of (A.31):

$$h_{j,t} = -\tilde{\eta}_t m_{j,t} + a^2 \varsigma \sigma_L \left(m_{j,t}^{\sigma_L + 1} - \mathring{l}_{j,t}^{\sigma_L + 1} \right) \tag{B.8}$$

and

$$\mathring{l}_{j,t}^{\sigma_L} = m_{j,t}^{\sigma_L} - \frac{1 + \tilde{\eta}_t}{\varsigma \left(1 + \sigma_L\right) a^2}. \tag{B.9}$$

Consider the j^{th} household that enjoys a level of household consumption and employment, C_t and $h_{j,t}$, respectively. Note that given (A.34), the j^{th} household's discounted utility is given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\ln \left(C_t - bC_{t-1} \right) - z(h_{j,t}; \tilde{\eta}_t) \right]. \tag{B.10}$$

Note that the utility function is additively separable, like the utility functions assumed for the workers. Additive separability is convenient because perfect consumption insurance at the level of households implies that consumption is not indexed by labor type, j.

B.3. Household Problem

The j^{th} household is the monopoly supplier of the j^{th} type of labor service. The household understands that when it arranges work incentives for its workers so that employment is $h_{j,t}$, then $W_{j,t}$ takes on the value implied by the demand for its type of labor, (B.7). The household therefore faces the standard monopoly problem of selecting $W_{j,t}$ to optimize the welfare, (B.10), of its workers. It does so, subject to the requirement that it satisfy the demand for labor, (B.7), in each period. We follow EHL in supposing that the household experiences Calvo-style frictions in its choice of $W_{j,t}$. In particular, with probability $1-\xi_w$ the j^{th} household has the opportunity to reoptimize its wage rate. With the complementary probability, the household must set its wage rate according to the following rule:

$$W_{j,t} = \tilde{\pi}_{w,t} W_{j,t-1} \tag{B.11}$$

$$\tilde{\pi}_{w,t} = (\pi_{t-1})^{\kappa_w} (\bar{\pi})^{(1-\kappa_w)} \mu_{z^+}, \tag{B.12}$$

where $\kappa_w \in (0,1)$. Note that in a non-stochastic steady state, non-optimizing households raise their real wage at the rate of growth of the economy. Because optimizing households also do this in steady state, it follows that in the steady state, the wage of each type of household is the same.

In principle, the presence of wage setting frictions implies that households have idiosyncratic levels of wealth and, hence, consumption. However, we follow EHL in supposing that each household has access to perfect consumption insurance. At the level of the household, there is no private information about consumption or employment. The private information and associated incentive problems all exist among the workers inside a household. Because of the additive separability of the household utility function, perfect consumption insurance at the level of households implies equal consumption across households. We have used this property of the equilibrium to simplify our notation and not include a subscript, j, on the j^{th} households's consumption. Of course, we hasten to add that although consumption is equated across households, it is not constant across households and workers.

The j^{th} household's period t budget constraint is as follows:

$$P_t \left(C_t + \frac{1}{\Psi_t} I_t \right) + B_{t+1} \leq W_{t,j} h_{t,j} + X_t^k \bar{K}_t + R_{t-1} B_t + a_{t,j}. \tag{B.13}$$

Here, B_{t+1} denotes the quantity of risk-free bonds purchased by the household, R_{t-1} denotes the gross nominal interest rate on bonds purchased in period $t-1$ which pay off in period

t, and $a_{t,j}$ denotes the payments and receipts associated with the insurance on the timing of wage reoptimization. Also, P_t denotes the aggregate price level and I_t denotes the quantity of investment goods purchased for augmenting the beginning-of-period $t+1$ stock of physical capital, $\bar{K}_{t+1}$. The price of investment goods is P_t/Ψ_t, where Ψ_t is the unit root process with positive drift specified in (B.4). This is our way of capturing the trend decline in the relative price of investment goods.[49]

The household owns the economy's physical stock of capital, $\bar{K}_t$, sets the utilization rate of capital and rents the services of capital in a competitive market. The household accumulates capital using the following technology:

$$\bar{K}_{t+1} = (1-\delta)\,\bar{K}_t + \left(1 - S\left(\frac{I_t}{I_{t-1}}\right)\right) I_t. \tag{B.14}$$

Here, S is a convex function, with S and S' equal to zero on a steady state growth path. The function, S, is defined in section B.6. The function has one free parameter, its second derivative in the neighborhood of steady state, which we denote simply by S''.

For each unit of $\bar{K}_{t+1}$ acquired in period t, the household receives X_{t+1}^k in net cash payments in period $t+1$,

$$X_{t+1}^k = u_{t+1}^k P_{t+1} r_{t+1}^k - \frac{P_{t+1}}{\Psi_{t+1}} a(u_{t+1}^k). \tag{B.15}$$

where u_t^k denotes the rate of utilization of capital. The first term in (B.15) is the gross nominal period $t+1$ rental income from a unit of $\bar{K}_{t+1}$. The household supply of capital services in period $t+1$ is:

$$K_{t+1} = u_{t+1}^k \bar{K}_{t+1}.$$

It is the services of capital that intermediate good producers rent and use in their production functions, (B.2). The second term to the right of the equality in (B.15) represents the cost of capital utilization, $a(u_{t+1}^k)P_{t+1}/\Psi_{t+1}$. See section B.6 for the functional form of the capital utilization cost function. This function is constructed so the steady state value of utilization is unity, and $u(1) = u'(1) = 0$. The function has one free parameter, which we denote by σ_a. Here, $\sigma_a = a''(1)/a'$ and corresponds to the curvature of u in steady state.

The household's problem is to select sequences, $\left\{C_t, I_t, u_t^k, W_{j,t}, B_{t+1}, \bar{K}_{t+1}\right\}$, to maximize (B.10) subject to (B.7), (B.11), (B.12), (B.13), (B.14), (B.15) and the mechanism determining when wages can be reoptimized. The equilibrium conditions associated with this maximization problem are standard, and appear in section B.11.2.

[49] We suppose that there is an underlying technology for converting final goods, Y_t, one-to-one into C_t and one to Ψ_t into investment goods. These technologies are operated by competitive firms which equate price to marginal cost. The marginal cost of C_t with this technology is P_t and the marginal cost of I_t is P_t/Ψ_t. We avoid a full description of this environment so as to not clutter the presentation, and simply impose these properties of equilibrium on the household budget constraint.

B.4. Aggregate Resource Constraint, Monetary Policy and Equilibrium

Goods market clearing dictates that the homogeneous output good is allocated among alternative uses as follows:

$$Y_t = G_t + C_t + \tilde{I}_t. \tag{B.16}$$

Here, C_t denotes household consumption, G_t denotes exogenous government consumption and $\tilde{I}_t$ is a homogenous investment good which is defined as follows:

$$\tilde{I}_t = \frac{1}{\Psi_t}\left(I_t + a\left(u_t^k\right)\bar{K}_t\right). \tag{B.17}$$

As discussed above, the investment goods, I_t, are used by the households to add to the physical stock of capital, $\bar{K}_t$, according to (B.14). The remaining investment goods are used to cover maintenance costs, $a\left(u_t^k\right)\bar{K}_t$, arising from capital utilization, u_t^k. Finally, Ψ_t in (B.17) denotes the unit root investment specific technology shock with positive drift discussed after (B.2).

We suppose that monetary policy follows a Taylor rule of the following form:

$$\ln\left(\frac{R_t}{R}\right) = \rho_R \ln\left(\frac{R_{t-1}}{R}\right) + (1-\rho_R)\left[r_\pi \ln\left(\frac{\pi_t}{\pi}\right) + r_y \ln\left(\frac{gdp_t}{gdp}\right)\right] + \frac{\sigma_R \varepsilon_{R,t}}{400}, \tag{B.18}$$

where $\varepsilon_{R,t}$ is an iid monetary policy shock. As in CEE and ACEL, we assume that period t realizations of ε_R are not included in the period t information set of households and firms. Further, gdp_t denotes scaled real GDP which is defined as:

$$gdp_t = \frac{G_t + C_t + I_t/\Psi_t}{z_t^+}, \tag{B.19}$$

and gdp denotes the nonstochastic steady state value of gdp_t.

To guarantee balanced growth in the nonstochastic steady state, we require that each element in $[\phi_t, G_t]$ grows at the same rate as z_t^+ in steady state. To this end, we adopt the following specification:

$$[\phi_t, G_t]' = [\phi, G]'\,\Omega_t. \tag{B.20}$$

Here, Ω_t is defined as follows:

$$\Omega_t = \left(z_{t-1}^+\right)^\theta \left(\Omega_{t-1}\right)^{1-\theta}, \tag{B.21}$$

where $0 < \theta \leq 1$ is a parameter to be estimated. With this specification, Ω_t/z_t^+ converges to a constant in nonstochastic steady state. When θ is close to zero, Ω_t is virtually unresponsive in the short-run to an innovation in either of the two technology shocks, a feature that we find attractive on *a priori* grounds. Given the specification of the exogenous processes in the model, $Y_t/z_t^+, C_t/z_t^+$ and $I_t/(\Psi_t z_t^+)$ converge to constants in nonstochastic steady state.

We assume that lump-sum transfers balance the government budget.

An equilibrium is a stochastic process for the prices and quantities having the property that the household and firm problems are satisfied, and goods and labor markets clear.

B.5. Scaling of Variables

We adopt the following scaling of variables. The neutral shock to technology is z_t and its growth rate is $\mu_{z,t}$:

$$\frac{z_t}{z_{t-1}} = \mu_{z,t}.$$

The variable, Ψ_t, is an embodied shock to technology and it is convenient to define the following combination of embodied and neutral technology:

$$\begin{aligned} z_t^+ &\equiv \Psi_t^{\frac{\alpha}{1-\alpha}} z_t, \\ \mu_{z^+,t} &\equiv \mu_{\Psi,t}^{\frac{\alpha}{1-\alpha}} \mu_{z,t}. \end{aligned} \tag{B.22}$$

Capital, $\bar{K}_t$, and investment, I_t, are scaled by $z_t^+ \Psi_t$. Consumption goods C_t, and the real wage, W_t/P_t are scaled by z_t^+. Also, υ_t is the multiplier on the nominal household budget constraint in the Lagrangian version of the household problem. That is, υ_t is the marginal utility of one unit of currency. The marginal utility of a unit of consumption is $\upsilon_t P_t$. The latter must be multiplied by z_t^+ to induce stationarity. Thus, our scaled variables are:

$$\begin{aligned} k_{t+1} &= \frac{K_{t+1}}{z_t^+ \Psi_t},\ \bar{k}_{t+1} = \frac{\bar{K}_{t+1}}{z_t^+ \Psi_t},\ i_t = \frac{I_t}{z_t^+ \Psi_t},\ c_t = \frac{C_t}{z_t^+},\ \bar{w}_t = \frac{W_t}{z_t^+ P_t} \\ \psi_t &= \upsilon_t P_t z_t^+,\ \tilde{y}_t = \frac{Y_t}{z_t^+},\ \tilde{p}_t = \frac{\tilde{P}_t}{P_t},\ w_t = \frac{\tilde{W}_t}{W_t}. \end{aligned}$$

The technology diffusion process (B.21) can be written in scaled form as follows:

$$\begin{aligned} \Omega_t &= \left(z_{t-1}^+\right)^\theta (\Omega_{t-1})^{1-\theta} \\ \frac{\Omega_t}{z_t^+} &= \left(\frac{z_{t-1}^+}{z_t^+}\right)^\theta \left(\frac{\Omega_{t-1}}{z_t^+}\right)^{1-\theta} \\ n_t &= \frac{n_{t-1}^{1-\theta}}{\mu_{z^+,t}} \end{aligned}$$

Government consumption is scaled as follows:

$$\frac{G_t}{z_t^+} = \frac{G_t}{\Omega_t} \frac{\Omega_t}{z_t^+} = G \times n_t$$

We define the scaled date t price of new installed physical capital for the start of period $t+1$ as $p_{k',t}$ and we define the scaled real rental rate of capital as $\bar{r}_t^k$:

$$p_{k',t} = \Psi_t P_{k',t},\ \bar{r}_t^k = \Psi_t r_t^k.$$

where $P_{k',t}$ is in units of the homogeneous good. We define the following inflation rates:

$$\pi_t = \frac{P_t}{P_{t-1}},\ \pi_t^i = \frac{P_t^i}{P_{t-1}^i}.$$

Here, P_t is the price of the homogeneous output good and P_t^i is the price of the domestic final investment good.

B.6. Functional Forms

We adopt the following functional form for the capacity utilization cost function a :

$$a(u_t^K) = \sigma_a \sigma_b (u_t^K)^2/2 + \sigma_b (1 - \sigma_a) u_t^K + \sigma_b (\sigma_a/2 - 1), \tag{B.23}$$

where σ_a and σ_b are the parameters of this function. For a given value of σ_a we select σ_b so that the steady state value of u_t^K is unity. The object, σ_a, is a parameter to be estimated.

We assume that the investment adjustment cost function takes the following form:

$$S(I_t/I_{t-1}) = \frac{1}{2}\left\{\exp\left[\sqrt{S''}(I_t/I_{t-1} - \mu_{z^+}\mu_\Psi)\right] + \exp\left[-\sqrt{S''}(I_t/I_{t-1} - \mu_{z^+}\mu_\Psi)\right] - 2\right\}. \tag{B.24}$$

Here, μ_{z^+} and μ_Ψ denote the unconditional growth rates of z_t^+ and Ψ_t. The value of I_t/I_{t-1} in nonstochastic steady state is $(\mu_{z^+} \times \mu_\Psi)$. In addition, S'' denotes the second derivative of $S(\cdot)$, evaluated at steady state. The object, S'', is a parameter to be estimated. It is straightforward to verify that $S(\mu_{z^+}\mu_\Psi) = S'(\mu_{z^+}\mu_\Psi) = 0$.

Finally, we assume the following functional form for the impact of aggregate economic conditions on the worker's probability to find a job:

$$\mathcal{M}(\bar{m}_t/\bar{m}_{t-1}) = 100\omega(\bar{m}_t/\bar{m}_{t-1} - 1).$$

In the estimation we adopt a standard normal prior for ω. That is, we are agnostic about the sign of ω. A posteriori it turns out that the data want $\omega < 0$. Recall that $\tilde{\eta}_t = \eta + \mathcal{M}(\bar{m}_t/\bar{m}_{t-1})$ and $p(e_{l,t}; \tilde{\eta}_t) = \tilde{\eta}_t + ae_{l,t}$. That is, $\omega < 0$ implies that an inflow of workers into the labor force reduces the probability of a worker to find a job. Importantly, it is the rate of change of the labor force that triggers the probability of a worker to fall. Intuitively, one might think about this as a bottleneck-type access to the labor market. When the labor force grows rapidly, many workers get 'stuck' in the process to find work. According to our specification, it is not the level of the labor force but its rate of change that affects the probability of a worker to find a job. Finally, note that $\mathcal{M}$ does not affect the steady state of our model.

Why does the data prefer $\omega < 0$? Consider the h-m relationship:

$$h_t = -\tilde{\eta}_t m_t + a^2 \varsigma \sigma_L \left(m_t^{\sigma_L+1} - \mathring{l}_t^{\sigma_L+1}\right).$$

The presence of $\omega < 0$ generates a procyclical wedge on the right hand side of the h-m relationship. Recall that η is negative. In a boom, the labor force grows so that with $\omega < 0$, $\tilde{\eta}_t$ becomes more negative. As a result, $-\tilde{\eta}_t$ in the h-m relationship increases which generates the procyclical wedge. The data want this procyclical wedge as the model tends to otherwise overstate quantitatively the raise in the labor force after e.g. a monetary policy shock. In other words, the procyclical wedge allows the model to generate a smaller expansion in the labor force dictated by the data. In addition, the dependence of $\mathcal{M}\left(\bar{m}_t/\bar{m}_{t-1}\right)$ on the lagged aggregate labor force also allows the model to generate the protracted and very delayed hump in the labor force after a monetary policy shock.

We hasten to emphasize that while the inflow of workers into the labor force in a boom decreases the individual worker's probability of finding work, in a boom workers also increase their effort. Our estimated model shows that on net, the probability to find work goes up in a boom, i.e. the individual work effort channel dominates the aggregate labor force channel in the determination of the probability of finding a job for the worker.

Making $p(e_{l,t};\tilde{\eta}_t)$ dependent on aggregate conditions in addition to individual worker effort is attractive to us on a priori grounds. While the dependence of $\tilde{\eta}_t$ on the change in the labor force may appear ad-hoc, it shares in spirit the many features that are adopted in medium-sized NK DSGE models to slow down the responses of variables such as investment adjustment cost, capacity adjustment cost, habit formation etc. We leave providing a possible microfoundation for $\mathcal{M}\left(\bar{m}_t/\bar{m}_{t-1}\right)$ to future research.

We have also experimented with alternative specifications for $\mathcal{M}$. For example, we have estimated the model under the assumption that $\mathcal{M}\left(\bar{m}_t;\bar{m}\right) = 100\omega\left(\bar{m}_t/\bar{m}-1\right)$. This specification also allows the model to match the VAR response of the labor force quantitatively well. The specification, however, cannot generate the very delayed hump in the labor force after a monetary policy shock as suggested by the VAR evidence.

Finally, note that up to a first order approximation of the model, making η, a or ς a function of the procyclical wedge is observationally equivalent. In experiments we also verified that the primary quantitative impact of $\tilde{\eta}_t$ in the model occurs in the h-m relationship. That is, the quantitative impact of $\tilde{\eta}_t$ in equation (B.9) that determines $\mathring{l}$ or the wage Phillips curve is quite small.

B.7. Aggregate Hours Worked

We will estimate the log-linearized model. Our assumptions imply that the steady state is undistorted by wage frictions, i.e. we have

$$\hat{h}_t = \hat{H}_t.$$

where $\hat{h}_t$ denotes household hours and $\hat{H}_t$ denotes aggregate homogenous hours (both in log deviations from steady state). Although this is a well known result (see, e.g., Yun (1996)), we derive it here for completeness. Recall,

$$h_t \equiv \int_0^1 h_{j,t}dj.$$

Invert the demand for labor, (B.7), to obtain an expression in terms of $h_{j,t}$. Substitute this into the expression for h_t to obtain:

$$h_t = H_t \int_0^1 \mathring{w}_{j,t}^{\frac{\lambda_w}{1-\lambda_w}} dj, \tag{B.25}$$

where

$$\mathring{w}_{j,t} \equiv \frac{W_{j,t}}{W_t}.$$

Here, W_t denotes the aggregate wage rate, which one obtains by substituting (B.6) into (B.7):

$$W_t = \left[\int_0^1 W_{j,t}^{\frac{1}{1-\lambda_w}} dj\right]^{1-\lambda_w}.$$

Because all households are identical in steady state (see the discussion after (B.11)), $\mathring{w}_j = 1$ for all j. Totally differentiating (B.25),

$$\hat{h}_t = \hat{H}_t + \int_0^1 \widehat{\mathring{w}}_{j,t} dj.$$

Thus, to determine the percent deviation of aggregate employment from steady state, we require the integral of the percent deviations of type j wages from the aggregate wage, over all j. We now show that this integral is, to first order, equal to zero.

Express the integral in (B.25) as follows:

$$h_t = \mathring{w}_t^{\frac{\lambda_w}{1-\lambda_w}} H_t,$$

say, where

$$\mathring{w}_t \equiv \left[\int_0^1 \mathring{w}_{j,t}^{\frac{\lambda_w}{1-\lambda_w}} dj\right]^{\frac{1-\lambda_w}{\lambda_w}}. \tag{B.26}$$

Pursuing logic that is standard in the Calvo price/wage setting literature we obtain:

$$W_t = \left[(1-\xi_w)\left(\tilde{W}_t\right)^{\frac{1}{1-\lambda_w}} + \xi_w \left(\tilde{\pi}_{w,t} W_{t-1}\right)^{\frac{1}{1-\lambda_w}}\right]^{1-\lambda_w} \tag{B.27}$$

$$\mathring{w}_t = \left[(1-\xi_w) w_t^{\frac{\lambda_w}{1-\lambda_w}} + \xi_w \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} \mathring{w}_{t-1}\right)^{\frac{\lambda_w}{1-\lambda_w}}\right]^{\frac{1-\lambda_w}{\lambda_w}}, \tag{B.28}$$

where:

$$w_t \equiv \frac{\tilde{W}_t}{W_t},\ \ \pi_{w,t} \equiv \frac{W_t}{W_{t-1}},$$

and $\tilde{W}_t$ denotes the wage set by the $1-\xi_w$ households that have the opportunity to reoptimize in the current period. Because all households are identical in steady state

$$w = \mathring{w} = \frac{\tilde{\pi}_w}{\pi_w} = 1, \tag{B.29}$$

where $\tilde{\pi}_{w,t}$ is defined in (B.11) and $\pi_{w,t}$ denotes wage inflation:

$$\pi_{w,t} \equiv \frac{W_t}{W_{t-1}}.$$

Dividing (B.27) by W_t and solving,

$$w_t = \left[\frac{1 - \xi_w \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} \right)^{\frac{1}{1-\lambda_w}}}{1 - \xi_w} \right]^{1-\lambda_w}. \tag{B.30}$$

Differentiating (B.28) and (B.30) in steady state:

$$\begin{aligned} \widehat{\mathring{w}}_t &= (1 - \xi_w)\, \hat{w}_t + \xi_w \left(\widehat{\tilde{\pi}}_{w,t} - \hat{\pi}_{w,t} + \widehat{\mathring{w}}_{t-1} \right) \qquad \text{(B.31)} \\ \hat{w}_t &= -\frac{\xi_w}{1 - \xi_w} \left(\widehat{\tilde{\pi}}_{w,t} - \hat{\pi}_{w,t} \right) \end{aligned}$$

Using the latter to substitute out for $\hat{w}_t$ in (B.31):

$$\widehat{\mathring{w}}_t = \xi_w \widehat{\mathring{w}}_{t-1}.$$

Thus, to first order the wage distortions evolve according to a stable first order difference equation, unperturbed by shocks. For this reason, we set

$$\widehat{\mathring{w}}_t = 0, \tag{B.32}$$

for all t.

Totally differentiating (B.26) and using (B.29), (B.32):

$$\int_0^1 \widehat{\mathring{w}}_{j,t} dj = 0.$$

That is, to first order, the integral of the percent deviations of individual wages from the aggregate is zero.

B.8. Aggregate Labor Force and Unemployment in Our Model

We now derive our model's implications for unemployment and the labor market. At the level of the j^{th} household, unemployment and the labor force are defined in the same way as in the previous section, except that the endogenous variables now have a j subscript (the parameters and shocks are the same across households). Thus, the j^{th} households's labor force, $m_{j,t}$, and total employment, $h_{j,t}$, are related by (B.8) and (B.9) which we repeat here for convenience:

$$
\begin{aligned}
h_{j,t} &= -\tilde{\eta}_t m_{j,t} + a^2 \varsigma \sigma_L \left(m_{j,t}^{\sigma_L+1} - \mathring{l}_{j,t}^{\sigma_L+1} \right) \\
\mathring{l}_{j,t}^{\sigma_L} &= m_{j,t}^{\sigma_L} - \frac{1+\tilde{\eta}_t}{\varsigma\left(1+\sigma_L\right) a^2} \\
\tilde{\eta}_t &= \eta + \mathcal{M}(\bar{m}_t/\bar{m}_{t-1})
\end{aligned}
$$

Log-linearizing gives:

$$
\begin{aligned}
h\hat{h}_{j,t} &= -\tilde{\eta} m \left(\widehat{\tilde{\eta}}_t + \hat{m}_{j,t} \right) + \left(\sigma_L+1\right) a^2 \varsigma \sigma_L \left(m^{\sigma_L+1} \hat{m}_{j,t} - \mathring{l}^{\sigma_L+1} \widehat{\mathring{l}}_{j,t} \right) \qquad \text{(B.33)} \\
\sigma_L \mathring{l}^{\sigma_L} \widehat{\mathring{l}_{j,t}} &= \sigma_L m^{\sigma_L} \hat{m}_{j,t} - \frac{\tilde{\eta}}{\varsigma\left(1+\sigma_L\right) a^2} \widehat{\tilde{\eta}}_t
\end{aligned}
$$

Variables without subscript denote steady state values in the j^{th} household. Because we have made assumptions which guarantee that each household is identical in steady state, we drop the j subscripts from all steady state labor market variables (see the discussion after (B.11)).

Aggregate household hours and the labor force are defined as follows:

$$
h_t \equiv \int_0^1 h_{j,t} dj, \ \bar{m}_t = m_t \equiv \int_0^1 m_{j,t} dj, \ \mathring{l}_t \equiv \int_0^1 \mathring{l}_{j,t} dj.
$$

Totally differentiating,

$$
\hat{h}_t = \int_0^1 \hat{h}_{j,t} dj, \ \hat{m}_t \equiv \int_0^1 \hat{m}_{j,t} dj, \ \widehat{\mathring{l}}_t \equiv \int_0^1 \widehat{\mathring{l}_{j,t}} dj.
$$

Using the fact that, to first order, type j wage deviations from the aggregate wage cancel, we obtain:

$$
\hat{h}_t = \hat{H}_t. \qquad \text{(B.34)}
$$

See section B.7 for a derivation. That is, to a first order approximation, the percent deviation of aggregate household hours from steady state coincides with the percent deviation of aggregate homogeneous hours from steady state. Integrating (B.33) over all j :

$$
\begin{aligned}
h\hat{h}_t &= -\tilde{\eta} m \left(\widehat{\tilde{\eta}}_t + \hat{m}_t \right) + \left(\sigma_L+1\right) a^2 \varsigma \sigma_L \left(m^{\sigma_L+1} \hat{m}_t - \mathring{l}^{\sigma_L+1} \widehat{\mathring{l}}_t \right) \\
\sigma_L \mathring{l}^{\sigma_L} \widehat{\mathring{l}}_t &= \sigma_L m^{\sigma_L} \hat{m}_t - \frac{\tilde{\eta}}{\varsigma\left(1+\sigma_L\right) a^2} \widehat{\tilde{\eta}}_t.
\end{aligned}
$$

Which after substituting $\widehat{\mathring{l}}_t$ and simplifications can be written as:

$$
h\hat{h}_t = \underbrace{\left(-\tilde{\eta} m + \left(\sigma_L+1\right) a^2 \varsigma \sigma_L \left(m - \mathring{l} \right) m^{\sigma_L} \right)}_{>0} \hat{m}_t \underbrace{- \tilde{\eta} \left[m - \mathring{l} \right]}_{>0} \widehat{\tilde{\eta}}_t.
$$

where $\widehat{\tilde{\eta}}_t = \frac{\tilde{\eta}_t - \tilde{\eta}}{\tilde{\eta}}$. Aggregate unemployment is defined as follows:

$$u_t \equiv \frac{m_t - h_t}{m_t}$$

so that

$$du_t = \frac{h}{m}\left(\hat{m}_t - \hat{h}_t\right).$$

Here, du_t denotes the deviation of unemployment from its steady state value, not the percent deviation.

B.9. The Standard Model

We derive the utility function used in the standard model as a special case of the household utility function in our involuntary unemployment model. In part, we do this to ensure consistency across models. In part, we do this as a way of emphasizing that we interpret the labor input in the utility function in the standard model as corresponding to the number of people working, not, say, the hours worked of a representative person. With our interpretation, the curvature of the labor disutility function corresponds to the (consumption compensated) elasticity with which people enter or leave the labor force in response to a change in the wage rate. In particular, this curvature does not correspond to the elasticity with which the typical person adjusts the quantity of hours worked in response to a wage change. Empirically, the latter elasticity is estimated to be small and it is fixed at zero in the model.

Another advantage of deriving the standard model from ours is that it puts us in position to exploit an insight by Galí (2010). In particular, Galí (2010) shows that the standard model already has a theory of unemployment implicit in it. The monopoly power assumed by EHL has the consequence that wages are on average higher than what they would be under competition. The number of workers for which the wage is greater than the cost of work exceeds the number of people employed. Galí suggests defining this excess of workers as 'unemployed'. The implied unemployment rate and labor force represent a natural benchmark to compare with our model.

Notably, deriving an unemployment rate and labor force in the standard model does not introduce any new parameters. Moreover, there is no change in the equilibrium conditions that determine non-labor market variables. Galí's insight in effect simply adds a block recursive system of two equations to the standard DSGE model which determine the size of the labor force and unemployment. Although the unemployment rate derived in this way does not satisfy all the criteria for unemployment that we described in the introduction, it nevertheless provides a natural benchmark for comparison with our model. An extensive comparison of the economics of our approach to unemployment versus the approach implicit in the standard model appears in the appendix of the paper.

We suppose that the household has full information about its workers and that workers which join the labor force automatically receive a job without having to exert any effort. As in the previous subsections, we suppose that corresponding to each type j of labor, there is a unit measure of workers which gather together into a household. At the beginning of each period, each worker draws a random variable, l, from a uniform distribution with support, $[0,1]$. The random variable, l, determines a workers's aversion to work according to (A.1). Workers with $l \leq h_{t,j}$ work and workers with $h_{t,j} \leq l \leq 1$ take leisure. The type j household allocation problem is to maximize the utility of its workers with respect to consumption for non-employed workers, $c_{t,j}^{nw}$, and consumption of employed workers, $c_{t,j}^{w}$, subject to (A.17), and the given values of $h_{t,j}$ and C_t. In Lagrangian form, the problem is:

$$u\left(C_t - bC_{t-1}, h_{j,t}\right) = \max_{c_{t,j}^{w}, c_{t,j}^{nw}} \int_0^{h_{t,j}} \left[\ln\left(c_{t,j}^{w} - bC_{t-1}\right) - \varsigma\left(1+\sigma_L\right) l^{\sigma_L}\right] dl$$

$$+ \int_{h_{t,j}}^{1} \ln\left(c_{t,j}^{nw} - bC_{t-1}\right) dl + \lambda_{j,t}\left[C_t - h_{t,j}c_{t,j}^{w} - \left(1-h_{t,j}\right)c_{t,j}^{nw}\right].$$

Here, $\lambda_{j,t} > 0$ denotes the multiplier on the resource constraint. The first order conditions imply $c_{t,j}^{w} = c_{t,j}^{nw} = C_t$. Imposing this result and evaluating the integral, we find:

$$u\left(C_t - bC_{t-1}, h_{j,t}\right) = \ln\left(C_t - bC_{t-1}\right) - \varsigma h_{t,j}^{1+\sigma_L}. \tag{B.35}$$

The problem of the household is identical to what it is in section B.3, with the sole exception that the utility function, (A.34), is replaced by (B.35).

A type j worker that draws work aversion index l is defined to be unemployed if the following two conditions are satisfied:

$$(a)\ l > h_{j,t},\ \ (b)\ \upsilon_t W_{j,t} > \varsigma\left(1+\sigma_L\right) l^{\sigma_L}. \tag{B.36}$$

Here, υ_t denotes the multiplier on the budget constraint, (B.13), in the Lagrangian representation of the household optimization problem. Expression (a) in (B.36) simply says that to be unemployed, the worker must not be employed. Expression (b) in (B.36) determines whether a non-employed worker is unemployed or not in the labor force. The object on the left of the inequality in (b) is the value assigned by the household to the wage, $W_{j,t}$. The object on the right of (b) is the fixed cost of going to work for the l^{th} worker. Galí (2010) suggests defining workers with l satisfying (B.36) as unemployed. This approach to unemployment does not satisfy properties (i) and (iii) in the introduction. The approach does not meet the official definition of unemployment because no one is exercising effort to find a job. In addition, the existence of perfect consumption insurance implies that unemployed workers enjoy higher utility than employed workers.

We use (B.36) to define the labor force, m_t, in the standard model. With m_t and aggregate employment, h_t, we obtain the unemployment rate as follows

$$u_t = \frac{m_t - h_t}{m_t},$$

or, after linearization about steady state:

$$du_t = \frac{h}{m}\left(\hat{m}_t - \hat{h}_t\right).$$

Here, $h < m$ because of the presence of monopoly power. The object, $\hat{h}_t$ may be obtained from (B.34) and the solution to the standard model. We now discuss the computation of the aggregate labor force, m_t. We have

$$m_t \equiv \int_0^1 m_{j,t} dj,$$

where $m_{j,t}$ is the labor force associated with the j^{th} type of labor and is defined by enforcing (b) in (B.36) at equality. After linearization,

$$\hat{m}_t \equiv \int_0^1 \hat{m}_{j,t} dj.$$

We compute $\hat{m}_{j,t}$ by linearizing the equation that defines $\hat{m}_{j,t}$. After scaling that equation, we obtain

$$\psi_t \bar{w}_t \mathring{w}_{j,t} = \varsigma \left(1 + \sigma_L\right) m_{j,t}^{\sigma_L}, \tag{B.37}$$

where

$$\psi_t \equiv \upsilon_t P_t z_t^+, \;\; \bar{w}_t \equiv \frac{W_t}{z_t^+ P_t}, \;\; \mathring{w}_{j,t} \equiv \frac{W_{j,t}}{W_t}.$$

Log-linearizing (B.37) about steady state and integrating the result over all $j \in (0, 1)$:

$$\hat{\psi}_t + \widehat{\bar{w}}_t + \int_0^1 \widehat{\mathring{w}}_{j,t} dj = \sigma_L \hat{m}_t.$$

From the result in section B.7, the integral in the above expression is zero, so that:

$$\hat{m}_t = \frac{\hat{\psi}_t + \widehat{\bar{w}}_t}{\sigma_L}.$$

B.10. Wage Setting by the Household

We consider the problem of a monopolist who represents households that supply the type j labor service. That monopolist optimizes the utility function of $j-$type households, (A.34) in case of our involuntary unemployment model or in (B.35) in case of the standard model, subject to Calvo frictions. With probability $1-\xi_w$ the monopolist reoptimizes the wage and with probability ξ_w the monopolist sets the current wage rate according to (B.11). In each period, type j households supply the quantity of labor dictated by demand, (B.7). Because the $j-$type household has perfect consumption insurance, the monopolist can take the $j-$type household's consumption as given. However, the monopolist does assign a weight to the revenues from $j-$type labor that corresponds to the value, υ_t, assigned to income by the household. Ignoring terms beyond the control of the monopolist the monopolist seeks to maximize:

$$E_t^j \sum_{i=0}^{\infty} \beta^i \left[-z\left(h_{t+i,j};\tilde{\eta}_{t+i}\right) + \upsilon_{t+i} W_{t+i,j} h_{t+i,j} \right].$$

Here, υ_t denotes the Lagrange multiplier on the type j household's time t flow budget constraint, (B.13). The function, z, is defined in (A.34) for our involuntary unemployment model or in (B.35) for the standard model (with the understanding that the object $\tilde{\eta}$ does not exist in the standard model).

Consider the monopoly wage setter, j, that has an opportunity to reoptimize the wage rate. The objective function with $h_{t+i,j}$ substituted out using labor demand, (B.7), and ignoring terms beyond the control of the monopolist, is as follows:

$$E_t \sum_{i=0}^{\infty} (\beta\xi_w)^i [-z\left(\left(\frac{\tilde{W}_t \tilde{\pi}_{w,t+i}\cdots\tilde{\pi}_{w,t+1}}{W_{t+i}}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i};\tilde{\eta}_{t+i}\right)$$

$$+\upsilon_{t+i}\tilde{W}_t\tilde{\pi}_{w,t+i}\cdots\tilde{\pi}_{w,t+1}\left(\frac{\tilde{W}_t \tilde{\pi}_{w,t+i}\cdots\tilde{\pi}_{w,t+1}}{W_{t+i}}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}],$$

where

$$\tilde{W}_t\tilde{\pi}_{w,t+i}\cdots\tilde{\pi}_{w,t+1}$$

is the nominal wage rate of the monopolist which sets wage $\tilde{W}_t$ in period t and cannot reoptimize again afterward. We adopt the following scaling convention:

$$w_t = \frac{\tilde{W}_t}{W_t},\ \bar{w}_t = \frac{W_t}{z_t^+ P_t},\ \psi_t = \upsilon_t P_t z_t^+.$$

With this notation, the objective can be written,

$$E_t \sum_{i=0}^{\infty} (\beta\xi_w)^i [-z\left(\left(\frac{w_t\bar{w}_t}{\bar{w}_{t+i}}X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i};\tilde{\eta}_{t+i}\right) + \psi_{t+i} w_t^{\frac{1}{1-\lambda_w}}\bar{w}_t X_{t,i}\left(\frac{\bar{w}_t}{\bar{w}_{t+i}}X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}],$$

where:

$$X_{t,i} = \frac{\tilde{\pi}_{w,t+i}\cdots\tilde{\pi}_{w,t+1}}{\pi_{t+i}\pi_{t+i-1}\cdots\pi_{t+1}\mu_{z^+,t+i}\cdots\mu_{z^+,t+1}}.$$

Differentiating with respect to w_t,

$$E_t \sum_{i=0}^{\infty} (\beta\xi_w)^i \, [-z_{h,t+i}^t \frac{\lambda_w}{1-\lambda_w} w_t^{\frac{\lambda_w}{1-\lambda_w}-1} \left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}$$

$$+ \frac{1}{1-\lambda_w} \psi_{t+i} w_t^{\frac{1}{1-\lambda_w}-1} \bar{w}_t X_{t,i} \left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}],$$

where

$$z_{h,t+i}^t \equiv z_h \left(\left(\frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}; \tilde{\eta}_{t+i} \right).$$

Here, $z_{h,t+i}^t$ denotes the marginal utility of labor in period $t+i$, for a monopolist who last reoptimized the wage rate in period t. Note that in steady state we get the standard condition equating the (marked up) marginal rate of substitution to real wage:

$$\lambda_w \frac{z_h}{\psi} = \bar{w}.$$

Dividing and rearranging the above first order condition gives,

$$E_t \sum_{i=0}^{\infty} (\beta\xi_w)^i \left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} [\psi_{t+i} w_t \bar{w}_t X_{t,i} - \lambda_w z_{h,t+i}^t] = 0. \tag{B.38}$$

The first object in square brackets is the marginal utility real wage in period $t+i$ and the second is a markup, λ_w, over the marginal utility cost of working. According to (B.38) the monopolist attempts to set a weighted average of the term in square brackets to zero. The structure of $z_{z,t+i}^t$ makes it difficult to express (B.38) in recursive form. This is because we have not found a way to express $z_{h,t+1}^t = Z_t z_{h,t+1}^{t+1}$, for some variable, Z_t. The expression, (B.38), is recursive after linearizing it about steady state. Thus,

$$\hat{z}_{h,t+i}^t \equiv \frac{dz_h \left(\left(\frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}; \tilde{\eta}_{t+i} \right)}{z_h \left(w^{\frac{\lambda_w}{1-\lambda_w}} H; \tilde{\eta} \right)},$$

where a variable without a time subscript denotes non-stochastic steady state. Expanding this expression:

$$\hat{z}_{h,t+i}^t = \sigma_{\tilde{\eta}} \widehat{\tilde{\eta}}_{t+i} + \alpha_{h,1} \left(\hat{w}_t + \widehat{\bar{w}}_t - \widehat{\bar{w}}_{t+i} + \hat{X}_{t,i} \right) + \sigma_z \hat{H}_{t+i},$$

where

$$\alpha_{h,1} \equiv \frac{\lambda_w}{1-\lambda_w} \sigma_z.$$

For the involuntary unemployment model we have:

$$\sigma_z \equiv \frac{z_{hh} H}{z_h}, \;\; \sigma_{\tilde{\eta}} \equiv \frac{z_{h\tilde{\eta}} \tilde{\eta}}{z_h}$$

where the partial derivatives of the z function can be obtained from observing that

$$z_h = -u_h,\ z_{hh} = -u_{hh},\ z_{h\tilde{\eta}} = -u_{h\tilde{\eta}}$$

and the derivatives of the utility function are provided in section A.9.2.

For the standard model, we have:

$$\begin{aligned} z_h &= (1+\sigma_L)\,\varsigma H^{\sigma_L} \\ z_{hh} &= \sigma_L\,(1+\sigma_L)\,\varsigma H^{\sigma_L - 1} \end{aligned}$$

So that

$$\sigma_z \equiv \frac{z_{hh}H}{z_h} = \sigma_L, \sigma_{\tilde{\eta}} \equiv 0.$$

Also,

$$\hat{X}_{t,i} = \hat{\tilde{\bar{\pi}}}_{w,t+i} + \cdots + \hat{\tilde{\bar{\pi}}}_{w,t+1} - \hat{\pi}_{t+i} - \hat{\pi}_{t+i-1} - \cdots - \hat{\pi}_{t+1} - \hat{\mu}_{z^+,t+i} - \cdots - \hat{\mu}_{z^+,t+1}.$$

However, note:

$$\hat{\tilde{\bar{\pi}}}_{w,t+1} = \kappa_w \hat{\pi}_t.$$

Then,

$$\hat{X}_{t,i} = -\Delta_{\kappa_w}\hat{\pi}_{t+i} - \Delta_{\kappa_w}\hat{\pi}_{t+i-1} - \cdots - \Delta_{\kappa_w}\hat{\pi}_{t+1} - \hat{\mu}_{z^+,t+i} - \cdots - \hat{\mu}_{z^+,t+1},$$

where

$$\Delta_{\kappa_w} \equiv 1 - \kappa_w L,$$

where L denotes the lag operator.

Write out (B.38) in detail:

$$\begin{aligned} &H_t[\psi_t w_t \bar{w}_t - \lambda_w z_{h,t}^t] \\ &+\beta\xi_w \left(\frac{\bar{w}_t}{\bar{w}_{t+1}} X_{t,1}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+1}[\psi_{t+1} w_t \bar{w}_t X_{t,1} - \lambda_w z_{h,t+1}^t] \\ &+(\beta\xi_w)^2 \left(\frac{\bar{w}_t}{\bar{w}_{t+2}} X_{t,2}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+2}[\psi_{t+2} w_t \bar{w}_t X_{t,2} - \lambda_w z_{h,t+2}^t] + ... = 0 \end{aligned}$$

In expanding this expression, we can simply set the terms outside the square brackets to their steady state values. The reason is that the term inside the brackets are equal to zero in steady state. Thus, the expansion of the previous expression about steady state:

$$\begin{aligned} &H[d\left(\psi_t w_t \bar{w}_t\right) - \lambda_w d\left(z_{h,t}^t\right)] \\ &+\beta\xi_w H[d\left(\psi_{t+1} w_t \bar{w}_t X_{t,1}\right) - \lambda_w d\left(z_{h,t+1}^t\right)] \\ &+(\beta\xi_w)^2 H[d\left(\psi_{t+2} w_t \bar{w}_t X_{t,2}\right) - \lambda_w d\left(z_{h,t+2}^t\right)] + ... = 0 \end{aligned}$$

or,

$$
\begin{aligned}
&H[\psi_{z^+}\bar{w}\left(\hat{\psi}_t+\hat{w}_t+\widehat{\overline{w}}_t\right)-\lambda_w z_h\hat{z}^t_{h,t}] \\
&+\beta\xi_w H[\psi_{z^+}\bar{w}\left(\hat{\psi}_{t+1}+\hat{w}_t+\widehat{\overline{w}}_t+\hat{X}_{t,1}\right)-\lambda_w z_h\hat{z}^t_{h,t+1}] \\
&+\left(\beta\xi_w\right)^2 H[\psi_{z^+}\bar{w}\left(\hat{\psi}_{t+2}+\hat{w}_t+\widehat{\overline{w}}_t+\hat{X}_{t,2}\right)-\lambda_w z_h\hat{z}^t_{h,t+2}]+...=0
\end{aligned}
$$

Note that in steady state, $\psi\bar{w}=\lambda_w z_h$, so that, after multiplying by $1/\left(H\psi\bar{w}\right)$, we obtain:

$$
\begin{aligned}
&\hat{\psi}_t+\hat{w}_t+\widehat{\overline{w}}_t-\hat{z}^t_{h,t} \\
&+\beta\xi_w[\hat{\psi}_{t+1}+\hat{w}_t+\widehat{\overline{w}}_t+\hat{X}_{t,1}-\hat{z}^t_{h,t+1}] \\
&+\left(\beta\xi_w\right)^2[\hat{\psi}_{t+2}+\hat{w}_t+\widehat{\overline{w}}_t+\hat{X}_{t,2}-\hat{z}^t_{h,t+2}]+...=0
\end{aligned}
$$

Substitute out for $\hat{z}^t_{h,t+i}$ and $\hat{X}_{t,i}$:

$$
\begin{aligned}
0 \;=\;& \hat{\psi}_t+\hat{w}_t+\widehat{\overline{w}}_t-\left[\sigma_{\tilde{\eta}}\widehat{\tilde{\eta}}_t+\alpha_{h,1}\hat{w}_t+\sigma_z\hat{H}_t\right] \\
& \\
&+\beta\xi_w[\hat{\psi}_{t+1}+\hat{w}_t+\widehat{\overline{w}}_t-\left(\Delta_{\kappa_w}\hat{\pi}_{t+1}+\hat{\mu}_{z^+,t+1}\right) \\
&-\left(\sigma_{\tilde{\eta}}\widehat{\tilde{\eta}}_{t+1}+\alpha_{h,1}\left(\hat{w}_t+\widehat{\overline{w}}_t-\widehat{\overline{w}}_{t+1}-\left(\Delta_{\kappa_w}\hat{\pi}_{t+1}+\hat{\mu}_{z^+,t+1}\right)\right)+\sigma_z\hat{H}_{t+1}\right)] \\
& \\
&+\left(\beta\xi_w\right)^2[\hat{\psi}_{t+2}+\hat{w}_t+\widehat{\overline{w}}_t-\left(\Delta_{\kappa_w}\hat{\pi}_{t+2}+\hat{\mu}_{z^+,t+2}\right)-\left(\Delta_{\kappa_w}\hat{\pi}_{t+1}+\hat{\mu}_{z^+,t+1}\right) \\
&-\left(\sigma_{\tilde{\eta}}\widehat{\tilde{\eta}}_{t+2}+\alpha_{h,1}\left(\begin{array}{c}\hat{w}_t+\widehat{\overline{w}}_t-\widehat{\overline{w}}_{t+2}\\ -\left(\Delta_{\kappa_w}\hat{\pi}_{t+2}+\hat{\mu}_{z^+,t+2}\right)-\left(\Delta_{\kappa_w}\hat{\pi}_{t+1}+\hat{\mu}_{z^+,t+1}\right)\end{array}\right)+\sigma_z\hat{H}_{t+2}\right)]+...
\end{aligned}
$$

Collecting terms:

$$
\begin{aligned}
0 \;=\;& \sum_{j=0}^{\infty}\left(\beta\xi_w\right)^j\left[\hat{\psi}_{t+j}-\left(\sigma_{\tilde{\eta}}\widehat{\tilde{\eta}}_{t+j}+\sigma_z\hat{H}_{t+j}\right)\right]+\frac{1-\alpha_{h,1}}{1-\beta\xi_w}\hat{w}_t \\
&+\frac{1-\alpha_{h,1}\beta\xi_w}{1-\beta\xi_w}\widehat{\overline{w}}_t+\alpha_{h,1}\sum_{j=1}^{\infty}\left(\beta\xi_w\right)^j\widehat{\overline{w}}_{t+j} \\
&-\left(1-\alpha_{h,1}\right)\beta\xi_w\left[\left(\Delta_{\kappa_w}\hat{\pi}_{t+1}+\hat{\mu}_{z^+,t+1}\right)\right] \\
&-\left(1-\alpha_{h,1}\right)\left(\beta\xi_w\right)^2\left[\left(\Delta_{\kappa_w}\hat{\pi}_{t+2}+\hat{\mu}_{z^+,t+2}\right)+\left(\Delta_{\kappa_w}\hat{\pi}_{t+1}+\hat{\mu}_{z^+,t+1}\right)\right] \\
&-...
\end{aligned}
$$

or,

$$
\begin{aligned}
&0=\sum_{j=0}^{\infty}\left(\beta\xi_w\right)^j\left[\hat{\psi}_{t+j}-\left(\sigma_{\tilde{\eta}}\widehat{\tilde{\eta}}_{t+j}+\sigma_z\hat{H}_{t+j}\right)\right]+\frac{1-\alpha_{h,1}}{1-\beta\xi_w}\hat{w}_t \\
&+\frac{1-\alpha_{h,1}\beta\xi_w}{1-\beta\xi_w}\widehat{\overline{w}}_t+\sum_{j=1}^{\infty}\left(\beta\xi_w\right)^j\left[\alpha_{h,1}\widehat{\overline{w}}_{t+j}-\frac{1-\alpha_{h,1}}{1-\beta\xi_w}\left(\Delta_{\kappa_w}\hat{\pi}_{t+j}+\hat{\mu}_{z^+,t+j}\right)\right].
\end{aligned}
$$

Note

$$\begin{aligned} S_t &= X_t + \beta\xi_w X_{t+1} + (\beta\xi_w)^2 X_{t+2} + ... \\ &= X_t + \beta\xi_w \overbrace{[X_{t+1} + \beta\xi_w X_{t+2} + ...]}^{S_{t+1}}, \end{aligned}$$

so that the log-linearized first order condition can be written:

$$0 = F_t + \frac{1-\alpha_{h,1}}{1-\beta\xi_w}\hat{w}_t + \frac{1-\alpha_{h,1}\beta\xi_w}{1-\beta\xi_w}\widehat{\bar{w}} + G_t, \tag{B.39}$$

where

$$\begin{aligned} F_t &= \sum_{j=0}^{\infty} (\beta\xi_w)^j \left[\hat{\psi}_{t+j} - \left(\sigma_{\tilde{\eta}}\widehat{\tilde{\eta}}_{t+j} + \sigma_z \hat{H}_{t+j}\right)\right] \\ &= \hat{\psi}_t - \left(\sigma_{\tilde{\eta}}\widehat{\tilde{\eta}}_t + \sigma_z \hat{H}_t\right) + \beta\xi_w F_{t+1} \\ G_t &= \sum_{j=1}^{\infty} (\beta\xi_w)^j \left[\alpha_{h,1}\widehat{\bar{w}}_{t+j} - \frac{1-\alpha_{h,1}}{1-\beta\xi_w}\left(\Delta_{\kappa_w}\hat{\pi}_{t+j} + \hat{\mu}_{z^+,t+j}\right)\right] \\ &= \beta\xi_w\alpha_{h,1}\widehat{\bar{w}}_{t+1} - \frac{(1-\alpha_{h,1})\beta\xi_w}{1-\beta\xi_w}\left(\Delta_{\kappa_w}\hat{\pi}_{t+1} + \hat{\mu}_{z^+,t+1}\right) + \beta\xi_w G_{t+1} \end{aligned}$$

Note:

$$\left(1-\beta\xi_w L^{-1}\right) F_t \equiv F_t - \beta\xi_w F_{t+1} = \hat{\psi}_t - \left(\sigma_{\tilde{\eta}}\widehat{\tilde{\eta}}_t + \sigma_z \hat{H}_t\right) \tag{B.40}$$

$$\left(1-\beta\xi_w L^{-1}\right) G_t \equiv G_t - \beta\xi_w G_{t+1} = \beta\xi_w\alpha_{h,1}\widehat{\bar{w}}_{t+1} - \frac{(1-\alpha_{h,1})\beta\xi_w}{1-\beta\xi_w}\left(\Delta_{\kappa_w}\hat{\pi}_{t+1} + \hat{\mu}_{z^+,t+1}\right)$$

We now obtain a second restriction on $\hat{w}_t$ using the relation between the aggregate wage rate and the wage rates of individual households:

$$W_t = \left[(1-\xi_w)\left(\tilde{W}_t\right)^{\frac{1}{1-\lambda_w}} + \xi_w\left(\tilde{\pi}_{w,t}W_{t-1}\right)^{\frac{1}{1-\lambda_w}}\right]^{1-\lambda_w}.$$

Dividing both sides by W_t :

$$1 = (1-\xi_w)(w_t)^{\frac{1}{1-\lambda_w}} + \xi_w\left(\frac{\tilde{\pi}_{w,t}W_{t-1}}{W_t}\right)^{\frac{1}{1-\lambda_w}}.$$

Note,

$$\pi_{w,t} \equiv \frac{W_t}{W_{t-1}} = \frac{\bar{w}_t z_t^+ P_t}{\bar{w}_{t-1} z_{t-1}^+ P_{t-1}} = \frac{\bar{w}_t \mu_{z^+,t}\pi_t}{\bar{w}_{t-1}},$$

so that

$$1 = (1-\xi_w)(w_t)^{\frac{1}{1-\lambda_w}} + \xi_w\left(\frac{\bar{w}_{t-1}\tilde{\pi}_{w,t}}{\bar{w}_t\mu_{z^+,t}\pi_t}\right)^{\frac{1}{1-\lambda_w}}.$$

Differentiate and make use of $w = 1$, $\tilde{\pi}_w = \mu_{z^+}\pi$:

$$0 = (1 - \xi_w)\frac{1}{1 - \lambda_w}\hat{w}_t + \xi_w \frac{1}{1 - \lambda_w}\left[\widehat{\bar{w}}_{t-1} + \widehat{\tilde{\pi}}_{w,t} - \widehat{\bar{w}}_t - \hat{\mu}_{z^+,t} - \hat{\pi}_t\right],$$

or,

$$\hat{w}_t = -\frac{\xi_w}{1 - \xi_w}\left[\widehat{\bar{w}}_{t-1} - \widehat{\bar{w}}_t - \hat{\mu}_{z^+,t} - \Delta_{\kappa_w}\hat{\pi}_t\right].$$

Use this expression to substitute out for $\hat{w}_t$ in (B.39):

$$\frac{1 - \alpha_{h,1}}{1 - \beta\xi_w}\frac{\xi_w}{1 - \xi_w}\left[\widehat{\bar{w}}_{t-1} - \widehat{\bar{w}}_t - \hat{\mu}_{z^+,t} - \Delta_{\kappa_w}\hat{\pi}_t\right] = F_t + \frac{1 - \beta\xi_w\alpha_{h,1}}{1 - \beta\xi_w}\widehat{\bar{w}}_t + G_t.$$

Multiply by $(1 - \beta\xi_w L^{-1})$ and use (B.40):

$$\begin{aligned}
& \frac{1 - \alpha_{h,1}}{1 - \beta\xi_w}\frac{\xi_w}{1 - \xi_w}\left(1 - \beta\xi_w L^{-1}\right)\left[\widehat{\bar{w}}_{t-1} - \widehat{\bar{w}}_t - \hat{\mu}_{z^+,t} - \Delta_{\kappa_w}\hat{\pi}_t\right] \\
= \; & \hat{\psi}_t - \left(\sigma_{\tilde{\eta}}\widehat{\tilde{\eta}}_t + \sigma_z \hat{H}_t\right) + \left(1 - \beta\xi_w L^{-1}\right)\frac{1 - \beta\xi_w\alpha_{h,1}}{1 - \beta\xi_w}\widehat{\bar{w}}_t \\
& + \beta\xi_w\alpha_{h,1}\widehat{\bar{w}}_{t+1} - \frac{(1 - \alpha_{h,1})\,\beta\xi_w}{1 - \beta\xi_w}\left(\Delta_{\kappa_w}\hat{\pi}_{t+1} + \hat{\mu}_{z^+,t+1}\right),
\end{aligned}$$

or,

$$\begin{aligned}
& \frac{1 - \alpha_{h,1}}{1 - \beta\xi_w}\frac{\xi_w}{1 - \xi_w}\left[\begin{array}{c} \widehat{\bar{w}}_{t-1} - \beta\xi_w\widehat{\bar{w}}_t - \widehat{\bar{w}}_t + \beta\xi_w\widehat{\bar{w}}_{t+1} - \hat{\mu}_{z^+,t} \\ + \beta\xi_w\hat{\mu}_{z^+,t+1} - \Delta_{\kappa_w}\hat{\pi}_t + \beta\xi_w\Delta_{\kappa_w}\hat{\pi}_{t+1} \end{array}\right] \\
= \; & \hat{\psi}_t - \left(\sigma_{\tilde{\eta}}\widehat{\tilde{\eta}}_t + \sigma_z \hat{H}_t\right) + \frac{1 - \beta\xi_w\alpha_{h,1}}{1 - \beta\xi_w}\left[\widehat{\bar{w}}_t - \beta\xi_w\widehat{\bar{w}}_{t+1}\right] \\
& + \beta\xi_w\alpha_{h,1}\widehat{\bar{w}}_{t+1} - \frac{(1 - \alpha_{h,1})\,\beta\xi_w}{1 - \beta\xi_w}\left(\Delta_{\kappa_w}\hat{\pi}_{t+1} + \hat{\mu}_{z^+,t+1}\right).
\end{aligned}$$

Note that the wage does not simply enter via nominal wage inflation. To see this, note

$$\widehat{\bar{w}}_t - \widehat{\bar{w}}_{t-1} = \hat{\pi}_{w,t} - \hat{\mu}_{z^+,t} - \hat{\pi}_t,$$

where $\hat{\pi}_{w,t}$ denotes nominal wage inflation. But, it is not simply $\widehat{\bar{w}}_t - \widehat{\bar{w}}_{t-1}$ that enters in this expression. That is, if we tried to express the above expression in terms of nominal wage inflation, we would simply add another variable to it, $\hat{\pi}_{w,t}$, without subtracting any, such as the real wage, $\widehat{\bar{w}}_t$. Collecting terms:

$$\begin{aligned}
0 \; = \; & E_t[\eta_0\widehat{\bar{w}}_{t-1} + \eta_1\widehat{\bar{w}}_t + \eta_2\widehat{\bar{w}}_{t+1} + \eta_3\hat{\pi}_{t-1} + \eta_4\hat{\pi}_t + \eta_5\hat{\pi}_{t+1} + \eta_6\hat{\mu}_{z^+,t} + \eta_7\hat{\mu}_{z^+,t+1} \qquad \text{(B.41)} \\
& + \eta_8\hat{\psi}_t + \eta_9\hat{H}_t + \eta_{10}\widehat{\tilde{\eta}}_t],
\end{aligned}$$

where

$$\begin{aligned}
\eta_0 &= \frac{1-\alpha_{h,1}}{1-\beta\xi_w}\frac{\xi_w}{1-\xi_w},\ \eta_1 = -\eta_0\left(1+\beta\xi_w\right) - \frac{\left(1-\beta\xi_w\alpha_{h,1}\right)}{1-\beta\xi_w}, \\
\eta_2 &= \beta\xi_w\left(\eta_0 + \frac{\left(1-\beta\xi_w\alpha_{h,1}\right)}{1-\beta\xi_w} - \alpha_{h,1}\right),\ \eta_3 = \eta_0\kappa_w, \\
\eta_4 &= -\eta_0\left(1+\kappa_w\beta\xi_w\right) - \frac{\left(1-\alpha_{h,1}\right)\beta\xi_w}{1-\beta\xi_w}\kappa_w, \\
\eta_5 &= \eta_0\beta\xi_w + \frac{\left(1-\alpha_{h,1}\right)\beta\xi_w}{1-\beta\xi_w}, \\
\eta_6 &= -\eta_0,\ \eta_7 = \eta_5,\ \eta_8 = -1,\ \eta_9 = \sigma_z,\ \eta_{10} = \sigma_{\tilde{\eta}}.
\end{aligned}$$

Note that (B.41) is the same for the standard model and for our model with involuntary unemployment except for the presence of $\sigma_{\tilde{\eta}}$ in our model and the difference in the construction of σ_z in both models.

The wage equation can be thought of, for computational purposes, as a nonlinear equation, if we treat

$$\widehat{\bar{w}}_t = \frac{\bar{w}_t - \bar{w}}{\bar{w}},$$

and the other hatted variables in the same way. Likewise:

$$\widehat{\tilde{\eta}}_t = \frac{\tilde{\eta}_t - \tilde{\eta}}{\tilde{\eta}}.$$

B.11. Remaining Equilibrium Conditions

B.11.1. Firms

We let s_t denote the firm's marginal cost, divided by the price of the homogeneous good. The standard formula, expressing this as a function of the factor inputs, is as follows:

$$s_t = \frac{\left(\frac{r_t^k P_t}{\alpha}\right)^{\alpha}\left(\frac{W_t R_t^f}{1-\alpha}\right)^{1-\alpha}}{P_t z_t^{1-\alpha}}.$$

When expressed in terms of scaled variables, this reduces to:

$$s_t = \left(\frac{\bar{r}_t^k}{\alpha}\right)^{\alpha}\left(\frac{\bar{w}_t R_t^f}{1-\alpha}\right)^{1-\alpha}. \tag{B.42}$$

Productive efficiency dictates that s_t is also equal to the ratio of the real cost of labor to the marginal product of labor:

$$s_t = \frac{\left(\mu_{\Psi,t}\right)^{\alpha}\bar{w}_t R_t^f}{(1-\alpha)\left(\frac{k_{i,t}}{\mu_{z^+,t}}/H_{i,t}\right)^{\alpha}}. \tag{B.43}$$

The only real decision taken by intermediate good firms is to optimize price when it is selected to do so under the Calvo frictions. The first order necessary conditions associated with price optimization are, after scaling:

$$E_t\left[\psi_t y_t + \left(\frac{\tilde{\pi}_{f,t+1}}{\pi_{t+1}}\right)^{\frac{1}{1-\lambda_f}} \beta\xi_p F_{t+1}^f - F_t^f\right] = 0 \tag{B.44}$$

$$E_t\left[\lambda_f \psi_t y_t s_t + \beta\xi_p \left(\frac{\tilde{\pi}_{f,t+1}}{\pi_{t+1}}\right)^{\frac{\lambda_f}{1-\lambda_f}} K_{t+1}^f - K_t^f\right] = 0, \tag{B.45}$$

$$\mathring{p}_t = \left[(1-\xi_p)\left(\frac{1-\xi_p\left(\frac{\tilde{\pi}_{f,t}}{\pi_t}\right)^{\frac{1}{1-\lambda_f}}}{1-\xi_p}\right)^{\lambda_f} + \xi_p\left(\frac{\tilde{\pi}_{f,t}}{\pi_t}\mathring{p}_{t-1}\right)^{\frac{\lambda_f}{1-\lambda_f}}\right]^{\frac{1-\lambda_f}{\lambda_f}}, \tag{B.46}$$

$$\left[\frac{1-\xi_p\left(\frac{\tilde{\pi}_{f,t}}{\pi_t}\right)^{\frac{1}{1-\lambda_f}}}{1-\xi_p}\right]^{(1-\lambda_f)} = \frac{K_t^f}{F_t^f}, \tag{B.47}$$

$$\tilde{\pi}_{f,t} \equiv (\pi_{t-1})^{\kappa f} (\pi)^{1-\kappa f}. \tag{B.48}$$

In terms of scaled variables, the law of motion for the capital stock is as follows:

$$\bar{k}_{t+1} = \frac{1-\delta}{\mu_{z^+,t}\mu_{\Psi,t}}\bar{k}_t + \Upsilon_t\left(1-\tilde{S}\left(\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}}\right)\right)i_t. \tag{B.49}$$

The aggregate production relation is:

$$y_t = (\mathring{p}_t)^{\frac{\lambda_f}{\lambda_f-1}}\left[\epsilon_t\left(\frac{1}{\mu_{\Psi,t}}\frac{1}{\mu_{z^+,t}}\bar{k}_t u_t\right)^{\alpha} H_t^{1-\alpha} - n_t\phi\right].$$

Finally, the resource constraint is:

$$y_t = n_t G + c_t + i_t + a(u_t)\frac{\bar{k}_t}{\mu_{\psi,t}\mu_{z^+,t}}.$$

B.11.2. Household

We now derive the equilibrium conditions associated with the household, apart from the wage condition, which was derived in a previous subsection. The Lagrangian representation of the household's problem is:

$$
\begin{aligned}
& E_0^j \sum_{t=0}^{\infty} \beta^t \{ [\ln (C_t - bC_{t-1}) - z(h_{t,j}; \tilde{\eta}_t)] \\
& \upsilon_t \left[\begin{array}{c} W_{t,j} h_{t,j} + X_t^k \bar{K}_t + R_{t-1} B_t \\ + a_{t,j} - P_t \left(C_t + \frac{1}{\Psi_t} I_t \right) - B_{t+1} - P_t P_{k',t} \Delta_t \end{array} \right] \\
& + \omega_t \left[\Delta_t + (1-\delta) \bar{K}_t + \left(1 - \tilde{S} \left(\frac{I_t}{I_{t-1}} \right) \right) I_t - \bar{K}_{t+1} \right] \}
\end{aligned}
$$

The first order condition with respect to C_t is:

$$
\frac{1}{C_t - bC_{t-1}} - E_t \frac{b\beta}{C_{t+1} - bC_t} = \upsilon_t P_t,
$$

or, after expressing this in scaled terms and multiplying by z_t^+:

$$
\psi_t = \frac{1}{c_t - b\frac{c_{t-1}}{\mu_{z^+,t}}} - \beta b E_t \frac{1}{c_{t+1}\mu_{z^+,t+1} - bc_t}. \tag{B.50}
$$

The first order condition with respect to Δ_t is, after rearranging:

$$
P_t P_{k',t} = \frac{\omega_t}{\upsilon_t}. \tag{B.51}
$$

The first order condition with respect to I_t is:

$$
\omega_t \left[1 - \tilde{S}\left(\frac{I_t}{I_{t-1}} \right) - \tilde{S}'\left(\frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} \right] + E_t \beta \omega_{t+1} \tilde{S}'\left(\frac{I_{t+1}}{I_t} \right) \left(\frac{I_{t+1}}{I_t} \right)^2 = \frac{P_t \upsilon_t}{\Psi_t}.
$$

Making use of (B.51), multiplying by $\Psi_t z_t^+$, rearranging and using the scaled variables,

$$
\begin{aligned}
& \psi_t p_{k',t} \left[1 - \tilde{S}\left(\frac{\mu_{z^+,t}\mu_{\Psi,t} i_t}{i_{t-1}} \right) - \tilde{S}'\left(\frac{\mu_{z^+,t}\mu_{\Psi,t} i_t}{i_{t-1}} \right) \frac{\mu_{z^+,t}\mu_{\Psi,t} i_t}{i_{t-1}} \right] \\
& + \beta \psi_{t+1} p_{k',t+1} \tilde{S}'\left(\frac{\mu_{z^+,t+1}\mu_{\Psi,t+1} i_{t+1}}{i_t} \right) \left(\frac{i_{t+1}}{i_t} \right)^2 \mu_{z^+,t+1}\mu_{\Psi,t+1} \quad = \quad \psi_t.
\end{aligned} \tag{B.52}
$$

Optimality of the choice of $\bar{K}_{t+1}$ implies the following first order condition:

$$
\omega_t = \beta E_t \upsilon_{t+1} X_{t+1}^k + \beta E_t \omega_{t+1} (1-\delta) = \beta E_t \upsilon_{t+1} \left[X_{t+1}^k + P_{t+1} P_{k',t+1} (1-\delta) \right],
$$

using (B.51). Using (B.51) again,

$$
\upsilon_t = E_t \beta \upsilon_{t+1} \left[\frac{X_{t+1}^k + P_{t+1} P_{k',t+1} (1-\delta)}{P_t P_{k',t}} \right] = E_t \beta \upsilon_{t+1} R_{t+1}^k, \tag{B.53}
$$

where R_{t+1}^k denotes the rate of return on capital:

$$
R_{t+1}^k \equiv \frac{X_{t+1}^k + P_{t+1} P_{k',t+1} (1-\delta)}{P_t P_{k',t}}
$$

Multiply (B.53) by $P_t z_t^+$ and express the results in scaled terms:

$$\psi_t = \beta E_t \psi_{t+1} \frac{R_{t+1}^k}{\pi_{t+1} \mu_{z^+,t+1}}. \tag{B.54}$$

Expressing the rate of return on capital, (B.15), in terms of scaled variables:

$$R_{t+1}^k = \frac{\pi_{t+1}}{\mu_{\Psi,t+1}} \frac{u_{t+1} \bar{r}_{t+1}^k - a(u_{t+1}) + (1-\delta) p_{k',t+1}}{p_{k',t}}. \tag{B.55}$$

The first order condition associated with capital utilization is:

$$\Psi_t r_t^k = a'(u_t),$$

or, in scaled terms,

$$\bar{r}_t^k = a'(u_t). \tag{B.56}$$

The first order condition with respect to B_{t+1} is:

$$\upsilon_t = \beta E_t \upsilon_{t+1} R_t.$$

Multiply by $z_t^+ P_t$:

$$\psi_t = \beta E_t \frac{\psi_{t+1}}{\mu_{z^+,t+1} \pi_{t+1}} R_t. \tag{B.57}$$

C. Equilibrium Equations of the Medium-Sized DSGE Model

Here we list the scaled dynamic equilibrium equations of the medium-sized DSGE model with involuntary unemployment as well as the standard labor market model. We also list the corresponding steady state equations.

C.1. Dynamic Equilibrium Equations

$$\text{Cons. FOC (1)} : \quad \psi_t = \left(c_t - b\frac{c_{t-1}}{\mu_{z^+,t}}\right)^{-1} - \beta b E_t\left(c_{t+1}\mu_{z^+,t+1} - bc_t\right)^{-1}$$

$$\text{Bond. FOC (2)} : \quad \psi_t = \beta E_t \frac{\psi_{t+1}}{\mu_{z^+,t+1}\pi_{t+1}} R_t$$

$$\text{Invest. FOC (3)} : \quad \psi_t p_{k',t}\left[1 - \tilde{S}_t - \tilde{S}'_t \frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}}\right] + \beta E_t \psi_{t+1} p_{k',t+1} \tilde{S}'_{t+1}\left(\frac{i_{t+1}}{i_t}\right)^2 \mu_{z^+,t+1}\mu_{\Psi,t+1} = \psi_t$$

$$\text{Capital FOC (4)} : \quad \psi_t = \beta E_t \psi_{t+1} \frac{R^k_{t+1}}{\pi_{t+1}\mu_{z^+,t+1}}$$

$$\text{LOM capital (5)} : \quad \bar{k}_{t+1} = \frac{1-\delta}{\mu_{z^+,t}\mu_{\Psi,t}}\bar{k}_t + \left(1 - \tilde{S}\left(\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}}\right)\right) i_t$$

$$\text{Cost. minim. (6)} : \quad 0 = a'\left(u^k_t\right) u^k_t \bar{k}_t / (\mu_{\Psi,t}\mu_{z^+,t}) - \alpha/(1-\alpha) w_t \left[\nu^f R_t + 1 - \nu^f\right] \mathring{w}_t^{\lambda_w/(\lambda_w-1)} h_t$$

$$\text{Production (7)} : \quad y_t = (\mathring{p}_t)^{\frac{\lambda_f}{\lambda_f - 1}} \left[\left(\frac{1}{\mu_{\Psi,t}}\frac{1}{\mu_{z^+,t}}\bar{k}_t u^k_t\right)^{\alpha}\left(\mathring{w}_t^{\lambda_w/(\lambda_w-1)} h_t\right)^{1-\alpha} - n_t\phi\right]$$

$$\text{Resources (8)} : \quad y_t = n_t G + c_t + i_t + a\left(u^k_t\right)\frac{\bar{k}_t}{\mu_{\psi,t}\mu_{z^+,t}}$$

$$\text{Taylor rule (9)} : \quad \ln\left(\frac{R_t}{R}\right) = \rho_R \ln\left(\frac{R_{t-1}}{R}\right) + (1-\rho_R)\left[r_\pi \ln\left(\frac{\pi_t}{\pi}\right) + r_y \ln\left(\frac{gdp_t}{gdp}\right)\right] + \frac{\sigma_R \varepsilon_{R,t}}{400}$$

$$\text{Pricing 1 (10)} : \quad F^f_t = \psi_t y_t + \beta\xi_p E_t \left(\frac{\tilde{\pi}_{f,t+1}}{\pi_{t+1}}\right)^{\frac{1}{1-\lambda_f}} F^f_{t+1}$$

$$\text{Pricing 2 (11)} : \quad K^f_t = \lambda_f \psi_t y_t s_t + \beta\xi_p E_t \left(\frac{\tilde{\pi}_{f,t+1}}{\pi_{t+1}}\right)^{\frac{\lambda_f}{1-\lambda_f}} K^f_{t+1}$$

$$\text{Pricing 3 (12)} : \quad \left(1-\xi_p\right)\left(K^f_t / F^f_t\right)^{1/\left(1-\lambda_f\right)} = 1 - \xi_p \left(\frac{\tilde{\pi}_{f,t}}{\pi_t}\right)^{\frac{1}{1-\lambda_f}}$$

$$\text{Price disp. (13)} : \quad \mathring{p}_t^{\frac{\lambda_f}{1-\lambda_f}} = \left(1-\xi_p\right)\left(\left(1 - \xi_p\left(\frac{\tilde{\pi}_{f,t}}{\pi_t}\right)^{\frac{1}{1-\lambda_f}}\right) / \left(1-\xi_p\right)\right)^{\lambda_f} + \xi_p \left(\frac{\tilde{\pi}_{f,t}}{\pi_t}\mathring{p}_{t-1}\right)^{\frac{\lambda_f}{1-\lambda_f}}$$

$$\text{Real GDP (14)} : \quad gdp_t = n_t G + c_t + i_t$$

$$\text{Unemp. rate (15)}: \quad u_t = \frac{m_t - h_t}{m_t}$$

$$\text{Wage inflation (16)} : \quad \pi_{w,t} = w_t \mu_{z^+,t}\pi_t / w_{t-1}$$

For the involuntary unemployment model we have the following further equations:

Wage Phillips Curve (17): $$0 = E_t[\eta_0 \widehat{\bar{w}}_{t-1} + \eta_1 \widehat{\bar{w}}_t + \eta_2 \widehat{\bar{w}}_{t+1} + \eta_3 \hat{\pi}_{t-1} + \eta_4 \hat{\pi}_t + \eta_5 \hat{\pi}_{t+1} + \eta_6 \hat{\mu}_{z^+,t} + \eta_7 \hat{\mu}_{z^+,t+1} + \eta_8 \hat{\psi}_t + \eta_9 \hat{h}_t + \frac{\eta_{10}}{\tilde{\eta}} (\tilde{\eta}_t - \tilde{\eta})]$$

Labor force (18): $$h\hat{h}_t = -m(\tilde{\eta}_t - \tilde{\eta}) - \tilde{\eta} m \hat{m}_t + (\sigma_L + 1) a^2 \varsigma \sigma_L \left(m^{\sigma_L+1} \hat{m}_t - \mathring{l}^{\sigma_L+1} \widehat{\mathring{l}}_t \right)$$

Workers with p(e)=1 (19): $$\sigma_L \mathring{l}^{\sigma_L} \widehat{\mathring{l}}_t = \sigma_L m^{\sigma_L} \hat{m}_t - \frac{1}{\varsigma (1+\sigma_L) a^2} (\tilde{\eta}_t - \tilde{\eta}).$$

Intercept in p(e) (20): $$\tilde{\eta}_t = \eta + 100\omega (m_t / m_{t-1} - 1)$$

where in the above equations, hatted variables are related to level variables as follows:

$$\widehat{\bar{w}}_t = \frac{\bar{w}_t - \bar{w}}{\bar{w}}, \hat{\pi}_t = \frac{\pi_t - \pi}{\pi}, \hat{\mu}_{z^+,t} = \frac{\mu_{z^+,t} - \hat{\mu}_{z^+}}{\hat{\mu}_{z^+}}, \hat{\psi}_t = \frac{\psi_t - \psi}{\psi},$$
$$\hat{h}_t = \frac{h_t - h}{h}, \hat{m}_t = \frac{m_t - m}{m}, \widehat{\mathring{l}}_t = \frac{\mathring{l}_t - \mathring{l}}{\mathring{l}}.$$

Further, the coefficients of the wage Phillips curve are defined as:

$$\eta_0 = \frac{1-\alpha_{h,1}}{1-\beta\xi_w} \frac{\xi_w}{1-\xi_w}, \ \eta_1 = -\eta_0 (1+\beta\xi_w) - \frac{(1-\beta\xi_w \alpha_{h,1})}{1-\beta\xi_w},$$
$$\eta_2 = \beta\xi_w \left(\eta_0 + \frac{(1-\beta\xi_w\alpha_{h,1})}{1-\beta\xi_w} - \alpha_{h,1} \right), \ \eta_3 = \eta_0 \kappa_w,$$
$$\eta_4 = -\eta_0 (1+\kappa_w \beta\xi_w) - \frac{(1-\alpha_{h,1})\beta\xi_w}{1-\beta\xi_w} \kappa_w,$$
$$\eta_5 = \eta_0 \beta\xi_w + \frac{(1-\alpha_{h,1})\beta\xi_w}{1-\beta\xi_w}, \eta_6 = -\eta_0, \ \eta_7 = \eta_5,$$
$$\eta_8 = -1, \ \eta_9 = \sigma_z = \frac{z_{hh} h}{z_h}, \ \eta_{10} = \sigma_{\tilde{\eta}} = \frac{z_{h\tilde{\eta}} \tilde{\eta}}{z_h}$$

For the standard model we have the following further equations:

Wage Phillips Curve (17): $$0 = E_t[\eta_0 \widehat{\bar{w}}_{t-1} + \eta_1 \widehat{\bar{w}}_t + \eta_2 \widehat{\bar{w}}_{t+1} + \eta_3 \hat{\pi}_{t-1} + \eta_4 \hat{\pi}_t + \eta_5 \hat{\pi}_{t+1} + \eta_6 \hat{\mu}_{z^+,t} + \eta_7 \hat{\mu}_{z^+,t+1} + \eta_8 \hat{\psi}_t + \eta_9 \hat{H}_t]$$

Labor force (18): $$\sigma_L \hat{m}_t = \hat{\psi}_t + \widehat{\bar{w}}_t$$

Workers with p(e)=1 (19): $$\widehat{\mathring{l}}_t = 0$$

Intercept in p(e) (20) : $$\tilde{\eta}_t = 0$$

Finally, both models have the following exogenous variables:

Comp. Tech. (21) : $$\ln \mu_{z^+,t} = \alpha/(1-\alpha) \ln \mu_{\Psi,t} + \ln \mu_{z,t}$$

Invest. Tech. (22) : $$\ln \mu_{\Psi,t} = (1-\rho_{\mu_\Psi}) \ln \mu_\Psi + \rho_{\mu_\Psi} \ln \mu_{\Psi,t-1} + \sigma_{\mu_\Psi} \varepsilon_{\mu_\Psi,t}/100$$

Neutr. Tech. (23) : $$\ln \mu_{z,t} = \ln \mu_z + \sigma_{\mu_z} \varepsilon_{\mu_z,t}/100$$

Tech. diffus. (24) : $$n_t = n_{t-1}^{1-\theta} \mu_{z^+,t}^{-1}$$

In the above two models we have a total of 24 equations in the following 24 variables:

$$\psi_t \; c_t \; R_t \; \pi_t \; p_{k',t} \; i_t \; u_t^k \; \bar{k}_t \; h_t \; y_t \; \mathring{p}_t \; F_t^f \; K_t^f \; \bar{w}_t \; \pi_{w,t} \; u_t \; gdp_t \; \mu_{z^+,t} \; \mu_{z,t} \; \mu_{\Psi,t} \; n_t \; m_t \; \mathring{l}_t \; \tilde{\eta}_t$$

In the above equations, it is useful to define several abbreviated variables that are functions of the endogenous variables. In particular,

$$\begin{aligned}
\text{Cap. util. cost. (25)} &: \quad a(u_t^k) = 0.5\sigma_b\sigma_a\left(u_t^k\right)^2 + \sigma_b\left(1-\sigma_a\right)u_t^k + \sigma_b\left((\sigma_a/2)-1\right) \\
\text{Cap. util. deriv. (26)} &: \quad a'(u_t^k) = \sigma_b\sigma_a u_t^k + \sigma_b\left(1-\sigma_a\right) \\
\text{Invest. adj. cost (27)} &: \quad \tilde{S}_t = 0.5\exp\left[\sqrt{\tilde{S}''}\left(\mu_{z^+,t}\mu_{\Psi,t}i_t/i_{t-1} - \mu_{z^+}\cdot\mu_\Psi\right)\right] \\
&\qquad +0.5\exp\left[-\sqrt{\tilde{S}''}\left(\mu_{z^+,t}\mu_{\Psi,t}i_t/i_{t-1} - \mu_{z^+}\cdot\mu_\Psi\right)\right] - 1 \\
\text{Inv. adj. deriv. (28)} &: \quad \tilde{S}_t' = 0.5\sqrt{\tilde{S}''}\exp\left[\sqrt{\tilde{S}''}\left(\mu_{z^+,t}\mu_{\Psi,t}i_t/i_{t-1} - \mu_{z^+}\cdot\mu_\Psi\right)\right] \\
&\qquad -0.5\sqrt{\tilde{S}''}\exp\left[-\sqrt{\tilde{S}''}\left(\mu_{z^+,t}\mu_{\Psi,t}i_t/i_{t-1} - \mu_{z^+}\cdot\mu_\Psi\right)\right] \\
\text{Capital return (29)} &: \quad R_t^k = \pi_t/(\mu_{\Psi,t}p_{k',t-1})\left(u_t^k a'\left(u_t^k\right) - a(u_t^k) + (1-\delta^k)p_{k',t}\right) \\
\text{Marginal cost (30)} &: \quad mc_t = \left(\mu_{\Psi,t}\mu_{z^+,t}\right)^\alpha w_t\left[\nu^f R_t + 1 - \nu^f\right]\left(u_t^k\bar{k}_{t-1}/\left(\mathring{w}_t^{\lambda_w/(\lambda_w-1)}h_t\right)\right)^{-\alpha}/(1-\alpha) \\
\text{Price indexation (31)} &: \quad \tilde{\pi}_t = \pi_{t-1}^{\kappa^f}\pi^{1-\kappa^f} \\
\text{Wage indexation (32)} &: \quad \tilde{\pi}_{w,t} = \pi_{t-1}^{\kappa^w}\pi^{1-\kappa^w}\mu_{z^+}
\end{aligned}$$

In the baseline specification described in the main text we set $\kappa^f = 0$, $\kappa^w = 1$ and $\nu^f = 1$.

C.2. Steady State

$$
\begin{aligned}
& && \text{IMPOSE } u^k = 1, \text{ solve (29) for } \sigma_b \\
(25) &&:\quad & a(1) = 0 \\
(21) &&:\quad & \mu_z = \mu_{z^+} / \left(\mu_\Psi\right)^{\alpha/(1-\alpha)} \\
(24) &&:\quad & n = \mu_{z^+}^{-\frac{1}{\theta_i}} \\
(22) &&:\quad & \varepsilon_{\mu_z} = 0 \\
(23) &&:\quad & \varepsilon_{\mu_\Psi} = 0 \\
(27) &&:\quad & \tilde{S} = 0 \\
(28) &&:\quad & \tilde{S}' = 0 \\
& && \text{IMPOSE } \pi, \text{ ``drop'' equation (9), i.e. } R = R \\
(2) &&:\quad & R = \pi \mu_{z^+} / \beta \\
(3) &&:\quad & p_{k'} = 1 \\
(4) &&:\quad & R^k = \pi \mu_{z^+} / \beta \\
(29) &&:\quad & \sigma_b = R^k \mu_\Psi p_{k'} / \pi - (1 - \delta^k) p_{k'} \\
(26) &&:\quad & a'(1) = \sigma_b \\
(31) &&:\quad & \tilde{\pi}_t = \pi_{t-1}^{\kappa^f} \pi^{1-\kappa^f} \\
(32) &&:\quad & \tilde{\pi}_{w,t} = \pi_{t-1}^{\kappa^w} \pi^{1-\kappa^w} \mu_{z^+}
\end{aligned}
$$

$$(10\text{-}12) : mc = \frac{1}{\lambda}\frac{1-\beta\xi\left(\tilde{\pi}/\pi\right)^{\lambda/(1-\lambda)}}{1-\beta\xi\left(\tilde{\pi}/\pi\right)^{1/(1-\lambda)}}\left[\frac{1-\xi\left(\tilde{\pi}/\pi\right)^{1/(1-\lambda)}}{1-\xi}\right]^{1-\lambda}$$

$$(13) : \mathring{p} = \left[\frac{1-\xi\left(\tilde{\pi}/\pi\right)^{1/(1-\lambda)}}{1-\xi}\right]^{1-\lambda} / \left[\frac{1-\xi\left(\tilde{\pi}/\pi\right)^{\lambda/(1-\lambda)}}{1-\xi}\right]^{(1-\lambda)/\lambda}$$

$$(6 \ \& \ 30) : kh = \bar{k}/(\mathring{w}^{\lambda_w/(\lambda_w-1)}l) = \left[\alpha\left(\mu_\Psi\mu_{z^+}\right)^{1-\alpha} mc/\sigma_b\right]^{1/(1-\alpha)}$$

$$(16) : \pi_w = \mu_{z^+}\pi$$

$$(14) : \mathring{w} = \left(\frac{1-\xi_w\left(\tilde{\pi}_w/\pi_w\right)^{1/(1-\lambda_w)}}{1-\xi_w}\right)^{1-\lambda_w} / \left(\frac{1-\xi_w\left(\tilde{\pi}_w/\pi_w\right)^{\lambda_w/(1-\lambda_w)}}{1-\xi_w}\right)^{\frac{1-\lambda_w}{\lambda_w}}$$

$$(30) : w = \frac{(1-\alpha)\,mc}{\left(\mu_\Psi\mu_{z^+}\right)^\alpha\left[\nu^f R + 1 - \nu^f\right]}(kh)^\alpha$$

IMPOSE h and solve for ς later

IMPOSE zero profits and solve for ϕ later

$$(7 \ \& \text{ zero profits}) : y = \frac{mc}{(\mathring{p}^{\lambda/(1-\lambda)}-1)\,mc+1}\left(kh/(\mu_{z^+}\mu_\Psi)\right)^\alpha \mathring{w}^{\lambda_w/(\lambda_w-1)}h$$

$$: \bar{k} = kh\cdot\mathring{w}^{\lambda_w/(\lambda_w-1)}h$$

$$(7) : \phi = \left[\left(kh/(\mu_{z^+}\mu_\Psi)\right)^\alpha \mathring{w}^{\lambda_w/(\lambda_w-1)}h - y\mathring{p}^{\lambda/(1-\lambda)}\right]/n$$

$$(5) : i = \left[1-(1-\delta)/(\mu_{z^+}\mu_\Psi)\right]\bar{k}$$

Assume G equals share η_g of y

$$(8) : c = (1-\eta_g)y - i \text{ for some given } \eta_g \ \rightarrow G = \eta_g y/n_g$$

$$(1) : \psi = (c - bc/\mu_{z^+})^{-1} - \beta b\left(c\mu_{z^+} - bc\right)^{-1}$$

$$(14) : gdp = n_g G + c + i$$

$$(11) : K^f = \frac{\lambda\cdot\psi\cdot y\cdot mc}{1-\beta\xi\left(\tilde{\pi}/\pi\right)^{\lambda/(1-\lambda)}}$$

$$(10) : F^f = \frac{\psi\cdot y}{1-\beta\xi\left(\tilde{\pi}/\pi\right)^{1/(1-\lambda)}}$$

C.2.1. Standard Model

For the standard model we proceed as follows:

$$\begin{aligned}
\sigma_L &= \frac{z_{hh} h}{z_h} = \sigma_z^{\text{target}} \\
(17) &: \quad z_h = \frac{\psi}{\lambda_w} \bar{w} \ \Rightarrow \varsigma = \left(\frac{\psi}{\lambda_w} \bar{w} \right) / \left((1+\sigma_L) h^{\sigma_L} \right) \\
(18) &: \quad m = \left(\frac{\psi \bar{w}}{\varsigma \left(1+\sigma_L\right)} \right)^{\frac{1}{\sigma_L}} \\
(15) &: \quad u = \frac{m-h}{m}
\end{aligned}$$

C.2.2. Involuntary Unemployment Model

For the involuntary unemployment model we proceed as follows:

$$\begin{aligned}
& \text{IMPOSE } m \text{ and solve later for } \eta \\
(15) \quad &: \quad u = \frac{m-h}{m}
\end{aligned}$$

We solve for the following objects using a nonlinear solver:

$$\varsigma \ a \ \mathring{l} \ \sigma_L$$

Conditional on $\varsigma \ a \ \mathring{l} \ \sigma_L$ we can pursue further

$$\begin{aligned}
(19) &: \quad \tilde{\eta} = \varsigma \left(1+\sigma_L\right) a^2 \left(m^{\sigma_L} - \mathring{l}^{\sigma_L} \right) - 1 \\
(20) &: \quad \eta = \tilde{\eta} \\
\tilde{r} &= e^{-\left(F + \varsigma(1+\sigma_L) m^{\sigma_L} - \frac{2}{a^2} \tilde{\eta} \right)} \\
r &= \frac{\left(c - hb/\mu_{z+} c\right) \tilde{r} + hb/\mu_{z+} c}{c - (1-h)\, b/\mu_{z+} c + (1-h)\, b/\mu_{z+} c\tilde{r}} \\
c^w &= \frac{c}{h + (1-h)\, r}, c^{nw} = r c^w
\end{aligned}$$

We adjust $\varsigma \ a \ \mathring{l} \ \sigma_L$ to make the following four equations hold:

$$\begin{aligned}
(\varsigma)\,(17) &: \quad z_h = \frac{\psi}{\lambda_w} \bar{w} \\
\left(\mathring{l}\right)(18) &: \quad h = -\tilde{\eta} m + a^2 \varsigma \sigma_L \left(m^{\sigma_L + 1} - \mathring{l}^{\sigma_L + 1} \right) \\
(a) &: \quad \frac{z_{hh} h}{z_h} = \sigma_z^{\text{target}} \\
(\sigma_L) &: \quad r = r^{\text{target}}
\end{aligned}$$

D. Estimation Results When Unemployment Rate and Labor Force Data are Included in Estimation of Standard Model

Technical Appendix Table A.1 contains the estimated parameters of the standard model with and without including data for the unemployment rate and the labor force in the estimation. The posterior mode and parameter distributions are based on a standard MCMC algorithm with a total of 2.5 million draws based on 10 chains. We use the first 20 percent of draws for burn-in. The acceptance rates are about 0.25 in each chain. Figures 1 through 4 in the appendix to the main text show the impulse responses of the estimated standard model when data for the unemployment rate and the labor force in the estimation evaluated at the posterior mode shown in Technical Appendix Table A.1.

Technical Appendix Table A.1: Sensitivity of Estimated Standard Model

Parameter		Prior Distribution [bounds]	Prior Mode [2.5% 97.5%]	Posterior Mode [2.5% 97.5%] **Baseline Model**	Posterior Mode [2.5% 97.5%] **Model with U. & Lab. Force**
Price Setting Parameters					
Price Stickiness	ξ_p	Beta	0.67	0.616	0.776
		[0, 1]	[0.45 0.83]	[0.55 0.71]	[0.73 0.81]
Price Markup	λ_f	Gamma	1.19	1.230	-
		[1.001, ∞]	[1.01 1.40]	[1.10 1.36]	-
Monetary Authority Parameters					
Taylor Rule: Int. Smoothing	ρ_R	Beta	0.76	0.873	0.785
		[0, 1]	[0.37 0.93]	[0.82 0.90]	[0.77 0.85]
Taylor Rule: Inflation Coef.	r_π	Gamma	1.68	1.395	1.015
		[1.001, ∞]	[1.41 2.00]	[1.19 1.65]	[1.00 1.76]
Taylor Rule: GDP Coef.	r_y	Gamma	0.07	0.077	0.005
		[0, ∞]	[0.02 0.21]	[0.03 0.14]	[0.00 0.09]
Preference Parameters					
Consumption Habit	b	Beta	0.75	0.761	0.755
		[0, 1]	[0.64 0.83]	[0.72 0.79]	[0.74 0.81]
Inverse Labor Supply Elast.	σ_z	Gamma	0.26	0.165	18.18
		[0, ∞]	[0.13 0.52]	[0.08 0.23]	[12.97 25.57]
Technology Parameters					
Capital Share	α	Beta	0.32	0.31	0.270
		[0, 1]	[0.28 0.37]	[0.25 0.33]	[0.21 0.28]
Technology diffusion	θ	Beta	0.50	0.052	0.006
		[0, 1]	[0.12 0.86]	[0.01 0.80]	[0.00 0.02]
Capacity Adj. Costs Curv.	σ_a	Gamma	0.31	0.462	0.019
		[0, ∞]	[0.09 1.22]	[0.21 0.56]	[0.00 0.08]
Investment Adj. Costs Curv.	S''	Gamma	7.50	11.56	10.32
		[0, ∞]	[4.56 12.29]	[8.46 14.92]	[7.72 15.09]
Shocks					
Autocorr. Invest. Tech.	ρ_ψ	Beta	0.78	0.703	0.612
		[0, 1]	[0.53 0.91]	[0.54 0.77]	[0.53 0.77]
Std.Dev. Neutral Tech. Shock	σ_n	Inv. Gamma	0.06	0.211	0.282
		[0, ∞]	[0.04 0.44]	[0.18 0.25]	[0.26 0.33]
Std.Dev. Invest. Tech. Shock	σ_ψ	Inv. Gamma	0.06	0.125	0.149
		[0, ∞]	[0.04 0.44]	[0.09 0.17]	[0.10 0.17]
Std.Dev. Monetary Shock	σ_R	Inv. Gamma	0.22	0.496	0.597
		[0, ∞]	[0.14 1.49]	[0.41 0.60]	[0.52 0.71]

E. Estimation Results of Involuntary Unemployment Model with Constant $\tilde{\eta}$ ($\omega = 0$)

Technical Appendix Table A.2 contains the estimated parameters of the baseline involuntary unemployment model as well as the involuntary unemployment model when ω is set to zero, i.e. $\tilde{\eta}$ is constant. The posterior mode and parameter distributions are based on a standard MCMC algorithm with a total of 2.5 million draws based on 10 chains. We use the first 20 percent of draws for burn-in. The acceptance rates are about 0.25 in each chain. Technical Appendix Figures 1 through 4 show the impulse responses of the estimated baseline involuntary unemployment model and the involuntary unemployment model with $\omega = 0$ when both models are evaluated at the posterior mode shown in Technical Appendix Table A.2.

Technical Appendix Table A.2: Sensitivity of Estimated Involuntary Unemployment Model

Parameter		Prior Distribution [bounds]	Mode [2.5% 97.5%]	Posterior Mode [2.5% 97.5%] **Baseline Model**	**Model with $\omega = 0$**
		Price Setting Parameters			
Price Stickiness	ξ_p	Beta	0.67	0.727	0.745
		[0, 1]	[0.45 0.83]	[0.67 0.78]	[0.65 0.79]
Price Markup	λ_f	Gamma	1.19	1.399	1.491
		[1.001, ∞]	[1.01 1.40]	[1.29 1.54]	[1.38 1.64]
		Monetary Authority Parameters			
Taylor Rule: Int. Smoothing	ρ_R	Beta	0.76	0.890	0.802
		[0, 1]	[0.37 0.93]	[0.85 0.91]	[0.77 0.86]
Taylor Rule: Inflation Coef.	r_π	Gamma	1.68	1.414	1.338
		[1.001, ∞]	[1.41 2.00]	[1.19 1.69]	[1.19 1.62]
Taylor Rule: GDP Coef.	r_y	Gamma	0.07	0.113	0.028
		[0, ∞]	[0.02 0.21]	[0.05 0.18]	[0.01 0.08]
		Preference Parameters			
Consumption Habit	b	Beta	0.75	0.776	0.728
		[0, 1]	[0.64 0.83]	[0.74 0.80]	[0.68 0.76]
Inverse Labor Supply Elast.	σ_z	Gamma	0.26	0.334	0.267
		[0, ∞]	[0.13 0.52]	[0.17 0.43]	[0.13 0.35]
Replacement Ratio	c^{nw}/c^w	Beta	0.75	0.7973	0.818
		[0, 1]	[0.69 0.79]	[0.76 0.82]	[0.78 0.85]
Labor Force Impact on $p(e,\tilde{\eta})$	ω	Normal	0.0	-0.533	-
		[-∞, ∞]	[-1.96 1.96]	[-0.74 -0.38]	-
		Technology Parameters			
Capital Share	α	Beta	0.32	0.31	0.289
		[0, 1]	[0.28 0.37]	[0.25 0.33]	[0.25 0.32]
Technology diffusion	θ	Beta	0.50	0.052	0.009
		[0, 1]	[0.12 0.86]	[0.01 0.80]	[0.00 0.04]
Capacity Adj. Costs Curv.	σ_a	Gamma	0.31	0.462	0.312
		[0, ∞]	[0.09 1.22]	[0.21 0.56]	[0.16 0.54]
Investment Adj. Costs Curv.	S''	Gamma	7.50	11.56	12.24
		[0, ∞]	[4.56 12.29]	[8.46 14.92]	[9.37 16.56]
		Shocks			
Autocorr. Invest. Tech.	ρ_ψ	Beta	0.78	0.704	0.690
		[0, 1]	[0.53 0.91]	[0.59 0.82]	[0.57 0.79]
Std.Dev. Neutral Tech. Shock	σ_n	Inv. Gamma	0.06	0.194	0.194
		[0, ∞]	[0.04 0.44]	[0.17 0.23]	[0.16 0.22]
Std.Dev. Invest. Tech. Shock	σ_ψ	Inv. Gamma	0.06	0.115	0.128
		[0, ∞]	[0.04 0.44]	[0.08 0.15]	[0.09 0.16]
Std.Dev. Monetary Shock	σ_R	Inv. Gamma	0.22	0.449	0.535
		[0, ∞]	[0.14 1.49]	[0.37 0.53]	[0.40 0.63]

Figure Tech.App.1: Dynamic Responses to a Monetary Policy Shock

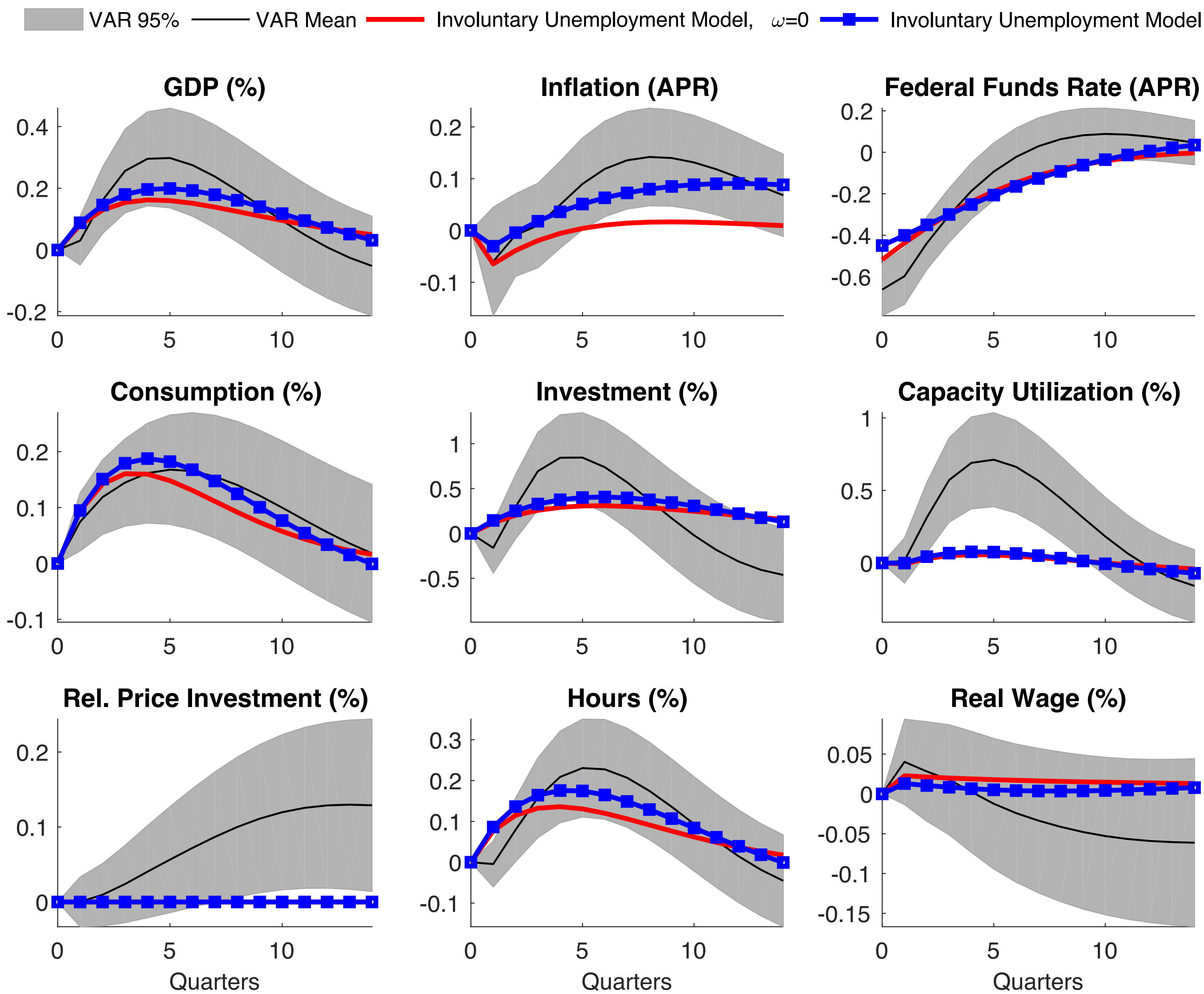

Figure Tech.App.2: Dynamic Responses to a Neutral Technology Shock

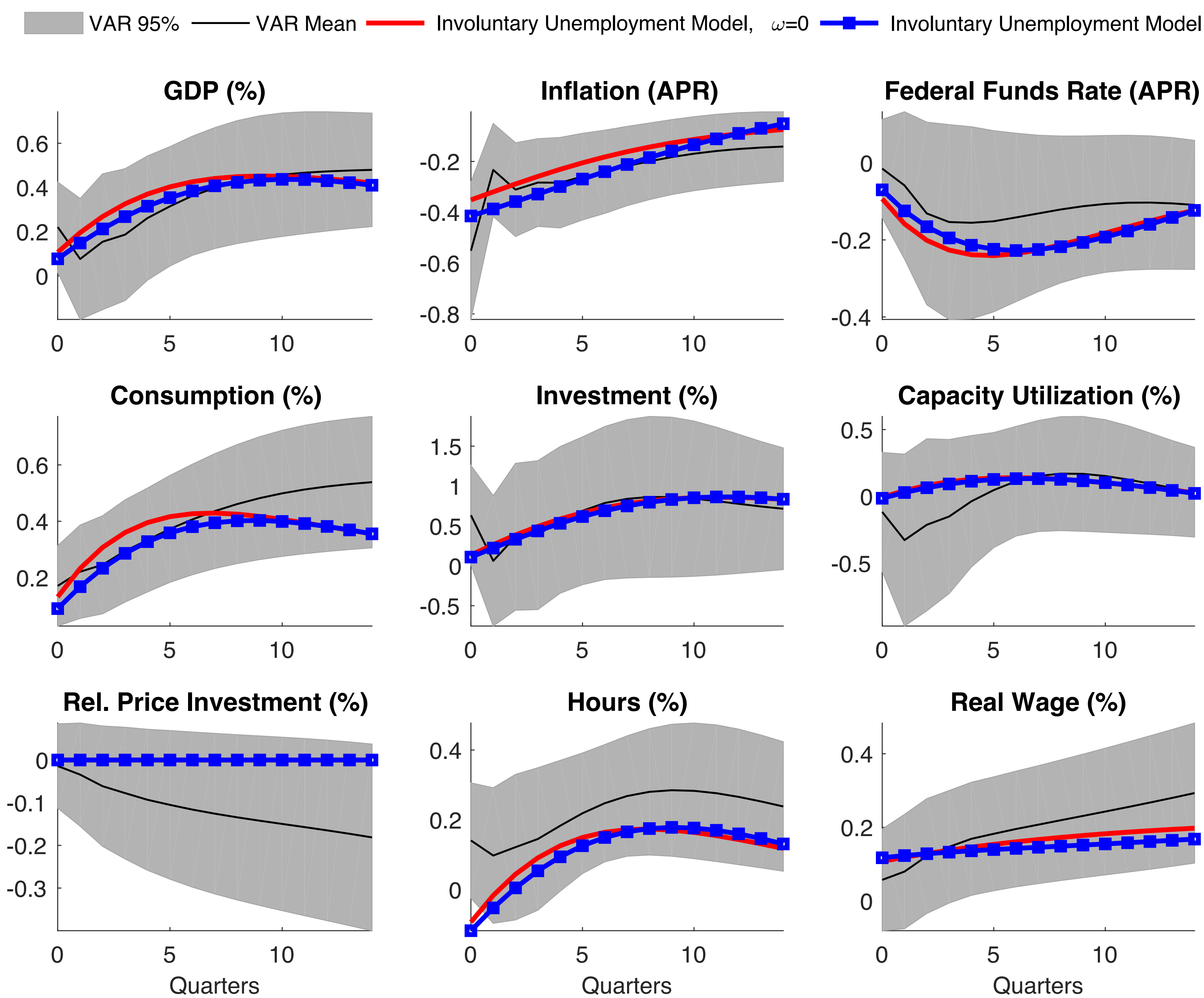

Figure Tech.App.3: Dynamic Responses to an Investment-Specific Technology Shock

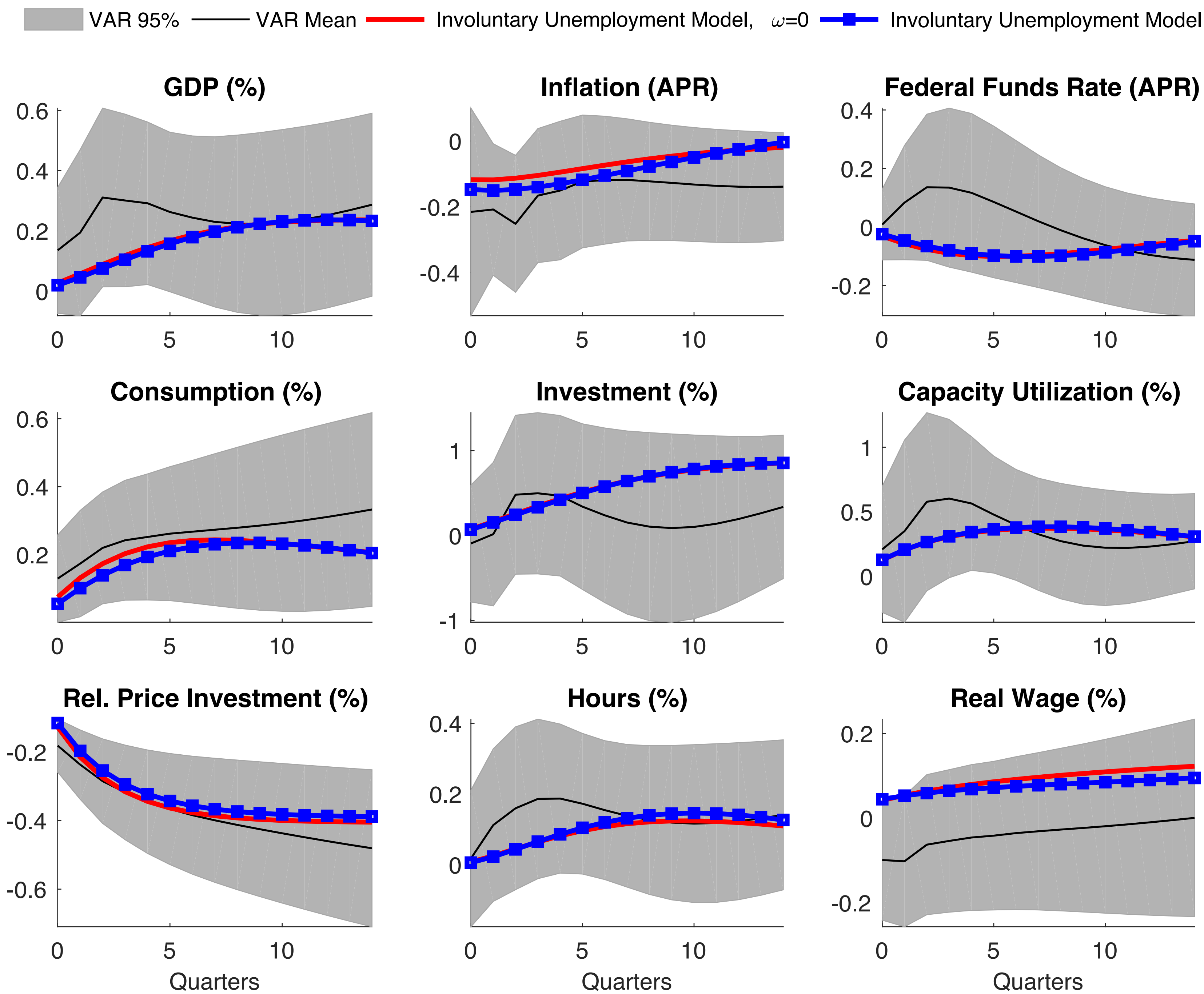

Figure Tech.App.4: Dynamic Responses of Unemployment and Labor Force to Three Shocks