Technical Appendix to "MAJA: A two-region DSGE model for Sweden and its main trading partners"

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July 2, 2020

Abstract

This Technical Appendix contains detailed derivations of the model presented in Corbo and Strid (2020). It contains the non-linear optimization problems and their solutions, the steady-state solution, and details on the scaling ang log-linearization of the model.

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1 Introduction

The model discussed in this document is an open-economy medium-sized DSGE model of the Swedish economy, presented in "MAJA: A two-region DSGE model for Sweden and its main trading partners". It draws heavily on the two previous DSGE models developed and used at the Riksbank, Ramses I and Ramses II, documented in Adolfson et al. (2005) and Adolfson et al. (2013), respectively. Ramses I was used as the main forecasting and simulation macro model at the Riksbank between 2005 and 2010. It was derived as an open-economy version of the Smets and Wouters (2003) model and estimated on data for the Swedish economy.[1] Ramses II was basically an extension of Ramses I, which included financial frictions in the accumulation of capital, search and matching on the labour market, and the use of imports as inputs in export production. It was originally developed and documented by Christiano, Trabandt, and Walentin (2011), and then adapted to policy purposes by the Modelling Division at the Riksbank.[2] Ramses II has beed the main macro model at the Riksbank since 2010, used for macroeconomic analysis, forecasting and alternative scenarios. The current model is the result of a project which aimed at evaluating and, ideally, improving on some dimensions of the previous model setups, among those the importance of foreign economic fluctuations for the Swedish economy.

To be able to evaluate the usefulness of alternative specifications and estimation methods, we need a benchmark to compare with. To this end, we derived and documented a version of the Ramses model which is closely in line with the Ramses II baseline model (Ramses II without financial frictions and search and matching) and very similar to Ramses I. We have chosen to keep the use of imports as inputs in export production and most of the minor changes made to the model between the Ramses I and II versions. Like its predecessors, our baseline model consists of a number of different firms, which import, export and combine different inputs in order to produce a number of different consumption and investment goods, and households, which consume, save in domestic and foreign bonds, and provide labour services to the firms. Firms and households are subject to price and wage setting frictions as in Calvo (1983), rendering monetary policy non-neutral. The central bank conducts monetary policy according to some interest rate rule, while government consumption is assumed to follow an exogenous process. With this as a starting point, we have made a number of alterations to the model structure. MAJA contains a structural model of the foreign economy, a more flexible model of the demand for Swedish exports, a differnet model of the labour market, a slow-moving interest rate trend (a 'neutral rate'), and a more disaggregated modelling of inflation, resulting from the an explicit treatment of energy prices. The specific differences and similarities to Ramses I and II are discussed more thoroghly throughout the document.

The rest of the document is organized as follows. We begin by introducing some definitions of growth rates, relative prices and inflation rates, needed for stationarizing the model. We then move on to describing the firms, starting with domestic and imported intermediate goods and then moving on to the production of final consumption, investment and export goods. We then consider the households, including wage setting, as households are assumed to be the ones setting the wages in our model economy. We then present our assumptions regarding the central bank and the government, and derive the aggregate resource constraint and the evolution of net foreign assets. The model is then completed with a despcription of the foreign economy block. Each section begins with the theoretical structure and the optimization problem of the respective agents. The derived conditions are then scaled to express the model in stationary form, and finally log-linearized. We present the scaling and log-linearization of each specific agent's problem in relation to the optimization problem, to ease reading. Towards the end of the document, we then include a separate section that summarizes the entire model in log-linear form. The documentation moreover includes the steady-state solution of

[1] In the first published documentations of Ramses I, Adolfson et al. (2007) and Adolfson et al. (2005), the model was estimated on data for the Euro Area. Model estimates of Ramses I on Swedish data are discussed.in Adolfson et al. (2008).

[2] See Adolfson et al. (2013) for a documentation of the policy version of Ramses II. In our work with the present model, we have also made use of earlier versions of the Christiano, Trabandt, and Walentin (2011) paper, some written in collaboration with the Modelling Division.

the model. Finally, we present a couple of alternative model setups, which were used in the model development stage.

2 Stationarization and linearization assumptions

In order to express all of the model variables in stationary form, we need to divide the quantities with the trend level of the neutral and, where applicable, investment-specific technologies. In the first part of this section, we specify how the scaling of the non-stationary variables is done. The second part of the section defines inflation rates and relative prices used in the model derivations. The final part contains a reminder of some log-linearization rules which we frequently use when log-linearizing the model.

2.1 Scaling of variables

This section specifies how the scaling of the non-stationary variables is done. We use the following scaling of variables, as in Christiano, Trabandt, and Walentin (2011) and Adolfson et al. (2013). The neutral shock of technology is denoted by z_t and has the following growth rate:

$$\frac{z_t}{z_{t-1}} = \mu_{z,t}. \tag{2.1}$$

In addition to the neutral technology shock, our model also includes an investment-specific technology shock, Ψ_t, with the following growth rate:

$$\frac{\Psi_t}{\Psi_{t-1}} = \mu_{\Psi,t}.$$

We define the following combination of investment-specific and neutral technology:

$$z_t^+ = \Psi_t^{\frac{\alpha}{1-\alpha}} z_t, \tag{2.2}$$

the growth rate of which is given by

$$\mu_{z^+,t} = \frac{z_t^+}{z_{t-1}^+} = \mu_{\Psi,t}^{\frac{\alpha}{1-\alpha}} \mu_{z,t}. \tag{2.3}$$

Capital and investment are scaled by $z_t^+ \Psi_t$.The inputs to the production of final investment, however, are scaled by z_t^+, as Ψ_t is defined as a shock to the technology employed in the aggregation of domestic and foreign intermediate investment goods into final goods. All consumption goods are scaled by z_t^+, including government consumption goods, and so are all export goods. The real wage and real foreign assets, where a bar above the variable denotes the real version of the corresponding nominal variable, are also scaled by z_t^+. The Lagrangian multiplier υ_t is the shadow value in utility terms of domestic currency, and $\psi_t \equiv \upsilon_t P_t^d$ is the shadow value of one consumption good (i.e. the marginal utility of consumption). This needs to be multiplied by z_t^+ to induce stationarity. Thus,

$$\begin{aligned}
k_{t+1} &= \frac{K_{t+1}}{z_t^+ \Psi_t},\ k_{t+1}^p = \frac{K_{t+1}^p}{z_t^+ \Psi_t},\ i_t = \frac{I_t}{z_t^+ \Psi_t}, \\
y_t &= \frac{Y_t}{z_t^+},\ c_t = \frac{C_t}{z_t^+},\ g_t = \frac{G_t}{z_t^+},\ x_t = \frac{X_t}{z_t^+}, \\
i_t^d &= \frac{I_t^d}{z_t^+},\ i_t^m = \frac{I_t^m}{z_t^+},\ c_t^d = \frac{C_t^d}{z_t^+},\ c_t^m = \frac{C_t^m}{z_t^+},\ x_t^m = \frac{X_t^m}{z_t^+}, \\
\bar{w}_t &= \frac{\bar{W}_t}{z_t^+} = \frac{W_t}{z_t^+ P_t^d},\ \bar{a}_t = \frac{\bar{A}_t}{z_t^+} = \frac{S_t B_{t+1}^F}{P_t^d z_t^+}, \\
\psi_{z^+,t} &= \upsilon_t P_t^d z_t^+.
\end{aligned}$$

We denote the scaled date-t price of physical capital installed for the start of period $t+1$ by $\breve{p}_{k',t}$, and the scaled rental rate of capital by r_t^k, so that

$$\begin{aligned}\breve{p}_{k',t} &= \Psi_t p_{k',t}, \\ r_t^k &= \Psi_t R_t^k,\end{aligned}$$

where $p_{k',t}$ is denoted in units of the domestic homogeneous good, i.e.

$$p_{k',t} = \frac{P_{k',t}}{P_t^d}.$$

Moreover, we denote by bars the real version of the corresponding nominal variable, so

$$\bar{r}_t^k = \frac{r_t^k}{P_t^d},\ \bar{R}_t^k = \frac{R_t^k}{P_t^d}, \bar{W}_t = \frac{W_t}{P_t^d}.$$

Note also that we define the real interest rate as follows:

$$\bar{R}_t = \frac{R_t}{E_t \pi_{t+1}^c}.$$

Finally, note that for the foreign economy, we have

$$\frac{z_t^*}{z_{t-1}^*} = \mu_{z^*,t},$$

$$\frac{\Psi_t^*}{\Psi_{t-1}^*} = \mu_{\Psi^*,t}$$

$$z_t^{+,*} = (\Psi_t^*)^{\frac{\alpha^*}{1-\alpha^*}} z_t^*,$$

$$\mu_{z^{+,*},t} = \frac{z_t^{+,*}}{z_{t-1}^{+,*}} = \left(\mu_{\Psi^*,t}\right)^{\frac{\alpha^*}{1-\alpha^*}} \mu_{z^*,t},$$

$$\begin{aligned}k_{t+1}^* &= \frac{K_{t+1}^*}{z_t^{+,*}\Psi_t^*},\ k_{t+1}^{p,*} = \frac{K_{t+1}^{p,*}}{z_t^{+,*}\Psi_t^*},\ i_t^* = \frac{I_t^*}{z_t^{+,*}\Psi_t^*}, \\ y_t^* &= \frac{Y_t^*}{z_t^{+,*}},\ c_t^* = \frac{C_t^*}{z_t^{+,*}},\ g_t^* = \frac{G_t^*}{z_t^{+,*}}, \\ \bar{w}_t^* &= \frac{\bar{W}_t^*}{z_t^{+,*}} = \frac{W_t^*}{z_t^{+,*}P_t^*} \\ \psi_{z^{+,*},t} &= v_t^* P_t^* z_t^{+,*},\end{aligned}$$

$$\begin{aligned}p_{k',t}^* &= \frac{P_{k',t}^*}{P_t^*}, \\ \breve{p}_{k',t}^* &= \Psi_t^* p_{k',t}^*,\end{aligned}$$

and

$$\begin{aligned}\bar{r}_t^{k,*} &= \frac{r_t^{k,*}}{P_t^*},\ \bar{R}_t^{k,*} = \frac{R_t^{k,*}}{P_t^*}, \\ r_t^k &= \Psi_t^* R_t^{k,*}.\end{aligned}$$

2.2 Definitions of inflation rates and relative prices

We define the following inflation rates:

$$\begin{aligned}
\pi_t^d &= \frac{P_t^d}{P_{t-1}^d},\ \pi_t^c = \frac{P_t^c}{P_{t-1}^c},\ \pi_t^{cxe} = \frac{P_t^{cxe}}{P_{t-1}^{cxe}},\\
\pi_t^{ce} &= \frac{P_t^{ce}}{P_{t-1}^{ce}},\ \pi_t^{d,ce} = \frac{P_t^{d,ce}}{P_{t-1}^{d,ce}},\\
\pi_t^i &= \frac{P_t^i}{P_{t-1}^i},\ \pi_t^{m,j} = \frac{P_t^{m,j}}{P_{t-1}^{m,j}},\ \pi_t^x = \frac{P_t^x}{P_{t-1}^x},
\end{aligned}$$

for $j = c, i, x, ce$. π_t^d is the rate of inflation of the domestic homogeneous good, π_t^c the rate of inflation of the final consumption good (i.e. the CPI inflation), π_t^{cxe} the rate of inflation of the aggregate non-energy consumption good, π_t^{ce} the rate of inflation of the aggregate energy consumption good, $\pi_t^{d,ce}$ the rate of inflation of the domestically-produced energy consumption good, π_t^i the rate of inflation of the final invesmtent good, and $\pi_t^{m,j}$ for $j = c, i, x, ce$ the rate of inflation of the import goods to be used in the production of consumption, investment, exports, and energy consumption, respectively. The corresponding prices are all in domestic currency. The price of exports is instead denoted in foreign currency units. π_t^x is the rate of inflation of the final export good. For the foreign economy, in an analogous way, we define:

$$\begin{aligned}
\pi_t^{d,*} &= \frac{P_t^{d,*}}{P_{t-1}^{d,*}},\ \pi_t^{c,*} = \frac{P_t^{c,*}}{P_{t-1}^{c,*}},\ \pi_t^{cxe,*} = \frac{P_t^{cxe,*}}{P_{t-1}^{cxe,*}},\\
\pi_t^{ce,*} &= \frac{P_t^{ce,*}}{P_{t-1}^{ce,*}},\ \pi_t^{i,*} = \frac{P_t^{i,*}}{P_{t-1}^{i,*}}.
\end{aligned}$$

We further define the following relative prices:

$$\begin{aligned}
p_t^c &= \frac{P_t^c}{P_t^d},\ p_t^{cxe} = \frac{P_t^{cxe}}{P_t^d},\ p_t^{ce} = \frac{P_t^{ce}}{P_t^d},\ p_t^{d,ce} = \frac{P_t^{d,ce}}{P_t^d},\\
p_t^i &= \frac{\Psi_t P_t^i}{P_t^d},\ p_t^x = \frac{P_t^x}{P_t^{d,*}}\left(\tilde{z}_t^{+,*}\right)^{-\frac{1}{\eta_f}},\\
p_t^{m,c} &= \frac{P_t^{m,c}}{P_t^d},\ p_t^{m,i} = \frac{P_t^{m,i}}{P_t^d},\ p_t^{m,x} = \frac{P_t^{m,x}}{P_t^d},\ p_t^{m,ce} = \frac{P_t^{m,ce}}{P_t^d}.
\end{aligned}$$

When the price is denominated in domestic currency units, we define a lower case price as the corresponding upper case price divided by the price of the homogeneous good. The handling of the price of final investment goods differs somewhat, however, from that of the prices of final consumption goods and imported goods. This is due to the shock to the final investment production technology Ψ_t, which makes the price of the final investment good grow at a rate slower than P_t^d. Thus, final investment prices are scaled by P_t^d/Ψ_t. Finally, as the export price is denominated in foreign currency units, we scale it by the price of the foreign homogeneous good. For the foreign economy, we define

$$p_t^{c,*} = \frac{P_t^{c,*}}{P_t^{d,*}},\ p_t^{cxe,*} = \frac{P_t^{cxe,*}}{P_t^{d,*}},\ p_t^{ce,*} = \frac{P_t^{ce,*}}{P_t^{d,*}},\ p_t^{i,*} = \frac{\Psi_t^* P_t^{i,*}}{P_t^{d,*}}.$$

We define also the relative optimal wages in the domestic and foreign economy:

$$\begin{aligned}
\tilde{w}_t &= \frac{\tilde{W}_t}{W_t},\\
\tilde{w}_t^* &= \frac{\tilde{W}_t^*}{W_t^*}.
\end{aligned}$$

Finally, we denote the growth rate of the nominal exchange rate by s_t, so that

$$s_t = \frac{S_t}{S_{t-1}},$$

and define the real exchange rate as

$$q_t = \frac{S_t P_t^{c,*}}{P_t^c}.$$

2.3 Log-linearization of the model

In what follows, variables without time subscript denote steady-state values, and variables with a hat denote log-deviation from their steady-state values. Consider a variable X_t. We define:

$$\hat{X}_t \equiv \log X_t - \log X.$$

Taking a Taylor expansion of order 1 of the right-hand side, we get:

$$\begin{aligned} \hat{X}_t &\approx \log X + \frac{\partial}{\partial X_t} \log X_t \big|_{X_t = X} (X_t - X) - \log X \\ &= \frac{X_t - X}{X}. \end{aligned}$$

Thus, for small $\hat{X}_t$, it can be interpretated as a percentage deviation of X_t from its steady-state value X. It is also useful to note that:

$$\hat{X}_t = \log\left(\frac{X_t}{X}\right) \Rightarrow X_t = X e^{\hat{X}_t} \approx X\left(e^0 + e^0\left(\hat{X}_t - 0\right)\right) = X\left(1 + \hat{X}_t\right).$$

Finally, recall that if f is a function of n arguments $(x_1, \ldots, x_n)$, a Taylor expansion of order 1 of f around $(a_1, \ldots, a_n)$ is:

$$f(x_1, \ldots, x_n) \approx f(a_1, \ldots, a_n) + \sum_{i=1}^{n} \frac{\partial}{\partial x_i} f(x_1, \ldots, x_n) \big|_{(x_1, \ldots, x_n = a_1, \ldots, a_n)} (x_i - a_i).$$

3 Firms

The production structure adopted in this model is similar to the one in Adolfson et al. (2013). Consumption, investment and exports are the three final goods. They are produced by combining the domestic homogeneous good with homogeneous goods derived from imports. Moreover, to arrive at the final consumption good, the combined domestic and imported final (non-energy) consumption good is combined with energy, which, in turn, is also a comination of domestic and imported energy. The domestic homogeneous good is produced by competitive retailers who buy their input, the domestic intermediate good, from the domestic intermediate goods producers. The domestic intermediate goods producers have monopoly power. Importing firms buy a foreign homogeneous good or foreign energy, respectively. They transform the former into a specialized imported intermediate good, which is supplied monopolistically to three different types of import retailers that produce the aggregate imported goods. In an analogous way, foreign energy is transformed into an imported energy good. Exporting firms produce a specialized export good combining domestic and imported intermediate inputs, sold to foreign competitive retailers which create a homogeneous good that is eventually sold to foreign agents. The firms are owned by the households in the economy, to which any firm profits will accrue.

Unlike in Adolfson et al. (2005) and Adolfson et al. (2013), in this version of the model we abstract from taxes. We choose to do so in order to reduce the number of variables and notational complexity of the model. As we wish to still include shocks to the Phillips curves, we instead assume that price and wage markups are varying. In the absence of price and wage distortions in steady state, this is isomorphic to including tax-like shocks which affect marginal costs, as mentioned in Adolfson et al. (2013).[3] Note that there are other taxes in Ramses I and II than the ones affecting marginal costs, such as taxes on capital holdings, which are omitted from our model for simplicity. These can be easily included in future versions if needed.

3.1 Production of domestic homogeneous goods

The domestic homogeneous good is produced using the following technology:

$$Y_t = \left[\int_0^1 (Y_{i,t})^{\frac{1}{\lambda_t^d}} di \right]^{\lambda_t^d}, \qquad 1 \leq \lambda_t^d \leq \infty, \tag{3.1}$$

where λ_t^d is a stochastic process determining the time-varying price markup in the domestic goods market, given by[4]

$$\log \lambda_t^d = \left(1 - \rho_{\lambda^d}\right) \log \lambda^d + \rho_{\lambda^d} \log \lambda_{t-1}^d + \sigma_{\lambda^d} \varepsilon_{\lambda^d,t}. \tag{3.2}$$

For calibration purposes, it is useful to note that λ^d is related to the elasticity of substitution between the different domestic goods, which we denote by η_d, in the following way: $\lambda^d = \frac{\eta_d}{\eta_d - 1}$. The domestic homogeneous good is produced by a representative, competitive firm which takes the price of output P_t^d and the price of inputs $P_{i,t}^d$ as given. The profit maximization problem writes:

$$\max_{Y_{i,t}} \quad P_t^d Y_t - \int_0^1 P_{i,t}^d Y_{i,t} di,$$

which yields the following first order condition:

$$Y_{i,t} = \left(\frac{P_{i,t}^d}{P_t^d} \right)^{\frac{\lambda_t^d}{1-\lambda_t^d}} Y_t. \tag{3.3}$$

Taking the integral over i in (3.3), and using the CES aggregator in (3.1) leads to the expression of the aggregate price level:

$$P_t^d = \left[\int_0^1 \left(P_{i,t}^d \right)^{\frac{1}{1-\lambda_t^d}} di \right]^{1-\lambda_t^d}. \tag{3.4}$$

Turning now to the intermediate goods producers, the i^{th} firm has the following production function:

$$Y_{i,t} = (z_t N_{i,t})^{1-\alpha} \epsilon_t K_{i,t}^{\alpha} - z_t^+ \phi^d, \tag{3.5}$$

where $K_{i,t}$ denotes the capital services rented by the i^{th} intermediate firm, and $N_{i,t}$ denotes homogeneous labour service hired by the same firm.[5] z_t is a technology shock whose first difference has a positive mean, ϵ_t is a stationary neutral technology shock, and ϕ^d denotes a fixed production cost. The fixed cost is assumed to grow at the same rate as consumption, the real wage and output in steady state, to ensure that profits remain zero. In general, the economy has two sources of growth: a

[3] Time-varying price-markups was also the assumption used in Adolfson et al. (2005). Calculations are simplified somewhat, however, by assuming tax-like shocks instead of time-varying price markups. This becomes particularly important if the model is not linearized (by hand), but rewritten recursively in its non-linear form (to be linearized by Dynare).

[4] The exogenous processes are discussed in more detail in Section 10.

[5] Note that $K_{i,t}$ may differ from the physical capital stock, K_t^p, since we allow for variable capital utilization in the model. This is discussed in more detail in Section 4.

positive drift in z_t and a positive drift in Ψ_t, where Ψ_t is the state of an investment-specific technology shock. The object z_t^+ in (3.5) is defined as follows:

$$z_t^+ = \Psi_t^{\frac{\alpha}{1-\alpha}} z_t.$$

The stationary neutral technology shock, ϵ_t, and the growth rates of z_t and Ψ_t, $\mu_{z,t}$ and $\mu_{\Psi,t}$, are assumed to evolve according to the following processes:

$$\log \epsilon_t = (1-\rho_\epsilon)\log \epsilon + \rho_\epsilon \log \epsilon_{t-1} + \sigma_\epsilon \varepsilon_{\epsilon,t}, \tag{3.6}$$

$$\log \mu_{z,t} = \left(1-\rho_{\mu_z}\right)\log \mu_{z^+} + \rho_{\mu_z}\log \mu_{z,t-1} + \sigma_{\mu_z}\varepsilon_{\mu_z,t}, \tag{3.7}$$

and

$$\log \mu_{\Psi,t} = \left(1-\rho_{\mu_\Psi}\right)\log \mu_{\Psi} + \rho_{\mu_\Psi}\log \mu_{\Psi,t-1} + \sigma_{\mu_\Psi}\varepsilon_{\mu_\Psi,t}. \tag{3.8}$$

Note that the growth rate of z_t^+ is then given by the following equation:

$$\mu_{z^+,t} = \frac{z_t^+}{z_{t-1}^+} = \mu_{\Psi,t}^{\frac{\alpha}{1-\alpha}}\mu_{z,t}. \tag{3.9}$$

The cost minimization problem of the i^{th} intermediate firm is to minimize total costs subject to the constraint of producing enough to meet demand:

$$\begin{cases} \min\limits_{N_{i,t},\,K_{i,t}} \quad N_{i,t}W_t R_t^{wc,d} + K_{i,t}R_t^k \\ s.t. \quad (z_t N_{i,t})^{1-\alpha}\epsilon_t K_{i,t}^{\alpha} - z_t^+\phi^d \geq \left(\frac{P_{i,t}^d}{P_t^d}\right)^{-\frac{\lambda_t^d}{\lambda_t^d-1}} Y_t, \end{cases}$$

where R_t^k is the gross nominal rental rate per unit of capital services, and W_t is the nominal wage rate per unit of aggregate homogeneous labour $N_{i,t}$, common to all intermediate firms. $R_t^{wc,d}$ denotes the gross effective nominal rate of interest faced by the firms, and it reflects the assumption that a fraction $\nu_t^{wc,d}$ of the firms' wage bill has to be financed in advance.[6] The end of period labour cost faced by the firm is $N_{i,t}W_tR_t^{wc,d}$, with $R_t^{wc,d}$ defined as follows:

$$R_t^{wc,d} \equiv \nu_t^{wc,d}R_t + 1 - \nu_t^{wc,d}, \tag{3.10}$$

where R_t denotes the gross nominal interest rate determined by the central bank.[7] We assume that $\nu_t^{wc,d}$ follows

$$\log \nu_t^{wc,d} = (1-\rho_{\nu^{wc,d}})\log \nu^{wc,d} + \rho_{\nu^{wc,d}}\log \nu_{t-1}^{wc,d} + \sigma_{\nu^{wc,d}}\varepsilon_{\nu^{wc,d},t}. \tag{3.11}$$

[6] Note that, given our labor market setup discussed further in Section 4, wages are expressed as wages per employee rather than wages per hour worked as was the case in Ramses I and II (see Adolfson et al. (2005) and Adolfson et al. (2013), respectively).

[7] Here, we have used the same definition of $R_t^{wc,d}$ as in Ramses II (see Christiano, Trabandt, and Walentin (2011) and Adolfson et al. (2013)) and Christiano, Eichenbaum, and Evans (2005), for being the more recent one (note that in previous versions of Ramses, $R_t^{wc,d}$ was denoted R_t^f). In Ramses I, $R_t^{wc,d}$ is instead given by

$$R_t^{wc,d} \equiv \nu_t^{wc,d}R_{t-1} + 1 - \nu_t^{wc,d},$$

which is motivated by the fact that households purchase one-period zero-coupon bonds with certain nominal payout in period $t+1$. This choice is not likely to be of much importance, however, as it has been found that the working capital channel does not noteably improve the model fit. In fact, Adolfson et al. (2005) demonstrate that a version of Ramses I that excludes the working capital channel is favoured by the data compared to the baseline. It is relevant to note that the model is estimated using Bayesian estimation techniques, and that a different estimation method (e.g. matching IRF) may yield different results. Nonetheless, in later versions of the model, this channel is entirely shut off.

We note also that the above applies to the definitions of all the gross effective nominal rates of interest faced by the different types of firms in the model.

The Lagrange multiplier associated with the constraint in the cost minimization problem will have the interpretation of the nominal marginal cost (i.e. the nominal cost of producing an additional unit of the domestic good). The first order conditions (henceforth FOC) with respect to $N_{i,t}$ and $K_{i,t}$ are given by:

$$W_t R_t^{wc,d} = MC_{i,t}^d (1-\alpha) z_t^{1-\alpha} \epsilon_t \left(\frac{K_{i,t}}{N_{i,t}}\right)^{\alpha} \tag{3.12}$$

$$R_t^k = MC_{i,t}^d \alpha z_t^{1-\alpha} \epsilon_t \left(\frac{K_{i,t}}{N_{i,t}}\right)^{-(1-\alpha)}. \tag{3.13}$$

Combining these two FOCs, one gets the following expression for the nominal marginal cost:

$$MC_t^d = \frac{\left(W_t R_t^{wc,d}\right)^{1-\alpha}}{\left(R_t^k\right)^{-\alpha}} \frac{z_t^{-(1-\alpha)}}{(1-\alpha)^{1-\alpha} \alpha^{\alpha} \epsilon_t}, \tag{3.14}$$

where the firm index i is dropped as all the variables entering the right-hand side are aggregate variables. We note that, using FOC (3.12), we could also write the expression for the nominal marginal cost as follows:

$$MC_t^d = \frac{W_t R_t^{wc,d}}{MPL_{i,t}} = \frac{W_t R_t^{wc,d}}{(1-\alpha) z_t^{1-\alpha} \epsilon_t \left(\frac{K_{i,t}}{N_{i,t}}\right)^{\alpha}}, \tag{3.15}$$

where $MPL_{i,t}$ denotes the marginal product of labour $(\partial Y_{i,t}/\partial N_{i,t})$ of the i^{th} intermediate producer. Using FOC (3.13), we could also write:

$$MC_t^d = \frac{R_t^k}{MPK_{i,t}} = \frac{R_t^k}{\alpha z_t^{1-\alpha} \epsilon_t \left(\frac{K_{i,t}}{N_{i,t}}\right)^{-(1-\alpha)}}, \tag{3.16}$$

where $MPK_{i,t}$ denotes the marginal product of capital $(\partial Y_{i,t}/\partial K_{i,t})$ of the i^{th} intermediate producer. We need only one additional expression in the final set of model equations. Note that (3.14) implies that the capital services to labour ratio is the same for all firms, as they all face the same factor prices. We note also that the marginal cost equals the average unit cost, as our assumption about constant return to scale implies that the marginal cost does not change with output. Combining (3.14) and (3.15), we obtain the solution for the nominal rental rate of capital services:

$$R_t^k = \frac{\alpha}{1-\alpha} W_t R_t^{wc,d} \frac{N_t}{K_t}. \tag{3.17}$$

The i^{th} firm is a monopolist in the production of the i^{th} intermediate goods and it sets its price in a staggered fashion, following Calvo (1983). Each intermediate firm faces a probability $(1-\xi_d)$ that it can reoptimize its price in any period, independent on the time that has passed since it was last able to reoptimize. If the firm is not able to reoptimize in period t, the price in period $t+1$ will be set according to the following indexation rule:

$$\begin{cases} P_{i,t}^d = \tilde{\pi}_t^d P_{i,t-1}^d \\ \tilde{\pi}_t^d \equiv \left(\pi_{t-1}^d\right)^{\kappa_d} \left(\bar{\pi}_t^c\right)^{1-\kappa_d-\varkappa_d} \left(\breve{\pi}\right)^{\varkappa_d}, \end{cases} \tag{3.18}$$

where κ_d, $\varkappa_d$ are parameters such that κ_d, $\varkappa_d$, $\kappa_d + \varkappa_d \in [0,1]$, π_{t-1}^d is the lagged domestic gross inflation rate, and $\bar{\pi}_t^c$ is a time-varying inflation trend or, alternatively, the time-varying central-bank target inflation rate. Note that we allow $\bar{\pi}_t^c$ to vary over time, to capture medium-term movement in inflation expectations (or, potentially, changes in policy makers' preferences). It is assumed to follow the process

$$\log \bar{\pi}_t^c = (1-\rho_{\bar{\pi}^c}) \log \bar{\pi}^c + \rho_{\bar{\pi}^c} \log \bar{\pi}_{t-1}^c + \sigma_{\bar{\pi}^c} \varepsilon_{\bar{\pi}^c,t}. \tag{3.19}$$

$\breve{\pi}$ is a scalar which allows to capture, among other things, cases in which non-optimizing firms do not change their prices at all ($\breve{\pi} = \varkappa_d = 1$) or index only to the steady-state inflation rate ($\breve{\pi} = \bar{\pi}$, $\varkappa_d = 1$).

Consider a firm that optimized its price in period t and has not been allowed to reoptimize during s periods ahead. Denoting by $\tilde{P}^d_{i,t}$ the reoptimized price in period t, the price that it will charge in $t+s$ is given by:

$$P^d_{i,t+s} = \prod_{j=1}^{s} \tilde{\pi}^d_{t+j} \tilde{P}^d_{i,t}. \tag{3.20}$$

Hence, when setting its price at time t, the firm i will maximize its future discounted profits, taking into account that it will not get to reoptimize the time-t price in $t+1$ with probability ξ_d, in $t+2$ with probability $(\xi_d)^2$ and so on. Thus, intermediate firm i faces the following optimization problem:

$$\begin{cases} \max\limits_{\tilde{P}^d_{i,t}} \quad E_t \sum\limits_{s=0}^{\infty} (\beta\xi_d)^s \zeta^\beta_{t+s} \upsilon_{t+s} \left(P^d_{i,t+s} Y_{i,t+s} - mc^d_{t+s} P^d_{t+s} Y_{i,t+s} \right) \\ \qquad s.t. \quad Y_{i,t} = \left(\frac{P^d_{i,t}}{P^d_t} \right)^{-\frac{\lambda^d_t}{\lambda^d_t - 1}} Y_t, \end{cases}$$

where υ_t is the multiplier on the household's budget constraint, and measures the marginal value to the household of one unit of profit in terms of currency. mc^d_t denotes the real marginal cost. Substituting in the demand constraint, derived in equation (3.3), and equation (3.20), and rearranging, the optimization problem becomes:

$$\max_{\tilde{P}^d_{i,t}} \quad E_t \sum_{s=0}^{\infty} (\beta\xi_d)^s \zeta^\beta_{t+s} \upsilon_{t+s} P^d_{t+s} Y_{t+s} \left[\left(\frac{\tilde{\pi}^d_{t+1} \dots \tilde{\pi}^d_{t+s}}{P^d_{t+s}} \tilde{P}^d_{i,t} \right)^{1-\frac{\lambda^d_{t+s}}{\lambda^d_{t+s}-1}} - mc^d_{t+s} \left(\frac{\tilde{\pi}^d_{t+1} \dots \tilde{\pi}^d_{t+s}}{P^d_{t+s}} \tilde{P}^d_{i,t} \right)^{-\frac{\lambda^d_{t+s}}{\lambda^d_{t+s}-1}} \right]. \tag{3.21}$$

The FOC associated with this problem directly yields the expression for the optimal price:

$$\tilde{P}^d_t = \frac{E_t \sum\limits_{s=0}^{\infty} (\beta\xi_d)^s \zeta^\beta_{t+s} \upsilon_{t+s} P^d_{t+s} Y_{t+s} \lambda^d_{t+s} mc^d_{t+s} \left(\frac{\tilde{\pi}^d_{t+1} \dots \tilde{\pi}^d_{t+s}}{P^d_{t+s}} \right)^{\frac{\lambda^d_{t+s}}{1-\lambda^d_{t+s}}}}{E_t \sum\limits_{s=0}^{\infty} (\beta\xi_d)^s \zeta^\beta_{t+s} \upsilon_{t+s} P^d_{t+s} Y_{t+s} \left(\frac{\tilde{\pi}^d_{t+1} \dots \tilde{\pi}^d_{t+s}}{P^d_{t+s}} \right)^{\frac{1}{1-\lambda^d_{t+s}}}}.$$

Note that we drop the subscript i in the expression of the optimal price. This reflects the fact that all firms face the same optimization problem and hence have the same solution. To rewrite in terms of relative prices, divide both sides by P^d_t to obtain

$$\tilde{p}^d_t = \frac{E_t \sum\limits_{s=0}^{\infty} (\beta\xi_d)^s \zeta^\beta_{t+s} \upsilon_{t+s} P^d_{t+s} Y_{t+s} \lambda^d_{t+s} mc^d_{t+s} \left(\frac{\tilde{\pi}^d_{t+1} \dots \tilde{\pi}^d_{t+s}}{\pi^d_{t+1} \dots \pi^d_{t+s}} \right)^{\frac{\lambda^d_{t+s}}{1-\lambda^d_{t+s}}}}{E_t \sum\limits_{s=0}^{\infty} (\beta\xi_d)^s \zeta^\beta_{t+s} \upsilon_{t+s} P^d_{t+s} Y_{t+s} \left(\frac{\tilde{\pi}^d_{t+1} \dots \tilde{\pi}^d_{t+s}}{\pi^d_{t+1} \dots \pi^d_{t+s}} \right)^{\frac{1}{1-\lambda^d_{t+s}}}}, \tag{3.22}$$

where $\tilde{p}^d_t = \frac{\tilde{P}^d_t}{P^d_t}$, and we have used that $\frac{1}{P^d_t} = \frac{\left(P^d_t\right)^{\frac{\lambda^d_{t+s}}{1-\lambda^d_{t+s}}}}{\left(P^d_t\right)^{\frac{1}{1-\lambda^d_{t+s}}}}$ and $\frac{P^d_{t+s}}{P^d_t} = \pi^d_{t+s} \dots \pi^d_{t+1}$.

We consider again the definition of the price index P^d_t in equation (3.4). Bearing in mind that a fraction ξ_d of firms index their price, while the remaining firms get to reoptimize, we can rewrite the price index as follows:

$$\left(P^d_t\right)^{\frac{1}{1-\lambda^d_t}} = \left(\tilde{\pi}^d_t\right)^{\frac{1}{1-\lambda^d_t}} \int_0^{\xi_d} \left(P^d_{i,t-1}\right)^{\frac{1}{1-\lambda^d_t}} di + (1-\xi_d) \left(\tilde{P}^d_t\right)^{\frac{1}{1-\lambda^d_t}}.$$

There is a continuum of firms in the economy. Due to the Calvo pricing assumption, the firms who get to update their prices are randomly chosen, and thus, the integral of individual prices over some subset of the unit interval will be proportional to the integral over the entire unit interval, where the proportion is equal to the subset over which the integral is taken. Hence,

$$\int_0^{\xi_d} \left(P_{i,t-1}^d\right)^{\frac{1}{1-\lambda_t^d}} di = \xi_d \int_0^1 \left(P_{i,t-1}^d\right)^{\frac{1}{1-\lambda_t^d}} di = \xi_d \left(P_{t-1}^d\right)^{\frac{1}{1-\lambda_t^d}}.$$

This implies that

$$\left(P_t^d\right)^{\frac{1}{1-\lambda_t^d}} = \xi_d \left(\tilde{\pi}_t^d P_{t-1}^d\right)^{\frac{1}{1-\lambda_t^d}} + (1-\xi_d)\left(\tilde{P}_t^d\right)^{\frac{1}{1-\lambda_t^d}}.$$

Dividing both sides by $\left(P_t^d\right)^{\frac{1}{1-\lambda_t^d}}$, and solving for $\tilde{p}_t^d$, we have

$$\tilde{p}_t^d = \left[\frac{1-\xi_d\left(\frac{\tilde{\pi}_t^d}{\pi_t^d}\right)^{\frac{1}{1-\lambda_t^d}}}{(1-\xi_d)}\right]^{1-\lambda_t^d}. \tag{3.23}$$

It is also useful to note, for later use, that we can obtain the total demand for homogeneous goods by integrating equation (3.3) over i. We then have

$$\mathring{Y}_t \equiv \int_0^1 Y_{i,t} di \tag{3.24}$$

$$= \int_0^1 \left(\frac{P_{i,t}^d}{P_t^d}\right)^{\frac{\lambda_t^d}{1-\lambda_t^d}} Y_t di$$

$$= Y_t \left(\frac{\mathring{P}_t^d}{P_t^d}\right)^{\frac{\lambda_t^d}{1-\lambda_t^d}}, \tag{3.25}$$

where $\mathring{P}_t^d$ denotes a measure of price dispersion, defined as follows:

$$\mathring{P}_t^d = \left[\int_0^1 \left(P_{i,t}^d\right)^{\frac{\lambda_t^d}{1-\lambda_t^d}} di\right]^{\frac{1-\lambda_t^d}{\lambda_t^d}}.$$

We can divide by P_t^d, to obtain the following expression in terms of relative prices:

$$\mathring{p}_t^d = \left[\int_0^1 \left(\frac{P_{i,t}^d}{P_t^d}\right)^{\frac{\lambda_t^d}{1-\lambda_t^d}} di\right]^{\frac{1-\lambda_t^d}{\lambda_t^d}}, \tag{3.26}$$

yielding

$$\mathring{Y}_t = Y_t \left(\mathring{p}_t^d\right)^{\frac{\lambda_t^d}{1-\lambda_t^d}}.$$

We can break this integral, using the Calvo assumption on price setting, and re-express it in terms of relative prices as follows:

$$\mathring{p}_t^d = \left[\xi_d \left(\frac{\tilde{\pi}_t^d}{\pi_t^d}\mathring{p}_{t-1}^d\right)^{\frac{\lambda_t^d}{1-\lambda_t^d}} + (1-\xi_d)\left(\tilde{p}_t^d\right)^{\frac{\lambda_t^d}{1-\lambda_t^d}}\right]^{\frac{1-\lambda_t^d}{\lambda_t^d}}.$$

Substituting $\tilde{p}_t^d$ using (3.23), we get:

$$\mathring{p}_t^d = \left[\xi_d \left(\frac{\tilde{\pi}_t^d}{\pi_t^d} \mathring{p}_{t-1}^d \right)^{\frac{\lambda_t^d}{1-\lambda_t^d}} + (1-\xi_d) \left(\frac{1-\xi_d \left(\frac{\tilde{\pi}_t^d}{\pi_t^d} \right)^{\frac{1}{1-\lambda_t^d}}}{1-\xi_d} \right)^{\lambda_t^d} \right]^{\frac{1-\lambda_t^d}{\lambda_t^d}}. \tag{3.27}$$

For the steady-state solutions in Section 12, it is also useful to derive an expression for firms' profits. Time-t profits, taking into account the fixed costs, of the i^{th} domestic intermediate goods producer are given by

$$PROFITS_{i,t}^d = P_{i,t}^d Y_{i,t} - MC_t^d \left(Y_{i,t} + z_t^+ \phi^d \right).$$

Using the demand curve for the i^{th} intermediate goods producer in equation (3.3), we have

$$PROFITS_{i,t}^d = \left(\frac{P_{i,t}^d}{P_t^d} \right)^{\frac{1}{1-\lambda_t^d}} P_t^d Y_t - MC_t^d \left(\left(\frac{P_{i,t}^d}{P_t^d} \right)^{\frac{\lambda_t^d}{1-\lambda_t^d}} Y_t + z_t^+ \phi^d \right). \tag{3.28}$$

The domestic intermediate good is allocated among alternative uses as follows:

$$Y_t = G_t + C_t^d + I_t^d + X_t^d, \tag{3.29}$$

where G_t denotes government spending; C_t^d denotes intermediate domestic consumption goods, used together with intermediate imported consumption goods to produce final consumption goods; I_t^d denotes domestic investment goods, used together with imported investment goods to produce final investment goods; and X_t^d denotes intermediate goods allocated to exports.[8]

3.1.1 Scaling of the domestic intermediate goods producers' optimal conditions

The domestic intermediate firm's real marginal cost is equal to the nominal marginal cost MC_t^d divided by the price of the homogeneous good. Scaling the non-stationary variables, equation (3.14) becomes:

$$mc_t^d = \left(\bar{w}_t R_t^{wc,d} \right)^{1-\alpha} \frac{\left(\bar{r}_t^k \right)^{\alpha}}{(1-\alpha)^{1-\alpha} \alpha^{\alpha} \epsilon_t}. \tag{3.30}$$

where $\bar{w}_t = \frac{\bar{W}_t}{z_t^+} = \frac{W_t}{z_t^+ P_t^d}$ denotes the scaled real wage, and $\bar{r}_t^k = \frac{\Psi_t R_t^k}{P_t^d}$ the scaled real capital rental rate. The second expression for the firm's marginal cost, (3.15), becomes, after scaling:

$$mc_t^d = \frac{\bar{w}_t R_t^{wc,d}}{\epsilon_t (1-\alpha) \left(\frac{k_t}{\mu_{z+,t} \mu_{\Psi,t} N_t} \right)^{\alpha}}, \tag{3.31}$$

[8] Note that Adolfson et al. (2013) includes another term in the equation for the domestic good allocation, so that

$$Y_t = G_t + C_t^d + I_t^d + X_t^d + D_t.$$

Here, D_t denotes the costs of the real frictions in the model. We do not include this term as our modelling of the labour market does not include any vacancy posting costs. Moreover, the capital utilization costs in the model are assumed to be paid for using the final investment good, implying that the share of the domestic good used to pay for this costs are already accounted for through the inclusion of I_t^d (see Section 3.4). The latter is not the case in Ramses I, where capital utilization costs enter the aggregate resource constraint explicitly. Finally, the investment adjustment costs are specified so that some fraction of the investment is lost in the transformation to physical capital (see Section 4.3). As such, they are already included through investment demand.

while the third can be written as

$$mc_t^d = \frac{\bar{r}_t^k}{\alpha\epsilon_t\left(\frac{k_t}{\mu_{z^+,t}\mu_{\Psi,t}N_t}\right)^{-(1-\alpha)}}. \tag{3.32}$$

We can also scale the expressions for the marginal costs of labour and capital. Using that

$$MPL_t = (1-\alpha)\, z_t^{1-\alpha}\epsilon_t\left(\frac{K_t}{N_t}\right)^{\alpha} \tag{3.33}$$

and

$$MPK_t = \alpha z_t^{1-\alpha}\epsilon_t\left(\frac{K_t}{N_t}\right)^{-(1-\alpha)}, \tag{3.34}$$

we can obtain

$$mpl_t = \frac{MPL_t}{z_t^+} = (1-\alpha)\,\epsilon_t\left(\frac{k_t}{\mu_{z^+,t}\mu_{\Psi,t}N_t}\right)^{\alpha}, \tag{3.35}$$

and

$$mpk_t = MPK_t\Psi_t = \alpha\epsilon_t\left(\frac{k_t}{\mu_{z^+,t}\mu_{\Psi,t}N_t}\right)^{-(1-\alpha)}. \tag{3.36}$$

Note that the marginal products of labour and of capital are scaled in the same way as wages and the rental rate of capital, respectively, as specified in Section 2.1 above. Recall that, using (3.35) and (3.36), we can also write the expression for marginal costs as follows:

$$mc_t^d = \frac{\bar{w}_t R_t^{wc,d}}{mpl_t}, \tag{3.37}$$

$$mc_t^d = \frac{\bar{r}_t^k}{mpk_t}. \tag{3.38}$$

Combining, we have

$$\frac{\bar{w}_t R_t^{wc,d}}{mpl_t} = \frac{\bar{r}_t^k}{mpk_t}$$

and

$$\frac{\bar{w}_t R_t^{wc,d}}{(1-\alpha)} = \frac{\bar{r}_t^k}{\alpha\left(\frac{k_t}{\mu_{z^+,t}\mu_{\Psi,t}N_t}\right)^{-1}}.$$

We can also directly rewrite the relationship for the rental rate of capital services, (3.17), in terms of stationary variables as follows:

$$\bar{r}_t^k = \frac{\alpha}{1-\alpha}\bar{w}_t R_t^{wc,d}\frac{N_t}{k_t}\mu_{z^+,t}\mu_{\Psi,t}. \tag{3.39}$$

We scale the optimal-price equation of the domestic intermediate goods producer, equation (3.22), by z_t^+ to obtain:

$$\tilde{p}_t^d = \frac{E_t\sum_{s=0}^{\infty}(\beta\xi_d)^s\zeta_{t+s}^{\beta}\psi_{z^+,t+s}y_{t+s}\lambda_{t+s}^d mc_{t+s}^d\left(\frac{\tilde{\pi}_{t+1}^d\ldots\tilde{\pi}_{t+s}^d}{\pi_{t+1}^d\ldots\pi_{t+s}^d}\right)^{\frac{\lambda_{t+s}^d}{1-\lambda_{t+s}^d}}}{E_t\sum_{s=0}^{\infty}(\beta\xi_d)^s\zeta_{t+s}^{\beta}\psi_{z^+,t+s}y_{t+s}\left(\frac{\tilde{\pi}_{t+1}^d\ldots\tilde{\pi}_{t+s}^d}{\pi_{t+1}^d\ldots\pi_{t+s}^d}\right)^{\frac{1}{1-\lambda_{t+s}^d}}}, \tag{3.40}$$

where we have used that:

$$\upsilon_{t+s}P_{t+s}^d Y_{t+s} = \upsilon_{t+s}P_{t+s}^d z_{t+s}^+\frac{Y_{t+s}}{z_{t+s}^+} = \psi_{z^+,t+s}y_{t+s}.$$

We can also scale the expression for profits in equation (3.28) to obtain

$$profits_{i,t}^{d} = \left(\frac{P_{i,t}^{d}}{P_{t}^{d}}\right)^{\frac{1}{1-\lambda_{t}^{d}}} P_{t}^{d} y_{t} - mc_{t}^{d} P_{t}^{d} y_{t} \left(\left(\frac{P_{i,t}^{d}}{P_{t}^{d}}\right)^{\frac{\lambda_{t}^{d}}{1-\lambda_{t}^{d}}} + \frac{\phi^{d}}{y_{t}}\right).$$

Integrating this expression over i, we get

$$\overline{profits}_{t}^{d} = y_{t}\left[1 - mc_{t}^{d}\left(\left(\mathring{p}_{t}^{d}\right)^{\frac{\lambda_{t}^{d}}{1-\lambda_{t}^{d}}} + \frac{\phi^{d}}{y_{t}}\right)\right], \tag{3.41}$$

where $\overline{profits}_{t}^{d}$ denotes real total profits, and where we have used the definition of the aggregate domestic price index (3.4) and the domestic price dispersion measure (6.3).

3.1.2 Log-linearization of the domestic intermediate goods producers' optimal conditions

We start with the working capital interest rate for domestic intermediate goods producers. Log-linearizing equation (3.10), we have

$$\hat{R}_{t}^{wc,d} = \frac{\nu^{wc,d}(R-1)}{R^{wc,d}}\hat{\nu}_{t}^{wc,d} + \frac{\nu^{wc,d}R}{R^{wc,d}}\hat{R}_{t}. \tag{3.42}$$

We next log-linearize the expressions for domestic intermediate goods producers' marginal costs, (3.30) and (3.31). Equation (3.30) gives

$$\widehat{mc}_{t}^{d} = (1-\alpha)\left(\hat{\bar{w}}_{t} + \hat{R}_{t}^{wc,d}\right) + \alpha\hat{\bar{r}}_{t}^{k} - \hat{\epsilon}_{t}. \tag{3.43}$$

From equation (3.31), we have

$$\widehat{mc}_{t}^{d} = \hat{\bar{w}}_{t} + \hat{R}_{t}^{wc,d} + \alpha\left(\hat{\mu}_{\Psi,t} - \hat{k}_{t} + \hat{\mu}_{z^{+},t} + \hat{N}_{t}\right) - \hat{\epsilon}_{t}. \tag{3.44}$$

We can combine equations (3.43) and (3.44) to obtain the following expression for the rental rate of capital:

$$\hat{\bar{r}}_{t}^{k} = \hat{\mu}_{z^{+},t} + \hat{\mu}_{\Psi,t} + \hat{\bar{w}}_{t} + \hat{R}_{t}^{wc,d} + \hat{N}_{t} - \hat{k}_{t}. \tag{3.45}$$

We could alternatively use equations (3.37) and (3.38) to characterize firms' marginal costs. Log-linearizing, we have

$$\widehat{mc}_{t}^{d} = \hat{\bar{w}}_{t} + \hat{R}_{t}^{wc,d} - \widehat{mpl}_{t}, \tag{3.46}$$

and

$$\widehat{mc}_{t}^{d} = \hat{\bar{r}}_{t}^{k} - \widehat{mpk}_{t}. \tag{3.47}$$

Moreover, log-linearizing (3.35) and (3.36), we obtain the following expressions for the marginal rate of labour and capital, respectively:

$$\widehat{mpl}_{t} = \alpha\widehat{\left(\frac{k}{N}\right)}_{t} + \hat{\epsilon}_{t}, \tag{3.48}$$

$$\widehat{mpk}_{t} = -(1-\alpha)\widehat{\left(\frac{k}{N}\right)}_{t} + \hat{\epsilon}_{t}, \tag{3.49}$$

where $\widehat{\left(\frac{k}{N}\right)}_{t}$ denotes the log-linearized capital-to-labour ratio given by

$$\widehat{\left(\frac{k}{N}\right)}_{t} = \hat{k}_{t} - \hat{N}_{t} - \left(\hat{\mu}_{z^{+},t} + \hat{\mu}_{\Psi,t}\right). \tag{3.50}$$

We also need to log-linearize the expression for the combination of investment-specific and neutral technology $\mu_{z^+,t}$. Log-linearization of equation (3.9) yields

$$\hat{\mu}_{z^+,t} = \frac{\alpha}{1-\alpha}\hat{\mu}_{\Psi,t} + \hat{\mu}_{z,t}. \tag{3.51}$$

We now consider the domestic intermediate goods producers' price setting. We start by log-linearizing the expression of the domestic intermediate goods producers' optimal price (3.40):

$$\begin{aligned}
&\tilde{p}_t^d E_t \sum_{s=0}^{\infty} (\beta\xi_d)^s \zeta_{t+s}^{\beta} \psi_{z^+,t+s} y_{t+s} \left(\frac{\tilde{\pi}_{t+1}^d \dots \tilde{\pi}_{t+s}^d}{\pi_{t+1}^d \dots \pi_{t+s}^d}\right)^{\frac{1}{1-\lambda_{t+s}^d}} \\
&= E_t \sum_{s=0}^{\infty} (\beta\xi_d)^s \zeta_{t+s}^{\beta} \psi_{z^+,t+s} y_{t+s} \lambda_{t+s}^d mc_{t+s}^d \left(\frac{\tilde{\pi}_{t+1}^d \dots \tilde{\pi}_{t+s}^d}{\pi_{t+1}^d \dots \pi_{t+s}^d}\right)^{\frac{\lambda_{t+s}^d}{1-\lambda_{t+s}^d}}.
\end{aligned}$$

$$\begin{aligned}
&\tilde{p}^d \hat{\tilde{p}}_t^d \zeta^{\beta} \psi_{z^+} y \sum_{s=0}^{\infty} (\beta\xi_d)^s \left(\frac{\tilde{\pi}^d}{\pi^d}\right)^{\frac{s}{1-\lambda^d}} + \tilde{p}^d E_t \sum_{s=0}^{\infty} (\beta\xi_d)^s \zeta^{\beta} \psi_{z^+} y \left(\frac{\tilde{\pi}^d}{\pi^d}\right)^{\frac{s}{1-\lambda^d}} \times \\
&\times \left[\hat{\zeta}_{t+s}^{\beta} + \hat{\psi}_{z^+,t+s} + \hat{y}_{t+s} - \frac{\lambda^d}{\left(1-\lambda^d\right)^2} \log\left(\frac{\tilde{\pi}^d}{\pi^d}\right)^s \hat{\lambda}_{t+s}^d + \frac{1}{1-\lambda^d}\left(\hat{\tilde{\pi}}_{t+1}^d + \dots + \hat{\tilde{\pi}}_{t+s}^d - \hat{\pi}_{t+1}^d - \dots - \hat{\pi}_{t+s}^d\right)\right] \\
= \; & E_t \sum_{s=0}^{\infty} (\beta\xi_d)^s \zeta^{\beta} \psi_{z^+} y \lambda^d mc^d \left(\frac{\tilde{\pi}^d}{\pi^d}\right)^{\frac{s}{1-\lambda^d}} \left[\hat{\zeta}_{t+s}^{\beta} + \hat{\psi}_{z^+,t+s} + \hat{y}_{t+s} + \left(1 + \frac{\lambda^d}{\left(1-\lambda^d\right)^2} \log\left(\frac{\tilde{\pi}^d}{\pi^d}\right)^s\right) \hat{\lambda}_{t+s}^d \right. \\
&\left. + \frac{\lambda^d}{1-\lambda^d}\left(\hat{\tilde{\pi}}_{t+1}^d + \dots + \hat{\tilde{\pi}}_{t+s}^d - \hat{\pi}_{t+1}^d - \dots - \hat{\pi}_{t+s}^d\right)\right],
\end{aligned}$$

where we have used the steady-state relationship:

$$\tilde{p}^d \sum_{s=0}^{\infty} (\beta\xi_d)^s \zeta^{\beta} \psi_{z^+} y \left(\frac{\tilde{\pi}^d}{\pi^d}\right)^{\frac{s}{1-\lambda^d}} = \sum_{s=0}^{\infty} (\beta\xi_d)^s \zeta^{\beta} \psi_{z^+} y \lambda^d mc^d \left(\frac{\tilde{\pi}^d}{\pi^d}\right)^{\frac{s\lambda^d}{1-\lambda^d}}.$$

Now turn to the expression of $\tilde{\pi}_t^d$:

$$\tilde{\pi}_t^d = \left(\pi_{t-1}^d\right)^{\kappa_d} \left(\bar{\pi}_t^c\right)^{1-\kappa_d-\varkappa_d} \left(\breve{\pi}\right)^{\varkappa_d},$$

which gives, in steady state:

$$\tilde{\pi}^d = \left(\pi^d\right)^{\kappa_d} \left(\bar{\pi}^c\right)^{1-\kappa_d-\varkappa_d} \left(\breve{\pi}\right)^{\varkappa_d}.$$

As discussed in Section 12 below, we assume that $\pi^d = \pi^c = \bar{\pi}^c$, which implies that $\tilde{\pi}^d = \left(\pi^d\right)^{1-\varkappa_d} \left(\breve{\pi}\right)^{\varkappa_d}$. Under the additional assumption that there is full indexation, i.e. $\varkappa_d = 0$ or, alternatively, that $\breve{\pi}$ is equal to the inflation target as well, we have $\tilde{\pi}^d = \pi^d$. Under the same assumption, we also have that $mc^d = \frac{1}{\lambda^d}$ (Section 12, equation (12.27)). Hence, we have:

$$\begin{aligned}
\hat{\tilde{p}}_t^d \zeta^{\beta} \psi_{z^+} y \sum_{s=0}^{\infty} (\beta\xi_d)^s &= E_t \zeta^{\beta} \psi_{z^+} y \sum_{s=0}^{\infty} (\beta\xi_d)^s \left[\widehat{mc}_{t+s}^d + \hat{\lambda}_{t+s}^d - \left(\hat{\tilde{\pi}}_{t+1}^d + \dots + \hat{\tilde{\pi}}_{t+s}^d - \hat{\pi}_{t+1}^d - \dots - \hat{\pi}_{t+s}^d\right)\right] \\
\Leftrightarrow \quad \hat{\tilde{p}}_t^d &= (1-\beta\xi_d) E_t \sum_{s=0}^{\infty} (\beta\xi_d)^s \left[\widehat{mc}_{t+s}^d + \hat{\lambda}_{t+s}^d - \left(\hat{\tilde{\pi}}_{t+1}^d + \dots + \hat{\tilde{\pi}}_{t+s}^d - \hat{\pi}_{t+1}^d - \dots - \hat{\pi}_{t+s}^d\right)\right].
\end{aligned}$$

$\widehat{\tilde{p}}_t^d$ can be written recursively as follows:

$$\widehat{\tilde{p}}_t^d = (1-\beta\xi_d)\left(\widehat{mc}_t^d + \hat{\lambda}_t^d\right) + (\beta\xi_d)\, E_t\widehat{\tilde{p}}_{t+1}^d + (1-\beta\xi_d)\, E_t\sum_{s=1}^{\infty}(\beta\xi_d)^s\left(\hat{\pi}_{t+1}^d - \widehat{\tilde{\pi}}_{t+1}^d\right), \tag{3.52}$$

where we have used that, for a variable X_t, $E_t\left[E_{t+1}X_t\right] = E_tX_t$.

We can log-linearize expression (3.23) to obtain another expression for $\widehat{\tilde{p}}_t^d$. Using the steady-state relationships

$$\begin{aligned} 1 &= \xi_d\left(\frac{\tilde{\pi}^d}{\pi^d}\right)^{\frac{1}{1-\lambda^d}} + (1-\xi_d)\left(\tilde{p}^d\right)^{\frac{1}{1-\lambda^d}}, \\ \tilde{p}^d &= 1, \qquad \frac{\tilde{\pi}^d}{\pi^d} = 1, \end{aligned}$$

we obtain

$$\widehat{\tilde{p}}_t^d = \frac{\xi_d}{1-\xi_d}\left(\hat{\pi}_t^d - \widehat{\tilde{\pi}}_t^d\right). \tag{3.53}$$

Combining (3.52) with (3.53), we have:

$$\frac{\xi_d}{1-\xi_d}\left(\hat{\pi}_t^d - \widehat{\tilde{\pi}}_t^d\right) = (1-\beta\xi_d)\left(\widehat{mc}_t^d + \hat{\lambda}_t^d\right) + \frac{\beta\xi_d}{1-\xi_d}E_t\left(\hat{\pi}_{t+1}^d - \widehat{\tilde{\pi}}_{t+1}^d\right) \tag{3.54}$$

Log-linearizing the expression of $\tilde{\pi}_t^d$ in equation (3.18) yields

$$\widehat{\tilde{\pi}}_t^d = \kappa_d\hat{\pi}_{t-1}^d + (1-\kappa_d)\,\widehat{\bar{\pi}}_t^c, \tag{3.55}$$

under the assumption that $\varkappa_d = 0$. Plugging into (3.54), we finally obtain the following Phillips curve relation:

$$\begin{aligned} \hat{\pi}_t^d - \widehat{\bar{\pi}}_t^c &= \frac{(1-\beta\xi_d)(1-\xi_d)}{\xi_d(1+\beta\kappa_d)}\left(\widehat{mc}_t^d + \hat{\lambda}_t^d\right) + \frac{\kappa_d}{1+\beta\kappa_d}\left(\hat{\pi}_{t-1}^d - \widehat{\bar{\pi}}_t^c\right) \\ &\quad + \frac{\beta}{1+\beta\kappa_d}E_t\left(\hat{\pi}_{t+1}^d - \widehat{\bar{\pi}}_{t+1}^c\right) - \frac{\beta\kappa_d}{1+\beta\kappa_d}E_t\left(\widehat{\bar{\pi}}_t^c - \widehat{\bar{\pi}}_{t+1}^c\right). \end{aligned} \tag{3.56}$$

For use in later sections, we need also to log-linearize the price dispersion equation (3.27). Rearranging,

$$\left(\mathring{p}_t^d\right)^{\frac{\lambda_t^d}{1-\lambda_t^d}} = \xi_d\left(\frac{\tilde{\pi}_t^d}{\pi_t^d}\mathring{p}_{t-1}^d\right)^{\frac{\lambda_t^d}{1-\lambda_t^d}} + (1-\xi_d)\left(\frac{1-\xi_d\left(\frac{\tilde{\pi}_t^d}{\pi_t^d}\right)^{\frac{1}{1-\lambda_t^d}}}{1-\xi_d}\right)^{\lambda_t^d}.$$

Log-linearizing, we get

$$\frac{\lambda^d}{1-\lambda^d}\left(\mathring{p}^d\right)^{\frac{\lambda^d}{1-\lambda^d}}\widehat{\mathring{p}}_t^d+\ln\left(\mathring{p}^d\right)\left(\mathring{p}^d\right)^{\frac{\lambda^d}{1-\lambda^d}}\frac{\lambda^d}{\left(1-\lambda^d\right)^2}\hat{\lambda}_t^d$$

$$\begin{aligned}
= \quad & \xi_d\frac{\lambda^d}{1-\lambda^d}\left(\frac{\tilde{\pi}^d}{\pi^d}\mathring{p}^d\right)^{\frac{\lambda^d}{1-\lambda^d}}\left[\widehat{\tilde{\pi}}_t^d-\hat{\pi}_t^d+\widehat{\mathring{p}}_{t-1}^d\right]+\xi_d\ln\left(\frac{\tilde{\pi}^d}{\pi^d}\mathring{p}^d\right)\left(\frac{\tilde{\pi}^d}{\pi^d}\mathring{p}^d\right)^{\frac{\lambda^d}{1-\lambda^d}}\frac{\lambda^d}{\left(1-\lambda^d\right)^2}\hat{\lambda}_t^d \\
& -\left(\frac{1-\xi_d\left(\frac{\tilde{\pi}^d}{\pi^d}\right)^{\frac{1}{1-\lambda^d}}}{1-\xi_d}\right)^{\lambda^d-1}\frac{\lambda^d}{1-\lambda^d}\xi_d\left(\frac{\tilde{\pi}^d}{\pi^d}\right)^{\frac{1}{1-\lambda^d}}\left[\widehat{\tilde{\pi}}_t^d-\hat{\pi}_t^d\right] \\
& +\left(1-\xi_d\right)\ln\left(\frac{1-\xi_d\left(\frac{\tilde{\pi}^d}{\pi^d}\right)^{\frac{1}{1-\lambda^d}}}{1-\xi_d}\right)\left(\frac{1-\xi_d\left(\frac{\tilde{\pi}^d}{\pi^d}\right)^{\frac{1}{1-\lambda^d}}}{1-\xi_d}\right)^{\lambda^d}\lambda^d\hat{\lambda}_t^d \\
& -\left(\frac{1-\xi_d\left(\frac{\tilde{\pi}^d}{\pi^d}\right)^{\frac{1}{1-\lambda^d}}}{1-\xi_d}\right)^{\lambda^d-1}\xi_d\ln\left(\frac{\tilde{\pi}^d}{\pi^d}\right)\left(\frac{\tilde{\pi}^d}{\pi^d}\right)^{\frac{1}{1-\lambda^d}}\left(\frac{\lambda^d}{1-\lambda^d}\right)^2\hat{\lambda}_t^d.
\end{aligned}$$

Dividing through by $\frac{\lambda^d}{1-\lambda^d}\left(\mathring{p}^d\right)^{\frac{\lambda^d}{1-\lambda^d}}$, we obtain

$$\begin{aligned}
\widehat{\mathring{p}}_t^d = & \xi_d\left(\frac{\tilde{\pi}^d}{\pi^d}\right)^{\frac{\lambda^d}{1-\lambda^d}}\left[\widehat{\tilde{\pi}}_t^d-\hat{\pi}_t^d+\widehat{\mathring{p}}_{t-1}^d\right]+\xi_d\ln\left(\frac{\tilde{\pi}^d}{\pi^d}\mathring{p}^d\right)\left(\frac{\tilde{\pi}^d}{\pi^d}\right)^{\frac{\lambda^d}{1-\lambda^d}}\frac{1}{1-\lambda^d}\hat{\lambda}_t^d-\ln\left(\mathring{p}^d\right)\frac{1}{1-\lambda^d}\hat{\lambda}_t^d \\
& -\frac{1}{\left(\mathring{p}^d\right)^{\frac{\lambda^d}{1-\lambda^d}}}\left(\frac{1-\xi_d\left(\frac{\tilde{\pi}^d}{\pi^d}\right)^{\frac{1}{1-\lambda^d}}}{1-\xi_d}\right)^{\lambda^d-1}\xi_d\left(\frac{\tilde{\pi}^d}{\pi^d}\right)^{\frac{1}{1-\lambda^d}}\left[\widehat{\tilde{\pi}}_t^d-\hat{\pi}_t^d\right] \\
& +\frac{1-\xi_d}{\left(\mathring{p}^d\right)^{\frac{\lambda^d}{1-\lambda^d}}}\ln\left(\frac{1-\xi_d\left(\frac{\tilde{\pi}^d}{\pi^d}\right)^{\frac{1}{1-\lambda^d}}}{1-\xi_d}\right)\left(\frac{1-\xi_d\left(\frac{\tilde{\pi}^d}{\pi^d}\right)^{\frac{1}{1-\lambda^d}}}{1-\xi_d}\right)^{\lambda^d}\left(1-\lambda^d\right)\hat{\lambda}_t^d \\
& -\left(\frac{1-\xi_d\left(\frac{\tilde{\pi}^d}{\pi^d}\right)^{\frac{1}{1-\lambda^d}}}{1-\xi_d}\right)^{\lambda^d-1}\frac{\xi_d}{\left(\mathring{p}^d\right)^{\frac{\lambda^d}{1-\lambda^d}}}\ln\left(\frac{\tilde{\pi}^d}{\pi^d}\right)\left(\frac{\tilde{\pi}^d}{\pi^d}\right)^{\frac{1}{1-\lambda^d}}\frac{\lambda^d}{1-\lambda^d}\hat{\lambda}_t^d.
\end{aligned}$$

We can now use the steady-state relationship $\tilde{\pi}^d=\pi^d$ to simplify the expression further. Note that this assumption also implies that

$$\mathring{p}^d=1,$$

and thus that $\ln\left(\mathring{p}^d\right)=0$. We then finally arrive at the following expression:

$$\widehat{\mathring{p}}_t^d=\xi_d\widehat{\mathring{p}}_{t-1}^d. \tag{3.57}$$

3.2 Production of imported intermediate goods

The import sector consists of domestic firms that buy a homogeneous good from foreign firms. There are four different types of importing firms: (i) those that turn the imported product into a specialized non-energy consumption good $C_{i,t}^m$, (ii) those that turn the imported product into a specialized investment good $I_{i,t}^m$, (iii) those that turn the imported product into a specialized good used as input by exporting firms $X_{i,t}^m$, and (iv) those that turn the imported energy into a specialized energy consumption good $C_{i,t}^{e,m}$. There is a continuum of importing firms in each category. They sell their specialized output to import retailers, which produce the final imported goods. Consequently, there are also four types of import retailers.

Consider the production of the homogeneous consumption good derived from imports, C_t^m. It is a composite of the specialized consumption goods $C_{i,t}^m$, and it is produced by domestic retailers according to the following technology:

$$C_t^m = \left[\int_0^1 (C_{i,t}^m)^{\frac{1}{\lambda_t^{m,c}}} di \right]^{\lambda_t^{m,c}}, \qquad 1 \leq \lambda_t^{m,c} \leq \infty, \tag{3.58}$$

where $\lambda_t^{m,c}$ is a time-varying price markup in the import consumption market. Specifically, for the four different types of importing firms, we assume that

$$\log \lambda_t^{m,j} = (1 - \rho_{\lambda^{m,j}}) \log \lambda^{m,j} + \rho_{\lambda^{m,j}} \log \lambda_{t-1}^{m,j} + \sigma_{\lambda^{m,j}} \varepsilon_{\lambda^{m,j},t}, \quad j = c, i, x, ce. \tag{3.59}$$

The retailers of the imported consumption goods operate under perfect competition and take the price of output $P_t^{m,c}$ and input $P_{i,t}^{m,c}$ as given. Profit maximization writes:

$$\max_{C_{i,t}^m} \quad P_t^{m,c} C_t^m - \int_0^1 P_{i,t}^{m,c} C_{i,t}^m di,$$

which yields

$$C_{i,t}^m = \left(\frac{P_{i,t}^{m,c}}{P_t^{m,c}} \right)^{\frac{\lambda_t^{m,c}}{1-\lambda_t^{m,c}}} C_t^m. \tag{3.60}$$

The i^{th} importing firm producing $C_{i,t}^m$ takes (3.60) as a demand curve. It buys the foreign homogeneous good and converts it into a differentiated consumption good through access to a differentiating technology (brand naming). The importing firm must pay the inputs at the beginning of the period in foreign currency, and as it doesn't have resources at the beginning of the period it must borrow those resources. The marginal cost of this producer is

$$MC_t^{m,c} = S_t P_t^{d,*} R_t^{wc,m} \tag{3.61}$$

where

$$R_t^{wc,m} = \nu_t^{wc,m} R_t^* + 1 - \nu_t^{wc,m}, \tag{3.62}$$

R_t^* is the foreign nominal interest rate, S_t the nominal exchange rate (domestic currency per units of foreign currency), and $P_t^{d,*}$ is the foreign currency price of the foreign homogeneous good. $\nu_t^{wc,m}$ denotes the fraction of the import retailers' costs that has to be financed in advance. We assume that $\nu_t^{wc,m}$ follows

$$\log \nu_t^{wc,m} = (1 - \rho_{\nu^{wc,m}}) \log \nu^{wc,m} + \rho_{\nu^{wc,m}} \log \nu_{t-1}^{wc,m} + \sigma_{\nu^{wc,m}} \varepsilon_{\nu^{wc,m},t}. \tag{3.63}$$

There is no risk for this firm because all shocks are realized at the beginning of the period, and thus there is no uncertainty about the realization of prices and exchange rates during the borrowing period.[9] For reasons related to the steady-state calculations, further discussed in Section 12, we will assume that each firm i, producing the speacilized consumption good $C_{i,t}^m$, needs to pay a fixed cost of

[9]This feature is commented on in an earlier documentation of Ramses II. In particular, the authors point out that they are somewhat uncomfortable with this feature of the model, as the fact that interest is due and matters indicates that some time evolves over the duration of the loan, and thus the assumption that no uncertainty is realized over a period of signicant duration of time seems implausible. They conjecture, however, that this does not affect the first order properties of the model.

production like the one assumed for the intermediate goods producers in Section 3.1 above. We thus separate between gross demand for consumption imports, which we denote by $\tilde{C}_t^m$, and the demand net of fixed costs. Specifically, we assume that

$$\tilde{C}_t^m = \int_0^1 C_{i,t}^m + z_t^+ \phi^{m,c}, \tag{3.64}$$

and that the firm pays for the fixed costs using its own imported good. Denoting by $\phi^{m,c}$ the fixed cost faced by the domestic importers of consumption goods, we derive the following expression for the firm's profits:

$$PROFITS_{i,t}^{m,c} = P_{i,t}^{m,c} C_{i,t}^m - MC_t^{m,c}\left(C_{i,t}^m + z_t^+ \phi^{m,c}\right).$$

Using the demand curve for the i^{th} intermediate goods producer in equation (3.60), we have

$$PROFITS_{i,t}^{m,c} = \left(\frac{P_{i,t}^{m,c}}{P_t^{m,c}}\right)^{\frac{1}{1-\lambda_t^{m,c}}} P_t^{m,c} C_t^m - MC_t^{m,c}\left(\left(\frac{P_{i,t}^{m,c}}{P_t^{m,c}}\right)^{\frac{\lambda_t^{m,c}}{1-\lambda_t^{m,c}}} C_t^m + z_t^+ \phi^{m,c}\right). \tag{3.65}$$

Now turn to the production of the homogeneous investment good derived from imports, I_t^m. It is a composite of the specialized investment goods, $I_{i,t}^m$, and it is produced by competitive domestic retailers according to:

$$I_t^m = \left[\int_0^1 \left(I_{i,t}^m\right)^{\frac{1}{\lambda_t^{m,i}}} di\right]^{\lambda_t^{m,i}}, \qquad 1 \leq \lambda_t^{m,i} \leq \infty. \tag{3.66}$$

The retailers of the imported investment goods take the price of output, $P_t^{m,i}$, and input, $P_{i,t}^{m,i}$, as given. As for consumption, profit maximization leads to:

$$I_{i,t}^m = \left(\frac{P_{i,t}^{m,i}}{P_t^{m,i}}\right)^{\frac{\lambda_t^{m,i}}{1-\lambda_t^{m,i}}} I_t^m. \tag{3.67}$$

The i^{th} importing firm producing $I_{i,t}^m$ takes (3.67) as a demand curve. It buys the foreign homogeneous good and converts it into a differentiated investment good. The marginal cost of this producer is the same as (3.61):

$$MC_t^{m,i} = S_t P_t^{d,*} R_t^{wc,m}. \tag{3.68}$$

This implies that the price met by this firm, $P_t^{d,*}$ (in terms of foreign currency), is the same as the cost of the importing firm producing $C_{i,t}^m$. This may seem inconsistent with the fact that domestically produced investment and consumption goods have different relative prices. From the aggregation technologies for consumption and investment goods, discussed in Sections 3.3 and 3.4, it will be clear that the efficiency of imported investment goods grows over time, relative that of the imported consumption good. As for imported consumption goods, we assume that gross demand for investment imports is given by

$$\tilde{I}_t^m = \int_0^1 I_{i,t}^m + z_t^+ \phi^{m,i}. \tag{3.69}$$

We also have the following expression for the investment importing firm's profits:

$$PROFITS_{i,t}^{m,i} = P_{i,t}^{m,i} I_{i,t}^m - MC_t^{m,i}\left(I_{i,t}^m + z_t^+ \phi^{m,i}\right).$$

$$PROFITS_{i,t}^{m,i} = \left(\frac{P_{i,t}^{m,i}}{P_t^{m,i}}\right)^{\frac{1}{1-\lambda_t^{m,i}}} P_t^{m,i} I_t^m - MC_t^{m,i}\left(\left(\frac{P_{i,t}^{m,i}}{P_t^{m,i}}\right)^{\frac{\lambda_t^{m,i}}{1-\lambda_t^{m,i}}} I_t^m + z_t^+ \phi^{m,i}\right). \tag{3.70}$$

Next, consider the production of the homogeneous imported input, X_t^m, used in the production of the specialized export good X_t. It is a composite of the specialized export goods $X_{i,t}^m$ and it is produced by competitive domestic retailers according to:

$$X_t^m = \left[\int_0^1 \left(X_{i,t}^m\right)^{\frac{1}{\lambda_t^{m,x}}} di\right]^{\lambda_t^{m,x}}, \qquad 1 \leq \lambda_t^{m,x} \leq \infty. \tag{3.71}$$

The retailers of X_t^m take the price of output, $P_t^{m,x}$, and input, $P_{i,t}^{m,x}$, as given. As for consumption and investment, profit maximization leads to the following demand curve for the producer of $X_{i,t}^m$:

$$X_{i,t}^m = \left(\frac{P_{i,t}^{m,x}}{P_t^{m,x}}\right)^{\frac{\lambda_t^{m,x}}{1-\lambda_t^{m,x}}} X_t^m. \tag{3.72}$$

The marginal cost associated with the production of $X_{i,t}^m$ is:

$$MC_t^{m,x} = S_t P_t^{d,*} R_t^{wc,m}. \tag{3.73}$$

Finally, we assume that gross demand for imports used in export production are given by

$$\tilde{X}_t^m = \int_0^1 X_{i,t}^m + z_t^+ \phi^{m,x}, \tag{3.74}$$

and derive the following expression for the profits of firm i importing goods for export production:

$$PROFITS_{i,t}^{m,x} = P_{i,t}^{m,x} X_{i,t}^m - MC_t^{m,x}\left(X_{i,t}^m + z_t^+ \phi^{m,x}\right).$$

$$PROFITS_{i,t}^{m,x} = \left(\frac{P_{i,t}^{m,x}}{P_t^{m,x}}\right)^{\frac{1}{1-\lambda_t^{m,x}}} P_t^{m,x} X_t^m - MC_t^{m,x}\left(\left(\frac{P_{i,t}^{m,x}}{P_t^{m,x}}\right)^{\frac{\lambda_t^{m,x}}{1-\lambda_t^{m,x}}} X_t^m + z_t^+ \phi^{m,x}\right). \tag{3.75}$$

Finally, we turn to the production of the homogeneous energy consumption good derived from imports, $C_t^{e,m}$. It is a composite of the specialized investment goods, $C_{i,t}^{e,m}$, and it is produced by competitive domestic retailers according to:

$$C_t^{e,m} = \left[\int_0^1 \left(C_{i,t}^{e,m}\right)^{\frac{1}{\lambda_t^{m,ce}}} di\right]^{\lambda_t^{m,ce}}, \qquad 1 \leq \lambda_t^{m,ce} \leq \infty. \tag{3.76}$$

The retailers of $C_t^{e,m}$ take the price of output, $P_t^{m,ce}$, and input, $P_{i,t}^{m,ce}$, as given. As for non-energy consumption, investment, and exports, profit maximization leads to the following demand curve for the producer of $C_{i,t}^{e,m}$:

$$C_{i,t}^{e,m} = \left(\frac{P_{i,t}^{m,ce}}{P_t^{m,ce}}\right)^{\frac{\lambda_t^{m,ce}}{1-\lambda_t^{m,ce}}} C_t^{e,m}. \tag{3.77}$$

The marginal cost associated with the production of $C_{i,t}^{e,m}$ is:

$$MC_t^{m,ce} = S_t P_t^{ce,*} R_t^{wc,m}. \tag{3.78}$$

Finally, we assume that gross demand for imports used in energy consumption production are given by

$$\tilde{C}_t^{e,m} = \int_0^1 C_{i,t}^{e,m} + z_t^+ \phi^{m,ce}, \tag{3.79}$$

and derive the following expression for the profits of firm i importing goods for export production:

$$PROFITS_{i,t}^{m,ce} = P_{i,t}^{m,ce} C_{i,t}^{e,m} - MC_t^{m,ce}\left(C_{i,t}^{e,m} + z_t^+ \phi^{m,ce}\right).$$

$$PROFITS_{i,t}^{m,ce} = \left(\frac{P_{i,t}^{m,ce}}{P_t^{m,ce}}\right)^{\frac{1}{1-\lambda_t^{m,ce}}} P_t^{m,ce} C_t^{e,m} - MC_t^{m,ce}\left(\left(\frac{P_{i,t}^{m,ce}}{P_t^{m,ce}}\right)^{\frac{\lambda_t^{m,ce}}{1-\lambda_t^{m,ce}}} C_t^{e,m} + z_t^+ \phi^{m,ce}\right). \tag{3.80}$$

Each of the four types of importing firms is a monopolist and is subject to Calvo price-setting frictions. Each importing firm faces a probability $\left(1-\xi_{m,j}\right)$ that it can reoptimizes its price in any period, for $j = c, i, x, ce$, independent on the time that has passed since it was last able to reoptimize. If the firm is not able to reoptimize in period t, the price in period $t+1$ will be set according to the following indexation rule:

$$\begin{cases} P_{i,t}^{m,j} = \tilde{\pi}_t^{m,j} P_{i,t-1}^{m,j} \\ \tilde{\pi}_t^{m,j} \equiv \left(\pi_{t-1}^{m,j}\right)^{\kappa_{m,j}} \left(\bar{\pi}_t^c\right)^{1-\kappa_{m,j}-\varkappa_{m,j}} \left(\breve{\pi}\right)^{\varkappa_{m,j}}, \end{cases} \quad j = c, i, x, \tag{3.81}$$

where $\kappa_{m,j}$, $\varkappa_{m,j}$ are parameters such that $\kappa_{m,j}, \varkappa_{m,j}, \kappa_{m,j} + \varkappa_{m,j} \in [0,1]$. In a similar way as what we had for the domestic producers of intermediate goods, when setting its price at time t, the i^{th} importing firm in each of the categories will maximize its future discounted profits. Denoting by $\tilde{P}_{i,t}^{m,j}(j = c, i, x, ce)$ the reoptimized price at period t, the firm faces the following optimization problem:

$$\begin{cases} \max\limits_{\tilde{P}_{i,t}^{m,j}} \quad E_t \sum\limits_{s=0}^{\infty} \left(\beta \xi_{m,j}\right)^s \zeta_{t+s}^{\beta} \upsilon_{t+s} \left(P_{i,t+s}^{m,j} Z_{i,t+s}^j - mc_{t+s}^{m,j} P_{t+s}^{m,j} Z_{i,t+s}^j\right) \\ s.t. \quad Z_{i,t}^j = \left(\frac{P_{i,t}^{m,j}}{P_t^{m,j}}\right)^{-\frac{\lambda_t^{m,j}}{\lambda_t^{m,j}-1}} Z_t^j \end{cases}, \quad Z_{i,t}^j = \begin{cases} C_{i,t}^m & if\ j = c \\ I_{i,t}^m & if\ j = i \\ X_{i,t}^m & if\ j = x \\ C_{i,t}^{e,m} & if\ j = ce \end{cases}.$$

The FOC yields the expression for the optimal price:

$$\tilde{p}_t^{m,j} = \frac{E_t \sum\limits_{s=0}^{\infty} (\beta\xi_{m,j})^s \zeta_{t+s}^{\beta} \upsilon_{t+s} P_{t+s}^{m,j} Z_{t+s}^j \lambda_{t+s}^{m,j} mc_{t+s}^{m,j} \left(\frac{\tilde{\pi}_{t+1}^{m,j} \dots \tilde{\pi}_{t+s}^{m,j}}{\pi_{t+1}^{m,j} \dots \pi_{t+s}^{m,j}}\right)^{\frac{\lambda_{t+s}^{m,j}}{1-\lambda_{t+s}^{m,j}}}}{E_t \sum\limits_{s=0}^{\infty} (\beta\xi_{m,j})^s \zeta_{t+s}^{\beta} \upsilon_{t+s} P_{t+s}^{m,j} Z_{t+s}^j \left(\frac{\tilde{\pi}_{t+1}^{m,j} \dots \tilde{\pi}_{t+s}^{m,j}}{\pi_{t+1}^{m,j} \dots \pi_{t+s}^{m,j}}\right)^{\frac{1}{1-\lambda_{t+s}^{m,j}}}}, \tag{3.82}$$

$$Z_t^j = \begin{cases} C_t^m & if\ j = c \\ I_t^m & if\ j = i \\ X_t^m & if\ j = x \\ C_{i,t}^{e,m} & if\ j = ce \end{cases}.$$

where $\tilde{p}_t^{m,j} = \frac{\tilde{P}_t^{m,j}}{P_t^d}$ for $j = c, i, x, ce$.

We can derive a second expression for $\tilde{p}_t^{m,j}$ from the expression of the aggregate price of imported goods, $P_t^{m,j}$, in a similar way as for domestic intermediate goods producers. We have:

$$\tilde{p}_t^{m,j} = \left[\frac{1 - \xi_{m,j}\left(\frac{\tilde{\pi}_t^{m,j}}{\pi_t^{m,j}}\right)^{\frac{1}{1-\lambda_t^{m,j}}}}{\left(1-\xi_{m,j}\right)}\right]^{1-\lambda_t^{m,j}}. \tag{3.83}$$

Note that, to obtain total demand for imports of non-energy consumption, investment, export and energy consumption input goods, we can integrate expressions (3.60), (3.67), (3.72) and (3.77), respectively. Starting with consumption we have

$$\begin{aligned}\int_0^1 C_{i,t}^m di &= \int_0^1 \left(\frac{P_{i,t}^{m,c}}{P_t^{m,c}}\right)^{\frac{\lambda_t^{m,c}}{1-\lambda_t^{m,c}}} C_t^m di \\ &= C_t^m \int_0^1 \left(\frac{P_{i,t}^{m,c}}{P_t^{m,c}}\right)^{\frac{\lambda_t^{m,c}}{1-\lambda_t^{m,c}}} di\end{aligned}$$

Defining a measure of price dispersion, $\mathring{p}_t^{m,j}$, as follows:

$$\mathring{p}_t^{m,j} = \left[\int_0^1 \left(\frac{P_{i,t}^{m,j}}{P_t^{m,j}}\right)^{\frac{\lambda_t^{m,j}}{1-\lambda_t^{m,j}}}\right]^{\frac{1-\lambda_t^{m,j}}{\lambda_t^{m,j}}}, \quad j = c, i, x, ce, \tag{3.84}$$

we have that total demand for imports of consumption goods is given by

$$\int_0^1 C_{i,t}^m di = C_t^m \left(\mathring{p}_t^{m,c}\right)^{\frac{\lambda_t^{m,c}}{1-\lambda_t^{m,c}}}. \tag{3.85}$$

Proceeding in the same way, we can derive total demand for imports of investment goods

$$\int_0^1 I_{i,t}^m di = I_t^m \left(\mathring{p}_t^{m,i}\right)^{\frac{\lambda_t^{m,i}}{1-\lambda_t^{m,i}}}, \tag{3.86}$$

total demand for imports used as inputs in export production

$$\int_0^1 X_{i,t}^m di = X_t^m \left(\mathring{p}_t^{m,x}\right)^{\frac{\lambda_t^{m,x}}{1-\lambda_t^{m,x}}}, \tag{3.87}$$

and total demand for imports of energy consumption goods

$$\int_0^1 C_{i,t}^{e,m} di = C_t^{e,m} \left(\mathring{p}_t^{m,ce}\right)^{\frac{\lambda_t^{m,ce}}{1-\lambda_t^{m,ce}}}. \tag{3.88}$$

As for domestic intermediate goods producers above, we can use the Calvo assumption to rewrite (3.84) in terms of relative prices as follows:

$$\mathring{p}_t^{m,j} = \left[\xi_{m,j}\left(\frac{\tilde{\pi}_t^{m,j}}{\pi_t^{m,j}}\mathring{p}_{t-1}^{m,j}\right)^{\frac{\lambda_t^{m,j}}{1-\lambda_t^{m,j}}} + \left(1-\xi_{m,j}\right)\left(\tilde{p}_t^{m,j}\right)^{\frac{\lambda_t^{m,j}}{1-\lambda_t^{m,j}}}\right]^{\frac{1-\lambda_t^{m,j}}{\lambda_t^{m,j}}}.$$

Substituting $\tilde{p}_t^{m,j}$ using (3.83), we get

$$\begin{aligned}\mathring{p}_t^{m,j} &= \left[\xi_{m,j}\left(\frac{\tilde{\pi}_t^{m,j}}{\pi_t^{m,j}}\mathring{p}_{t-1}^{m,j}\right)^{\frac{\lambda_t^{m,j}}{1-\lambda_t^{m,j}}} + \left(1-\xi_{m,j}\right)\left(\frac{1-\xi_{m,j}\left(\frac{\tilde{\pi}_t^{m,j}}{\pi_t^{m,j}}\right)^{\frac{1}{1-\lambda_t^{m,j}}}}{1-\xi_{m,j}}\right)^{\lambda_t^{m,j}}\right]^{\frac{1-\lambda_t^{m,j}}{\lambda_t^{m,j}}}, \\ j &= c, i, x, ce.\end{aligned} \tag{3.89}$$

3.2.1 Scaling of the imported intermediate goods producers' optimal conditions

The real marginal cost of the imported (non-energy) consumption, investment and export intermediate goods producers is given by (dividing expressions (3.61), (3.68) and (3.73) by the corresponding price of imported goods):

$$mc_t^{m,j} = \frac{S_t P_t^{d,*} R_t^{wc,m}}{P_t^{m,j}} = \frac{S_t P_t^{c,*}}{P_t^c} \frac{P_t^c}{P_t^d} \frac{P_t^{d,*}}{P_t^{c,*}} \frac{P_t^d}{P_t^{m,j}} R_t^{wc,m} = \frac{q_t p_t^c}{p_t^{c,*} p_t^{m,j}} R_t^{wc,m}, \quad for\ j = c, i, x. \tag{3.90}$$

For the imported energy goods producers, it is given by (dividing expression (3.78) by the price of the imported energy goods):

$$mc_t^{m,ce} = \frac{S_t P_t^{ce,*} R_t^{wc,m}}{P_t^{m,ce}} = \frac{S_t P_t^{c,*}}{P_t^c} \frac{P_t^c}{P_t^d} \frac{P_t^{ce,*}}{P_t^{d,*}} \frac{P_t^{d,*}}{P_t^{c,*}} \frac{P_t^d}{P_t^{m,ce}} R_t^{wc,m} = \frac{q_t p_t^c p_t^{ce,*}}{p_t^{c,*} p_t^{m,ce}} R_t^{wc,m}. \tag{3.91}$$

Scaling equation (3.82) by z_t^+, the optimal-price condition of the importing firms becomes

$$\tilde{p}_t^{m,j} = \frac{E_t \sum_{s=0}^{\infty} (\beta \xi_{m,j})^s \zeta_{t+s}^{\beta} \psi_{z^+,t+s} p_t^{m,j} z_{t+s}^j \lambda_{t+s}^{m,j} mc_{t+s}^{m,j} \left(\frac{\tilde{\pi}_{t+1}^{m,j} \ldots \tilde{\pi}_{t+s}^{m,j}}{\pi_{t+1}^{m,j} \ldots \pi_{t+s}^{m,j}} \right)^{\frac{\lambda_{t+s}^{m,j}}{1-\lambda_{t+s}^{m,j}}}}{E_t \sum_{s=0}^{\infty} (\beta \xi_{m,j})^s \zeta_{t+s}^{\beta} \psi_{z^+,t+s} p_t^{m,j} z_{t+s}^j \left(\frac{\tilde{\pi}_{t+1}^{m,j} \ldots \tilde{\pi}_{t+s}^{m,j}}{\pi_{t+1}^{m,j} \ldots \pi_{t+s}^{m,j}} \right)^{\frac{1}{1-\lambda_{t+s}^{m,j}}}}, \tag{3.92}$$

$$z_t^j = \begin{cases} c_t^m & if\ j = c \\ i_t^m & if\ j = i \\ x_t^m & if\ j = x \\ c_t^{e,m} & if\ j = ce \end{cases},$$

where we have used that

$$v_{t+s} P_{t+s}^{m,j} Z_{t+s} = v_{t+s} z_{t+s}^+ P_{t+s}^d \frac{P_{t+s}^{m,j}}{P_{t+s}^d} \frac{Z_{t+s}^j}{z_{t+s}^+} = \psi_{z^+,t+s} p_{t+s}^{m,j} z_{t+s}^j.$$

We can also scale the expression for profits in equation (3.65) to obtain:

$$profits_{i,t}^{m,c} = P_t^{m,c} c_t^m \left[\left(\frac{P_{i,t}^{m,c}}{P_t^{m,c}} \right)^{\frac{1}{1-\lambda_t^{m,c}}} - mc_t^{m,c} \left(\left(\frac{P_{i,t}^{m,c}}{P_t^{m,c}} \right)^{\frac{\lambda_t^{m,c}}{1-\lambda_t^{m,c}}} + \frac{\phi^{m,c}}{c_t^m} \right) \right].$$

Integrating this expression over the whole set of firms importing consumption goods, we get:

$$profits_t^{m,c} = P_t^{m,c} c_t^m \left[\left(\frac{1}{P_t^{m,c}} \right)^{\frac{1}{1-\lambda_t^{m,c}}} (P_t^{m,c})^{\frac{1}{1-\lambda_t^{m,c}}} - mc_t^{m,c} \left(\left(\mathring{p}_t^{m,j} \right)^{\frac{\lambda_t^{m,c}}{1-\lambda_t^{m,c}}} + \frac{\phi^{m,c}}{c_t^m} \right) \right]$$

or, defining real profits as $\overline{profits}_t^{m,c} = \frac{profits_t^{m,c}}{P_t^{m,c}}$, one gets in real terms:

$$\overline{profits}_t^{m,c} = c_t^m \left[1 - mc_t^{m,c} \left((\mathring{p}_t^{m,c})^{\frac{\lambda_t^{m,c}}{1-\lambda_t^{m,c}}} + \frac{\phi^{m,c}}{c_t^m} \right) \right]. \tag{3.93}$$

Following the same reasoning for the other three types of importing firms (investment and export goods, and energy consumption), we obtain:

$$\overline{profits}_t^{m,i} = i_t^m \left[1 - mc_t^{m,i} \left(\left(\mathring{p}_t^{m,i} \right)^{\frac{\lambda_t^{m,i}}{1-\lambda_t^{m,i}}} + \frac{\phi^{m,i}}{i_t^m} \right) \right], \tag{3.94}$$

$$\overline{profits}_t^{m,x} = x_t^m \left[1 - mc_t^{m,x} \left((\mathring{p}_t^{m,x})^{\frac{\lambda_t^{m,x}}{1-\lambda_t^{m,x}}} + \frac{\phi^{m,x}}{x_t^m} \right) \right], \tag{3.95}$$

$$\overline{profits}_t^{m,ce} = c_t^{e,m} \left[1 - mc_t^{m,ce} \left((\mathring{p}_t^{m,ce})^{\frac{\lambda_t^{m,ce}}{1-\lambda_t^{m,ce}}} + \frac{\phi^{m,ce}}{c_t^{e,m}} \right) \right]. \tag{3.96}$$

3.2.2 Log-linearization of the imported intermediate goods producers' optimal conditions

Log-linearization of the expression for the working capital interest rate for importing firms in equation (3.62) yields

$$\hat{R}_t^{wc,m} = \frac{\nu^{wc,m}(R^* - 1)}{R^{wc,m}} \hat{\nu}_t^{wc,m} + \frac{\nu^{wc,m} R^*}{R^{wc,m}} \hat{R}_t^*. \tag{3.97}$$

Moving on to the importing firms' marginal cost, we can log-linearize equation (3.90) to obtain

$$\widehat{mc}_t^{m,j} = \hat{q}_t + \hat{p}_t^c - \hat{p}_t^{c,*} - \hat{p}_t^{m,j} + \hat{R}_t^{wc,m}, \quad for\ j = c, i, x. \tag{3.98}$$

We can also log-linearize equation (3.91) to obtain

$$\widehat{mc}_t^{m,ce} = \hat{q}_t + \hat{p}_t^c + \hat{p}_t^{ce,*} - \hat{p}_t^{c,*} - \hat{p}_t^{m,ce} + \hat{R}_t^{wc,m}. \tag{3.99}$$

We now turn to the import goods producers' price setting. Starting from equation (3.92), we can proceed with the derivations in the same way as for the for the domestic intermediate goods producers. Assuming that $\varkappa_{m,j} = 0$, for $j = c, i, x, ce$, we get the following Phillips curves for the importing firms (corresponding to equation (3.56) for the domestic intermediate goods producers):

$$\begin{aligned} \hat{\pi}_t^{m,j} - \hat{\bar{\pi}}_t^c &= \frac{(1-\beta\xi_{m,j})(1-\xi_{m,j})}{\xi_{m,j}(1+\beta\kappa_{m,j})} \left(\widehat{mc}_t^{m,j} + \hat{\lambda}_t^{m,j} \right) + \frac{\kappa_{m,j}}{1+\beta\kappa_{m,j}} \left(\hat{\pi}_{t-1}^{m,j} - \hat{\bar{\pi}}_t^c \right) \\ &+ \frac{\beta}{1+\beta\kappa_{m,j}} E_t \left(\hat{\pi}_{t+1}^{m,j} - \hat{\bar{\pi}}_{t+1}^c \right) - \frac{\beta\kappa_{m,j}}{1+\beta\kappa_{m,j}} E_t \left(\hat{\bar{\pi}}_t^c - \hat{\bar{\pi}}_{t+1}^c \right), \\ & j = c, i, x, ce, \end{aligned} \tag{3.100}$$

where we have used the log-linearized version of the price indexation rule for the importing firms in equation (3.81):

$$\hat{\tilde{\pi}}_t^{m,j} = \kappa_{m,j} \hat{\pi}_{t-1}^{m,j} + (1-\kappa_{m,j}) \hat{\bar{\pi}}_t^c, \quad j = c, i, x, ce. \tag{3.101}$$

For use in later sections, we need also to log-linearize the price dispersion equation (3.89). We proceed in the same way as for the domestic price dispersion term in Section 3.1.2. We rearrange (3.89) to obtain

$$\begin{aligned} \left(\mathring{p}_t^{m,j} \right)^{\frac{\lambda_t^{m,j}}{1-\lambda_t^{m,j}}} &= \xi_{m,j} \left(\frac{\tilde{\pi}_t^{m,j}}{\pi_t^{m,j}} \mathring{p}_{t-1}^{m,j} \right)^{\frac{\lambda_t^{m,j}}{1-\lambda_t^{m,j}}} + (1-\xi_{m,j}) \left(\frac{1 - \xi_{m,j} \left(\frac{\tilde{\pi}_t^{m,j}}{\pi_t^{m,j}} \right)^{\frac{1}{1-\lambda_t^{m,j}}}}{1-\xi_{m,j}} \right)^{\lambda_t^{m,j}}, \\ j &= c, i, x, ce, \end{aligned}$$

and log-linearize, which gives

$$\hat{\mathring{p}}_t^{m,j} = \xi_{m,j}\left(\frac{\tilde{\pi}^{m,j}}{\pi^{m,j}}\right)^{\frac{\lambda^{m,j}}{1-\lambda^{m,j}}}\left[\hat{\tilde{\pi}}_t^{m,j} - \hat{\pi}_t^{m,j} + \hat{\mathring{p}}_{t-1}^{m,j}\right]$$

$$+\xi_{m,j}\ln\left(\frac{\tilde{\pi}^{m,j}}{\pi^{m,j}}\mathring{p}^{m,j}\right)\left(\frac{\tilde{\pi}^{m,j}}{\pi^{m,j}}\right)^{\frac{\lambda^{m,j}}{1-\lambda^{m,j}}}\frac{1}{1-\lambda^{m,j}}\hat{\lambda}_t^{m,j} - \ln\left(\mathring{p}^{m,j}\right)\frac{1}{1-\lambda^{m,j}}\hat{\lambda}_t^{m,j}$$

$$-\frac{1}{\left(\mathring{p}^{m,j}\right)^{\frac{\lambda^{m,j}}{1-\lambda^{m,j}}}}\left(\frac{1-\xi_{m,j}\left(\frac{\tilde{\pi}^{m,j}}{\pi^{m,j}}\right)^{\frac{1}{1-\lambda^{m,j}}}}{1-\xi_{m,j}}\right)^{\lambda^{m,j}-1}\xi_{m,j}\left(\frac{\tilde{\pi}^{m,j}}{\pi^{m,j}}\right)^{\frac{1}{1-\lambda^{m,j}}}\left[\hat{\tilde{\pi}}_t^{m,j} - \hat{\pi}_t^{m,j}\right]$$

$$+\frac{1-\xi_{m,j}}{\left(\mathring{p}^{m,j}\right)^{\frac{\lambda^{m,j}}{1-\lambda^{m,j}}}}\ln\left(\frac{1-\xi_{m,j}\left(\frac{\tilde{\pi}^{m,j}}{\pi^{m,j}}\right)^{\frac{1}{1-\lambda^{m,j}}}}{1-\xi_{m,j}}\right)\left(\frac{1-\xi_{m,j}\left(\frac{\tilde{\pi}^{m,j}}{\pi^{m,j}}\right)^{\frac{1}{1-\lambda^{m,j}}}}{1-\xi_{m,j}}\right)^{\lambda^{m,j}}\left(1-\lambda^{m,j}\right)\hat{\lambda}_t^{m,j}$$

$$-\left(\frac{1-\xi_{m,j}\left(\frac{\tilde{\pi}^{m,j}}{\pi^{m,j}}\right)^{\frac{1}{1-\lambda^{m,j}}}}{1-\xi_{m,j}}\right)^{\lambda^{m,j}-1}\frac{\xi_{m,j}}{\left(\mathring{p}^{m,j}\right)^{\frac{\lambda^{m,j}}{1-\lambda^{m,j}}}}\ln\left(\frac{\tilde{\pi}^{m,j}}{\pi^{m,j}}\right)\left(\frac{\tilde{\pi}^{m,j}}{\pi^{m,j}}\right)^{\frac{1}{1-\lambda^{m,j}}}\frac{\lambda^{m,j}}{1-\lambda^{m,j}}\hat{\lambda}_t^{m,j}.$$

We can now use the steady-state relationship $\tilde{\pi}^{m,j} = \pi^{m,j}$, and thus $\mathring{p}^{m,j} = 1$, to obtain

$$\hat{\mathring{p}}_t^{m,j} = \xi_{m,j}\hat{\mathring{p}}_{t-1}^{m,j}, \quad j = c, i, x, ce. \tag{3.102}$$

3.2.3 Marginal costs with exchange rate lags

The marginal costs for the producers of imported consumption goods was given by equation (3.61) above. Considering that many importing firms may hedge against unexpected exchange rate movements, and that the exchange rate thus may affect the marginal cost only with some lag, we could alternatively assume that the marginal cost of the importing goods producers is given by[10]

$$MC_t^{m,j} = S_t\tilde{s}_tP_t^{d,*}R_t^{wc,m}, \quad j = c, i, x \tag{3.103}$$

where

$$\tilde{s}_t = S_t^{-\varsigma_1-\varsigma_2-\varsigma_3}S_{t-1}^{\varsigma_1}S_{t-2}^{\varsigma_2}S_{t-3}^{\varsigma_3}.$$

For importers of the energy consumption good, we could similarly have

$$MC_t^{m,ce} = S_t\tilde{s}_tP_t^{ce,*}R_t^{wc,m}. \tag{3.104}$$

Note that the steady-state value of $\tilde{s}_t$ equals one, so that the steady-state computations are not affected by this alternative marginal cost specification. We can rewrite $\tilde{s}_t$ as follows:

$$\begin{aligned}\tilde{s}_t &= S_t^{-\varsigma_1-\varsigma_2-\varsigma_3}S_{t-1}^{\varsigma_1}S_{t-2}^{\varsigma_2}S_{t-3}^{\varsigma_3}\\ &= \left(\frac{S_{t-1}}{S_t}\right)^{\varsigma_1}\left(\frac{S_{t-2}}{S_t}\right)^{\varsigma_2}\left(\frac{S_{t-3}}{S_t}\right)^{\varsigma_3}\\ &= \left(\frac{S_t}{S_{t-1}}\right)^{-\varsigma_1}\left(\frac{S_t}{S_{t-1}}\frac{S_{t-1}}{S_{t-2}}\right)^{-\varsigma_2}\left(\frac{S_t}{S_{t-1}}\frac{S_{t-1}}{S_{t-2}}\frac{S_{t-2}}{S_{t-3}}\right)^{-\varsigma_3}\\ \tilde{s}_t &= s_t^{-\varsigma_1}\left(s_ts_{t-1}\right)^{-\varsigma_2}\left(s_ts_{t-1}s_{t-2}\right)^{-\varsigma_3} \qquad (3.105)\end{aligned}$$

[10] This could be implemented for all import goods or only for the consumption goods, as that is the only import sector for which we include prices as an observable.

Scaling (3.103), we obtain

$$
\begin{aligned}
mc_t^{m,j} &= \frac{S_t \tilde{s}_t P_t^{d,*} R_t^{wc,m}}{P_t^{m,j}} = \frac{S_t P_t^{d,*} P_t^{c,*} P_t^c P_t^d R_t^{wc,m}}{P_t^{m,j} P_t^{c,*} P_t^c P_t^d} \tilde{s}_t \\
&= \frac{q_t p_t^c \tilde{s}_t}{p_t^{c,*} p_t^{m,j}} R_t^{wc,m}, \quad for\ j = c, i, x.
\end{aligned} \tag{3.106}
$$

Scaling (3.104), we obtain

$$
\begin{aligned}
mc_t^{m,ce} &= \frac{S_t \tilde{s}_t P_t^{ce,*} R_t^{wc,m}}{P_t^{m,ce}} = \frac{S_t P_t^{c,*}}{P_t^c} \frac{P_t^c}{P_t^d} \frac{P_t^{d,*}}{P_t^{c,*}} \frac{P_t^d}{P_t^{m,j}} \tilde{s}_t R_t^{wc,m} \\
&= \frac{q_t p_t^c p_t^{ce,*} \tilde{s}_t}{p_t^{c,*} p_t^{m,ce}} R_t^{wc,m}.
\end{aligned} \tag{3.107}
$$

Log-linearizing, we get

$$
\widehat{mc}_t^{m,j} = \hat{q}_t + \hat{p}_t^c + \hat{\tilde{s}}_t - \hat{p}_t^{c,*} - \hat{p}_t^{m,j} + \hat{R}_t^{wc,m}, \quad for\ j = c, i, x, \tag{3.108}
$$

$$
\widehat{mc}_t^{m,ce} = \hat{q}_t + \hat{p}_t^c + \hat{p}_t^{ce,*} + \hat{\tilde{s}}_t - \hat{p}_t^{c,*} - \hat{p}_t^{m,ce} + \hat{R}_t^{wc,m}, \tag{3.109}
$$

and

$$
\hat{\tilde{s}}_t = -\varsigma_1 \hat{s}_t - \varsigma_2 \left(\hat{s}_t + \hat{s}_{t-1}\right) - \varsigma_3 \left(\hat{s}_t + \hat{s}_{t-1} + \hat{s}_{t-2}\right). \tag{3.110}
$$

An alternative could be to include also lags of the price of the homogeneous foreign good which is used as input in the import production. For the import goods producers, we would then have the following alternative specification of $\tilde{s}_t$:

$$
\begin{aligned}
\tilde{s}_t &= \left(S_t P_t^{d,*}\right)^{-\varsigma_1 - \varsigma_2 - \varsigma_3} \left(S_{t-1} P_{t-1}^{d,*}\right)^{\varsigma_1} \left(S_{t-2} P_{t-2}^{d,*}\right)^{\varsigma_2} \left(S_{t-3} P_{t-3}^{d,*}\right)^{\varsigma_3} \\
&= \left(\frac{S_{t-1}}{S_t} \frac{P_{t-1}^{d,*}}{P_t^{d,*}}\right)^{\varsigma_1} \left(\frac{S_{t-2}}{S_t} \frac{P_{t-2}^{d,*}}{P_t^{d,*}}\right)^{\varsigma_2} \left(\frac{S_{t-3}}{S_t} \frac{P_{t-3}^{d,*}}{P_t^{d,*}}\right)^{\varsigma_3} \\
&= \left(\frac{S_t}{S_{t-1}} \frac{P_t^{d,*}}{P_{t-1}^{d,*}}\right)^{-\varsigma_1} \left(\frac{S_t}{S_{t-1}} \frac{S_{t-1}}{S_{t-2}} \frac{P_t^{d,*}}{P_{t-1}^{d,*}} \frac{P_{t-1}^{d,*}}{P_{t-2}^{d,*}}\right)^{-\varsigma_2} \times \\
&\quad \left(\frac{S_t}{S_{t-1}} \frac{S_{t-1}}{S_{t-2}} \frac{S_{t-2}}{S_{t-3}} \frac{P_t^{d,*}}{P_{t-1}^{d,*}} \frac{P_{t-1}^{d,*}}{P_{t-2}^{d,*}} \frac{P_{t-2}^{d,*}}{P_{t-3}^{d,*}}\right)^{-\varsigma_3} \\
\tilde{s}_t &= \left(s_t \pi_t^{d,*}\right)^{-\varsigma_1} \left(s_t s_{t-1} \pi_t^{d,*} \pi_{t-1}^{d,*}\right)^{-\varsigma_2} \left(s_t s_{t-1} s_{t-2} \pi_t^{d,*} \pi_{t-1}^{d,*} \pi_{t-2}^{d,*}\right)^{-\varsigma_3}.
\end{aligned} \tag{3.111}
$$

Log-linearizing,

$$
\begin{aligned}
\hat{\tilde{s}}_t &= -\varsigma_1 \left(\hat{s}_t + \hat{\pi}_t^{d,*}\right) - \varsigma_2 \left(\hat{s}_t + \hat{s}_{t-1} + \hat{\pi}_t^{d,*} + \hat{\pi}_{t-1}^{d,*}\right) \\
&\quad -\varsigma_3 \left(\hat{s}_t + \hat{s}_{t-1} + \hat{s}_{t-2} + \hat{\pi}_t^{d,*} + \hat{\pi}_{t-1}^{d,*} + \hat{\pi}_{t-2}^{d,*}\right).
\end{aligned} \tag{3.112}
$$

For the importers of the energy consumption good, we would then instead have

$$
\widehat{mc}_t^{m,ce} = \hat{q}_t + \hat{p}_t^c + \hat{p}_t^{ce,*} + \hat{\tilde{s}}_t^{ce} - \hat{p}_t^{c,*} - \hat{p}_t^{m,ce} + \hat{R}_t^{wc,m}, \tag{3.113}
$$

where

$$
\begin{aligned}
\hat{\tilde{s}}_t^{ce} &= -\varsigma_1 \left(\hat{s}_t + \hat{\pi}_t^{ce,*}\right) - \varsigma_2 \left(\hat{s}_t + \hat{s}_{t-1} + \hat{\pi}_t^{ce,*} + \hat{\pi}_{t-1}^{ce,*}\right) \\
&\quad -\varsigma_3 \left(\hat{s}_t + \hat{s}_{t-1} + \hat{s}_{t-2} + \hat{\pi}_t^{ce,*} + \hat{\pi}_{t-1}^{ce,*} + \hat{\pi}_{t-2}^{ce,*}\right).
\end{aligned} \tag{3.114}
$$

3.3 Production of final consumption goods

Final consumption goods are purchased by households. The goods are produced by a representative competitive firm that combines domestically produced and imported goods and energy according to the following technologies:

$$C_t = \left[(1-\omega_e)^{1/\eta_e}\left(C_t^{xe}\right)^{(\eta_e-1)/\eta_e} + \omega_e^{1/\eta_e}\left(C_t^{e}\right)^{(\eta_e-1)/\eta_e}\right]^{\eta_e/(\eta_e-1)}, \tag{3.115}$$

$$C_t^{xe} = \left[(1-\omega_c)^{1/\eta_c}\left(C_t^{d}\right)^{(\eta_c-1)/\eta_c} + \omega_c^{1/\eta_c}\left(C_t^{m}\right)^{(\eta_c-1)/\eta_c}\right]^{\eta_c/(\eta_c-1)}, \tag{3.116}$$

$$C_t^{e} = \left[(1-\omega_{em})^{1/\eta_{em}}\left(C_t^{e,d}\right)^{(\eta_{em}-1)/\eta_{em}} + \omega_{em}^{1/\eta_{em}}\left(C_t^{e,m}\right)^{(\eta_{em}-1)/\eta_{em}}\right]^{\eta_{em}/(\eta_{em}-1)}. \tag{3.117}$$

Here, C_t^{xe} denotes consumption excluding energy and C_t^e is consumption of energy. C_t^d is a one-for-one transformation of the domestic homogeneous good Y_t, and C_t^m is the homogeneous composite of specialized consumption imported goods discussed in Section 3.2. $C_t^{e,d}$ is the consumption of domestically produced energy, and $C_t^{e,m}$ is the imported energy consumption. ω_e determines the steady-state share of energy in consumption and η_e is the elasticity of substitution between energy and non-energy consumption. ω_c determines the steady-state share of imports in non-energy consumption, and η_c the elasticity of substitution between domestic and imported consumption goods. ω_{em} determines the steady-state share of imports in energy consumption, and η_{em} the elasticity of substitution between domestic and imported energy. The introduction of energy consumption will allow us to include both headline and core inflation as observables. We assume that production of energy requires some use of the domestic homogenous good, just as all the other goods. We think of this as there being an endowment of energy, which requires labour and capital in the same proportions as all other goods in order to be turned into consumable energy. Note that we will treat the prices of the domestic energy consumption in the two countries as stochastic processes.

The representative firm takes the price of output P_t^c and the prices of inputs P_t^d, $P_t^{m,c}$, $P_t^{d,ce}$ and $P_t^{m,ce}$ as given. It faces the following three budget constraints:

$$P_t^{cxe}C_t^{xe} + P_t^{ce}C_t^{e} = P_t^{c}C_t, \tag{3.118}$$

$$P_t^{d}C_t^{d} + P_t^{m,c}C_t^{m} = P_t^{cxe}C_t^{xe}, \tag{3.119}$$

$$P_t^{d,ce}C_t^{e,d} + P_t^{m,ce}C_t^{e,m} = P_t^{ce}C_t^{e}. \tag{3.120}$$

We define the aggregate price index as the inverse of the Lagrange multiplier on the expenditures, i.e. $P_t^c \equiv 1/\lambda_t^c$. Also, for non-energy consumption we have that $P_t^{cxe} \equiv 1/\lambda_t^{cxe}$, while for energy consumption $P_t^{ce} \equiv 1/\lambda_t^{ce}$. This allows us to write total expenditures as $P_t^c C_t$ and expenditures on non-energy goods and energy as $P_t^{cxe}C_t^{xe}$ and $P_t^{ce}C_t^{e}$, respectively. We assume that the agent first solves the partial problems of how to allocate the shares they spend on non-energy goods and energy, respectively, between domestic and imported goods.

First, we derive expressions for C_t^d and C_t^m for a given expenditure level EXP_t^{xe}, by solving the following problem:

$$\max_{C_t^d, C_t^m}\left[(1-\omega_c)^{\frac{1}{\eta_c}}\left(C_t^{d}\right)^{\frac{\eta_c-1}{\eta_c}} + \omega_c^{\frac{1}{\eta_c}}\left(C_t^{m}\right)^{\frac{\eta_c-1}{\eta_c}}\right]^{\frac{\eta_c}{\eta_c-1}} - \lambda_t^{cxe}\left[P_t^d C_t^d + P_t^{m,c}C_t^m - EXP_t^{cxe}\right]$$

Optimization yields the following demand functions for C_t^d and C_t^m:

$$C_t^d = (1-\omega_c)\left[\frac{P_t^d}{P_t^{cxe}}\right]^{-\eta_c} C_t^{xe}, \tag{3.121}$$

$$C_t^m = \omega_c \left[\frac{P_t^{m,c}}{P_t^{cxe}} \right]^{-\eta_c} C_t^{xe}. \tag{3.122}$$

Here, P_t^d denotes the domestic price level, $P_t^{m,c}$ the price level of imported consumption, and P_t^{cxe} the aggregate consumption price index excluding energy (CPIxe). Substituting (3.121) and (3.122) into (3.119), we obtain the following expression for the CPI excluding energy:

$$P_t^{cxe} = \left[(1-\omega_c) \left(P_t^d \right)^{1-\eta_c} + \omega_c \left(P_t^{m,c} \right)^{1-\eta_c} \right]^{1/(1-\eta_c)}. \tag{3.123}$$

Next, we derive expressions for $C_t^{e,d}$ and $C_t^{e,m}$ for a given expenditure level EXP_t^e, by solving the following problem:

$$\max_{C_t^{e,d}, C_t^{e,m}} \left[(1-\omega_{em})^{\frac{1}{\eta_{em}}} \left(C_t^{e,d} \right)^{\frac{\eta_{em}-1}{\eta_{em}}} + \omega_{em}^{\frac{1}{\eta_{em}}} \left(C_t^{e,m} \right)^{\frac{\eta_{em}-1}{\eta_{em}}} \right]^{\frac{\eta_{em}}{\eta_{em}-1}}$$
$$-\lambda_t^{ce} \left[P_t^{d,ce} C_t^{e,d} + P_t^{m,ce} C_t^{e,m} - EXP_t^{ce} \right]$$

Optimization yields the following demand functions for $C_t^{e,d}$ and $C_t^{e,m}$:

$$C_t^{e,d} = (1-\omega_{em}) \left[\frac{P_t^{d,ce}}{P_t^{ce}} \right]^{-\eta_{em}} C_t^e, \tag{3.124}$$

$$C_t^{e,m} = \omega_{em} \left[\frac{P_t^{m,ce}}{P_t^{ce}} \right]^{-\eta_{em}} C_t^e. \tag{3.125}$$

Here, $P_t^{d,ce}$ denotes the price level of the domestic energy component, $P_t^{m,ce}$ the price level of the imported energy component, and P_t^{ce} the aggregate energy price index. Substituting (3.124) and (3.125) into (3.120), we obtain the following expression for the the energy price index:

$$P_t^{ce} = \left[(1-\omega_{em}) \left(P_t^{d,ce} \right)^{1-\eta_{em}} + \omega_{em} \left(P_t^{m,ce} \right)^{1-\eta_{em}} \right]^{1/(1-\eta_{em})}. \tag{3.126}$$

Finally, we solve the problem for the aggregate consumption good. We derive expressions for C_t^{xe} and C_t^e for a given total expenditure level EXP_t^c, by solving the following problem:

$$\max_{C_t^{xe}, C_t^e} \left[(1-\omega_e)^{\frac{1}{\eta_e}} \left(C_t^{xe} \right)^{\frac{\eta_e-1}{\eta_e}} + (\omega_e)^{\frac{1}{\eta_c}} \left(C_t^e \right)^{\frac{\eta_e-1}{\eta_e}} \right]^{\frac{\eta_e}{\eta_e-1}} - \lambda_t^c \left[P_t^{cxe} C_t^{xe} + P_t^{ce} C_t^e - EXP_t^c \right]$$

Optimization yields the following demand functions for C_t^{xe} and C_t^e:

$$C_t^{xe} = (1-\omega_e) \left[\frac{P_t^{cxe}}{P_t^c} \right]^{-\eta_e} C_t, \tag{3.127}$$

$$C_t^e = \omega_e \left[\frac{P_t^{ce}}{P_t^c} \right]^{-\eta_e} C_t. \tag{3.128}$$

Here, P_t^{ce} denotes the price of the energy consumption good and P_t^c the aggregate consumption price index (CPI). Substituting (3.127) and (3.128) into (3.118), we obtain the following expression for CPI:

$$P_t^c = \left[(1-\omega_e) \left(P_t^{cxe} \right)^{1-\eta_e} + \omega_e \left(P_t^{ce} \right)^{1-\eta_e} \right]^{1/(1-\eta_e)}. \tag{3.129}$$

Defining the relative prices of final consumption and imported consumption (relative to the domestic price) as

$$p_t^c = \frac{P_t^c}{P_t^d} \tag{3.130}$$

$$p_t^{m,c} = \frac{P_t^{m,c}}{P_t^d}, \tag{3.131}$$

the relative prices of non-energy and energy consumption as

$$p_t^{cxe} = \frac{P_t^{cxe}}{P_t^d}, \tag{3.132}$$

$$p_t^{ce} = \frac{P_t^{ce}}{P_t^d}, \tag{3.133}$$

and the relative price of domestically produced and imported energy consumption as

$$p_t^{d,ce} = \frac{P_t^{d,ce}}{P_t^d}, \tag{3.134}$$

$$p_t^{m,ce} = \frac{P_t^{m,ce}}{P_t^d}, \tag{3.135}$$

we can rewrite (3.123), (3.126) and (3.129) as follows

$$p_t^{cxe} = \left[(1-\omega_c) + \omega_c \left(p_t^{m,c}\right)^{1-\eta_c}\right]^{1/(1-\eta_c)}, \tag{3.136}$$

$$p_t^{ce} = \left[(1-\omega_{em}) \left(p_t^{d,ce}\right)^{1-\eta_{em}} + \omega_{em} \left(p_t^{m,ce}\right)^{1-\eta_{em}}\right]^{1/(1-\eta_{em})}, \tag{3.137}$$

and

$$p_t^c = \left[(1-\omega_e) \left(p_t^{cxe}\right)^{1-\eta_e} + \omega_e \left(p_t^{ce}\right)^{1-\eta_e}\right]^{1/(1-\eta_e)}. \tag{3.138}$$

The rate of inflation of the aggregate consumption good excluding energy is then given by

$$\pi_t^{cxe} = \frac{P_t^{cxe}}{P_{t-1}^{cxe}} = \left[\frac{(1-\omega_c)\left(P_t^d\right)^{1-\eta_c} + \omega_c \left(P_t^{m,c}\right)^{1-\eta_c}}{(1-\omega_c)\left(P_{t-1}^d\right)^{1-\eta_c} + \omega_c \left(P_{t-1}^{m,c}\right)^{1-\eta_c}}\right]^{1/(1-\eta_c)}, \tag{3.139}$$

or, in terms of relative prices,

$$\begin{aligned}
\pi_t^{cxe} &= \left(\frac{P_t^{cxe}}{P_{t-1}^{cxe}} \frac{P_{t-1}^d}{P_t^d}\right) \frac{P_t^d}{P_{t-1}^d} = \frac{p_t^{cxe}}{p_{t-1}^{cxe}} \pi_t^d \\
&= \pi_t^d \left[\frac{(1-\omega_c) + \omega_c \left(p_t^{m,c}\right)^{1-\eta_c}}{(1-\omega_c) + \omega_c \left(p_{t-1}^{m,c}\right)^{1-\eta_c}}\right]^{1/(1-\eta_c)},
\end{aligned} \tag{3.140}$$

where $\pi_t^d = P_t^d / P_{t-1}^d$ denotes the rate of inflation of the domestically produced goods. Similarly, the rate of inflation of aggregate energy consumption is then given by

$$\pi_t^{ce} = \frac{P_t^{ce}}{P_{t-1}^{ce}} = \left[\frac{(1-\omega_{em})\left(P_t^{d,ce}\right)^{1-\eta_{em}} + \omega_{em}\left(P_t^{m,ce}\right)^{1-\eta_{em}}}{(1-\omega_{em})\left(P_{t-1}^{d,ce}\right)^{1-\eta_{em}} + \omega_{em}\left(P_{t-1}^{m,ce}\right)^{1-\eta_{em}}}\right]^{1/(1-\eta_{em})}, \tag{3.141}$$

or, in terms of relative prices,

$$\begin{aligned}\pi_t^{ce} &= \left(\frac{P_t^{ce}}{P_{t-1}^{ce}}\frac{P_{t-1}^d}{P_t^d}\right)\frac{P_t^d}{P_{t-1}^d} = \frac{p_t^{ce}}{p_{t-1}^{ce}}\pi_t^d \\ &= \pi_t^d\left[\frac{(1-\omega_{em})\left(p_t^{d,ce}\right)^{1-\eta_{em}} + \omega_{em}\left(p_t^{m,ce}\right)^{1-\eta_{em}}}{(1-\omega_{em})\left(p_{t-1}^{d,ce}\right)^{1-\eta_{em}} + \omega_{em}\left(p_{t-1}^{m,ce}\right)^{1-\eta_{em}}}\right]^{1/(1-\eta_{em})}.\end{aligned} \tag{3.142}$$

Finally, the rate of inflation of the final consumption good is given by

$$\pi_t^c = \frac{P_t^c}{P_{t-1}^c} = \left[\frac{(1-\omega_e)\left(P_t^{cxe}\right)^{1-\eta_e} + \omega_e\left(P_t^{ce}\right)^{1-\eta_e}}{(1-\omega_e)\left(P_{t-1}^{cxe}\right)^{1-\eta_e} + \omega_e\left(P_{t-1}^{ce}\right)^{1-\eta_e}}\right]^{1/(1-\eta_e)}, \tag{3.143}$$

or, in terms of of relative prices,

$$\begin{aligned}\pi_t^c &= \left(\frac{P_t^c}{P_{t-1}^c}\frac{P_{t-1}^d}{P_t^d}\right)\frac{P_t^d}{P_{t-1}^d} = \frac{p_t^c}{p_{t-1}^c}\pi_t^d \\ &= \pi_t^d\left[\frac{(1-\omega_e)\left(p_t^{cxe}\right)^{1-\eta_e} + \omega_e\left(p_t^{ce}\right)^{1-\eta_e}}{(1-\omega_e)\left(p_{t-1}^{cxe}\right)^{1-\eta_e} + \omega_e\left(p_{t-1}^{ce}\right)^{1-\eta_e}}\right]^{\frac{1}{1-\eta_e}}.\end{aligned} \tag{3.144}$$

3.3.1 Scaling of the final consumption goods producers' optimal conditions

Scaling equations (3.121), (3.122), (3.124), (3.125), (3.127) and (3.128) by z_t^+, we obtain the following demand functions:

$$c_t^d = (1-\omega_c)\left(p_t^{cxe}\right)^{\eta_c} c_t^{xe}, \tag{3.145}$$

$$c_t^m = \omega_c\left(\frac{p_t^{m,c}}{p_t^{cxe}}\right)^{-\eta_c} c_t^{xe}, \tag{3.146}$$

$$c_t^{e,d} = (1-\omega_{em})\left(\frac{p_t^{d,ce}}{p_t^{ce}}\right)^{-\eta_{em}} c_t^e \tag{3.147}$$

$$c_t^{e,m} = \omega_{em}\left(\frac{p_t^{m,ce}}{p_t^{ce}}\right)^{-\eta_{em}} c_t^e \tag{3.148}$$

$$c_t^{xe} = (1-\omega_e)\left(\frac{p_t^{cxe}}{p_t^c}\right)^{-\eta_e} c_t, \tag{3.149}$$

$$c_t^e = \omega_e\left(\frac{p_t^{ce}}{p_t^c}\right)^{-\eta_e} c_t, \tag{3.150}$$

where we have used the definitions of relative prices p_t^c, $p_t^{m,c}$, p_t^{cxe}, p_t^{ce}, $p_t^{d,ce}$, and $p_t^{m,ce}$specified in Section 2.2.

3.3.2 Log-linearization of the final consumption goods producers' optimal conditions

From equation (3.145), we obtain the following log-linear demand for domestic consumption goods:

$$\hat{c}_t^d = \eta_c\hat{p}_t^{cxe} + \hat{c}_t^{xe}. \tag{3.151}$$

From equation (3.146), we have the following log-linear demand for imported consumption goods:

$$\hat{c}_t^m = -\eta_c\left(\hat{p}_t^{m,c} - \hat{p}_t^{cxe}\right) + \hat{c}_t^{xe}. \tag{3.152}$$

Equation (3.147), next, yields the following log-linear demand for consumption of domestically produced energy:

$$\hat{c}_t^{e,d} = -\eta_{em}\left(\hat{p}_t^{d,ce} - \hat{p}_t^{ce}\right) + \hat{c}_t^{e}. \tag{3.153}$$

Equation (3.148), in turn, yields the following log-linear demand for imported energy consumption

$$\hat{c}_t^{e,m} = -\eta_{em}\left(\hat{p}_t^{m,ce} - \hat{p}_t^{ce}\right) + \hat{c}_t^{e}. \tag{3.154}$$

Equation (3.149) yields the following log-linear demand for non-energy consumption:

$$\hat{c}_t^{xe} = -\eta_{e}\left(\hat{p}_t^{cxe} - \hat{p}_t^{c}\right) + \hat{c}_t. \tag{3.155}$$

Finally, equation (3.150) yields the following log-linear demand for energy consumption:

$$\hat{c}_t^{e} = -\eta_{e}\left(\hat{p}_t^{ce} - \hat{p}_t^{c}\right) + \hat{c}_t. \tag{3.156}$$

To obtain a log-linear expression for the consumer price index excluding energy, we can log-linearize equation (3.123) in levels to obtain

$$\hat{P}_t^{cxe} = (1-\omega_c)\left(\frac{1}{p^{cxe}}\right)^{1-\eta_c}\hat{P}_t^{d} + \omega_c\left(\frac{p^{m,c}}{p^{cxe}}\right)^{1-\eta_c}\hat{P}_t^{m,c},$$

where we have used the definitions of relative prices p_t^{cxe} and $p_t^{m,c}$, specified in Section 2.2. Lagging one period and differencing, and using that the definitions of the inflation rates in Section 2.2 imply that

$$\begin{aligned}
\hat{\pi}_t^{cxe} &= \hat{P}_t^{cxe} - \hat{P}_{t-1}^{cxe}, \\
\hat{\pi}_t^{d} &= \hat{P}_t^{d} - \hat{P}_{t-1}^{d}, \\
\hat{\pi}_t^{m,c} &= \hat{P}_t^{m,c} - \hat{P}_{t-1}^{m,c},
\end{aligned}$$

we obtain the following log-linear expression for the CPIxe inflation in terms of domestic and imported inflation:

$$\hat{\pi}_t^{cxe} = (1-\omega_c)\left(\frac{1}{p^{cxe}}\right)^{1-\eta_c}\hat{\pi}_t^{d} + \omega_c\left(\frac{p^{m,c}}{p^{cxe}}\right)^{1-\eta_c}\hat{\pi}_t^{m,c}. \tag{3.157}$$

Proceeding in the same way as for the CPIxe above, we can start from equation (3.126) in order to obtain an expression for the rate of inflation of the energy price index. Log-linearizing, we get

$$\hat{P}_t^{ce} = (1-\omega_{em})\left(\frac{p^{d,ce}}{p^{ce}}\right)^{1-\eta_{em}}\hat{P}_t^{d,ce} + \omega_{em}\left(\frac{p^{m,ce}}{p^{ce}}\right)^{1-\eta_{em}}\hat{P}_t^{m,ce},$$

where we have used the definitions of relative prices $p_t^{d,ce}$, $p_t^{m,ce}$ and p_t^{ce}, specified in Section 2.2. Lagging one period and differencing, and using that the definitions of the inflation rates in Section 2.2 imply that

$$\begin{aligned}
\hat{\pi}_t^{ce} &= \hat{P}_t^{ce} - \hat{P}_{t-1}^{ce}, \\
\hat{\pi}_t^{d,ce} &= \hat{P}_t^{d,ce} - \hat{P}_{t-1}^{d,ce}, \\
\hat{\pi}_t^{m,ce} &= \hat{P}_t^{m,ce} - \hat{P}_{t-1}^{m,ce},
\end{aligned}$$

we obtain the following log-linear expression for the consumer energy price inflation in terms of domestic and imported energy inflation:

$$\hat{\pi}_t^{ce} = (1-\omega_{em})\left(\frac{p^{d,ce}}{p^{ce}}\right)^{1-\eta_{em}}\hat{\pi}_t^{c,ce} + \omega_{em}\left(\frac{p^{m,ce}}{p^{ce}}\right)^{1-\eta_{em}}\hat{\pi}_t^{m,ce}. \tag{3.158}$$

Finally, in an analogous way to the above, from equation (3.129) we can obtain an expression for the rate of inflation of the aggregate total consumption goods. Log-linearizing, we get

$$\hat{P}_t^c = (1-\omega_e)\left(\frac{p^{cxe}}{p^c}\right)^{1-\eta_e}\hat{P}_t^{cxe} + \omega_e\left(\frac{p^{ce}}{p^c}\right)^{1-\eta_e}\hat{P}_t^{ce}.$$

Lagging one period and differencing, and using that the definitions of the inflation rates in Section 2.2 imply that

$$\begin{aligned}\hat{\pi}_t^c &= \hat{P}_t^c - \hat{P}_{t-1}^c,\\ \hat{\pi}_t^{cxe} &= \hat{P}_t^{cxe} - \hat{P}_{t-1}^{cxe},\\ \hat{\pi}_t^{ce} &= \hat{P}_t^{ce} - \hat{P}_{t-1}^{ce},\end{aligned}$$

we obtain the following log-linear expression for the aggregate consumer price inflation in terms of non-energy and energy goods price inflation:

$$\hat{\pi}_t^c = (1-\omega_e)\left(\frac{p^{cxe}}{p^c}\right)^{1-\eta_e}\hat{\pi}_t^{cxe} + \omega_e\left(\frac{p^{ce}}{p^c}\right)^{1-\eta_e}\hat{\pi}_t^{ce}. \tag{3.159}$$

We finally log-linearize the expressions for relative prices in equations (3.136), (3.137) and (3.138), which yields the following expressions:

$$\hat{p}_t^{cxe} = \omega_c\left(\frac{p^{m,c}}{p^{cxe}}\right)^{1-\eta_c}\hat{p}_t^{m,c}, \tag{3.160}$$

$$\hat{p}_t^{ce} = (1-\omega_{em})\left(\frac{p^{d,ce}}{p^{ce}}\right)^{1-\eta_{em}}\hat{p}_t^{d,ce} + \omega_{em}\left(\frac{p^{m,ce}}{p^{ce}}\right)^{1-\eta_{em}}\hat{p}_t^{m,ce}, \tag{3.161}$$

$$\hat{p}_t^c = (1-\omega_e)\left(\frac{p^{cxe}}{p^c}\right)^{1-\eta_e}\hat{p}_t^{cxe} + \omega_e\left(\frac{p^{ce}}{p^c}\right)^{1-\eta_e}\hat{p}_t^{ce}. \tag{3.162}$$

Note that we assume that the relative price of domestic energy evolves as an exogenous process, as given by the following equation

$$\log p_t^{d,ce} = \left(1-\rho_{p^{d,ce}}\right)\log p^{d,ce} + \rho_{p^{d,ce}}\log p_{t-1}^{d,ce} + \sigma_{p^{d,ce}}\varepsilon_{p^{d,ce},t}. \tag{3.163}$$

3.4 Production of final investment goods

As for consumption, final investment goods are produced by a representative competitive firm that combines domestically produced and imported goods. Total investment is, however, defined as the sum of investment goods used in the accumulation of physical capital, I_t, plus investment goods used in capital maintenance, $a(u_t)K_t^p$. Moreover, to accomodate the observation that the price of investment goods relative to the price of consumption goods is declining over time, the investment production technology includes a unit root process with a positive drift, denoted by Ψ_t. Specifically, the investment production technology is given by

$$I_t + a(u_t)K_t^p = \Psi_t\left[(1-\omega_i)^{1/\eta_i}\left(I_t^d\right)^{(\eta_i-1)/\eta_i} + \omega_i^{1/\eta_i}\left(I_t^m\right)^{(\eta_i-1)/\eta_i}\right]^{\eta_i/(\eta_i-1)}, \tag{3.164}$$

where I_t^d is a one-for-one transformation of the domestic homogeneous good Y_t, and I_t^m is the homogeneous composite of specialized investment imported goods discussed in Section 3.2. ω_i determines the steady-state share of imports in investment, and η_i the elasticity of substitution between domestic and imported investment goods. The representative firm takes the price of output, P_t^i, and the prices of inputs, P_t^d and $P_t^{m,i}$, as given. Proceding in the same way as for consumption above, i.e. maximizing equation (3.164) subject to the investment expenditure budget constraint, we obtain the following demand functions for I_t^d and I_t^m:

$$I_t^d = (1-\omega_i)\left[\frac{P_t^d}{P_t^i}\right]^{-\eta_i} \Psi_t^{\eta_i - 1}\left(I_t + a\left(u_t\right) K_t^p\right), \tag{3.165}$$

$$I_t^m = \omega_i \left[\frac{P_t^{m,i}}{P_t^i}\right]^{-\eta_i} \Psi_t^{\eta_i - 1}\left(I_t + a\left(u_t\right) K_t^p\right), \tag{3.166}$$

where $P_t^{m,i}$ denotes the price of imported investment, and P_t^i the aggregate investment price index. Note that the prices of the domestically produced consumption and investment goods are assumed to be the same. As for the CPI, we can use the derived demand functions to obtain an expression for the aggregate investment price index. Using equations (3.165) and (3.166) in the investment expenditure budget constraint, we obtain

$$P_t^i = \frac{1}{\Psi_t}\left[(1-\omega_i)\left(P_t^d\right)^{1-\eta_i} + \omega_i\left(P_t^{m,i}\right)^{1-\eta_i}\right]^{1/(1-\eta_i)}. \tag{3.167}$$

Defining the relative prices of final investment and imported investment relative to the domestic price as

$$p_t^i = \Psi_t \frac{P_t^i}{P_t^d}, \tag{3.168}$$

$$p_t^{m,i} = \frac{P_t^{m,i}}{P_t^d}, \tag{3.169}$$

we can rewrite (3.167) as follows

$$p_t^i = \left[(1-\omega_i) + \omega_i\left(p_t^{m,i}\right)^{1-\eta_i}\right]^{1/(1-\eta_i)}. \tag{3.170}$$

The rate of inflation of the final investment good is then given by

$$\pi_t^i = \frac{P_t^i}{P_{t-1}^i} = \frac{\Psi_{t-1}}{\Psi_t}\left[\frac{(1-\omega_i)\left(P_t^d\right)^{1-\eta_i} + \omega_i\left(P_t^{m,i}\right)^{1-\eta_i}}{(1-\omega_i)\left(P_{t-1}^d\right)^{1-\eta_i} + \omega_i\left(P_{t-1}^{m,i}\right)^{1-\eta_i}}\right]^{1/(1-\eta_i)}, \tag{3.171}$$

or, in terms of relative prices,

$$\begin{aligned}\pi_t^i &= \left(\frac{P_t^i}{P_{t-1}^i}\frac{P_{t-1}^d}{P_t^d}\frac{\Psi_t}{\Psi_{t-1}}\right)\frac{P_t^d}{P_{t-1}^d}\frac{\Psi_{t-1}}{\Psi_t} = \frac{p_t^i}{p_{t-1}^i}\frac{\pi_t^d}{\mu_{\Psi,t}} \\ &= \frac{\pi_t^d}{\mu_{\Psi,t}}\left[\frac{(1-\omega_i) + \omega_i\left(p_t^{m,i}\right)^{1-\eta_i}}{(1-\omega_i) + \omega_i\left(p_{t-1}^{m,i}\right)^{1-\eta_i}}\right]^{1/(1-\eta_i)},\end{aligned} \tag{3.172}$$

where

$$\mu_{\Psi,t} \equiv \frac{\Psi_t}{\Psi_{t-1}} \tag{3.173}$$

denotes the growht rate of the investment-specific technology shock Ψ_t.

3.4.1 Scaling of the final investment goods producers' optimal conditions

The demand functions for imported and domestically produced investment goods (3.165) and (3.166) in scaled form are given by:

$$i_t^d = (1-\omega_i)\left(\frac{1}{p_t^i}\right)^{-\eta_i}\left(i_t + a(u_t)\frac{k_t^p}{\mu_{z^+,t}\mu_{\Psi,t}}\right), \tag{3.174}$$

$$i_t^m = \omega_i\left(\frac{p_t^{m,i}}{p_t^i}\right)^{-\eta_i}\left(i_t + a(u_t)\frac{k_t^p}{\mu_{z^+,t}\mu_{\Psi,t}}\right), \tag{3.175}$$

where we have used the definitions of relative prices p_t^i and $p_t^{m,i}$ specified in Section 2.2.

3.4.2 Log-linearization of the final investment goods producers' optimal conditions

We begin by log-linearizing the demand for domestically produced investment goods in equation (3.174):

$$\hat{i}_t^d = \eta_i\hat{p}_t^i + \frac{1}{i + a(u)\frac{k^p}{\mu_{z^+}\mu_\Psi}}\left(i\hat{i}_t + \frac{a'(u)k^p}{\mu_{z^+}\mu_\Psi}u\hat{u}_t + \frac{a(u)k^p}{\mu_{z^+}\mu_\Psi}\left(\hat{k}_t^p - \hat{\mu}_{z^+,t} - \hat{\mu}_{\Psi,t}\right)\right).$$

Using that $a(u) = 0$, $a'(u) = \sigma_b$, and $u = 1$ (see Section 4.2.1, specifically equations (4.17) and (4.16)), we finally get

$$\hat{i}_t^d = \eta_i\hat{p}_t^i + \hat{i}_t + \frac{1}{i}\frac{\sigma_b k^p}{\mu_{z^+}\mu_\Psi}\hat{u}_t. \tag{3.176}$$

Similarly, log-linearizing the demand for imported investment goods in equation (3.175), we get

$$\hat{i}_t^m = -\eta_i\left(\hat{p}_t^{m,i} - \hat{p}_t^i\right) + \frac{1}{i + a(u)\frac{k^p}{\mu_{z^+}\mu_\Psi}}\left(i\hat{i}_t + \frac{a'(u)k^p}{\mu_{z^+}\mu_\Psi}u\hat{u}_t + \frac{a(u)k^p}{\mu_{z^+}\mu_\Psi}\left(\hat{k}_t^p - \hat{\mu}_{z^+,t} - \hat{\mu}_{\Psi,t}\right)\right).$$

Using again that $a(u) = 0$, $a'(u) = \sigma_b$, and $u = 1$, we obtain

$$\hat{i}_t^m = -\eta_i\left(\hat{p}_t^{m,i} - \hat{p}_t^i\right) + \hat{i}_t + \frac{1}{i}\frac{\sigma_b k^p}{\mu_{z^+}\mu_\Psi}\hat{u}_t. \tag{3.177}$$

As for consumption inflation, we can log-linearize equation (3.167) in levels to obtain

$$\hat{P}_t^i = (1-\omega_i)\left(p^i\right)^{\eta_i - 1}\hat{P}_t^d + \omega_i\left(\frac{p^{m,i}}{p^i}\right)^{1-\eta_i}\hat{P}_t^{m,i} - \frac{1}{1-\eta_i}\hat{\Psi}_t,$$

where we have used the definitions of relative prices p_t^i and $p_t^{m,i}$, specified in Section 2.2. Lagging one period and differencing, and using that the definitions of the inflation rates in Section 2.2 imply that

$$\begin{aligned}\hat{\pi}_t^i &= \hat{P}_t^i - \hat{P}_t^i,\\ \hat{\pi}_t^d &= \hat{P}_t^d - \hat{P}_{t-1}^d,\\ \hat{\pi}_t^{m,i} &= \hat{P}_t^{m,i} - \hat{P}_t^{m,i},\end{aligned}$$

and that the growth rate of the investment-specific technology shock, specified in Section 2.1, in log deviations is given by

$$\hat{\mu}_{\Psi,t} = \hat{\Psi}_t - \hat{\Psi}_{t-1},$$

we obtain the following log-linear expression for the aggregate investment price index inflation in terms of domestic and imported inflation:

$$\hat{\pi}_t^i = (1-\omega_i)\left(p^i\right)^{\eta_i - 1}\hat{\pi}_t^d + \omega_i\left(\frac{p^{m,i}}{p^i}\right)^{1-\eta_i}\hat{\pi}_t^{m,i} - \frac{1}{1-\eta_i}\hat{\mu}_{\Psi,t}. \tag{3.178}$$

We can also directly log-linearize the expression for the investment relative price (3.170). Rearranging:

$$\begin{aligned}
\left(p_t^i\right)^{1-\eta_i} &= (1-\omega_i) + \omega_i \left(p_t^{m,i}\right)^{1-\eta_i} \\
\left(p^i\right)^{1-\eta_i} + (1-\eta_i)\left(p^i\right)^{-\eta_i}\left(p_t^i - p^i\right) &= (1-\omega_i) + \omega_i \left(p^{m,i}\right)^{1-\eta_i} \\
&\quad + (1-\eta_i)\,\omega_i \left(p^{m,i}\right)^{-\eta_i}\left(p_t^{m,i} - p^{m,i}\right).
\end{aligned}$$

Substracting the steady-state relationship $\left(p^i\right)^{1-\eta_i} = (1-\omega_i) + \omega_i \left(p^{m,i}\right)^{1-\eta_i}$ and simplifying, we have:

$$\hat{p}_t^i = \omega_i \left(\frac{p^{m,i}}{p^i}\right)^{1-\eta_i} \hat{p}_t^{m,i}. \tag{3.179}$$

3.5 Production of final export goods

The goods X_t are produced by a representative, competitive, foreign retailer using specialized inputs as follows:

$$X_t = \left[\int_0^1 \left(X_{k,t}\right)^{\frac{1}{\lambda_t^x}} dk\right]^{\lambda_t^x}, \quad 1 \leq \lambda_t^x \leq \infty, \tag{3.180}$$

where $X_{k,t}$ are exports of specialized goods defined below, and λ_t^x is a time-varying price markup in the export market given by the following process

$$\log \lambda_t^x = (1-\rho_{\lambda^x}) \log \lambda^x + \rho_{\lambda^x} \log \lambda_{t-1}^x + \sigma_{\lambda^x} \varepsilon_{\lambda^x,t}. \tag{3.181}$$

The producer of X_t takes the price of output, P_t^x, and the prices of inputs, $P_{k,t}^x$, as given.[11] It maximizes profits according to:

$$\max_{X_{k,t}} \quad P_t^x X_t - \int_0^1 P_{k,t}^x X_{k,t} dk$$

which yields the following demand for exports of specialized goods:

$$X_{k,t} = \left(\frac{P_{k,t}^x}{P_t^x}\right)^{\frac{\lambda_t^x}{1-\lambda_t^x}} X_t. \tag{3.182}$$

Integrating (3.182) and using (3.180), we obtain the expression for the index of export prices:

$$P_t^x = \left[\int_0^1 \left(P_{k,t}^x\right)^{\frac{1}{1-\lambda_t^x}} dk\right]^{1-\lambda_t^x}. \tag{3.183}$$

The producer of the k^{th} specialized export good, $X_{k,t}$, is a monopolist and uses the following technology:

$$X_{k,t} = \left[\omega_x^{\frac{1}{\eta_x}} \left(X_{k,t}^m\right)^{\frac{\eta_x-1}{\eta_x}} + (1-\omega_x)^{\frac{1}{\eta_x}} \left(X_{k,t}^d\right)^{\frac{\eta_x-1}{\eta_x}}\right]^{\frac{\eta_x}{\eta_x-1}}, \tag{3.184}$$

where $X_{k,t}^m$ and $X_{k,t}^d$ are the demand by prodycer k for the imported and domestically produced intermediate goods used in the production of exports, X_t^m and X_t^d respectively, and ϕ^x denotes a fixed production cost. The cost minimization problem of the producer of the k^{th} specialized export good is to minimize total costs subject to the constraint of producing enough to meet demand:

$$\begin{cases}
\min\limits_{X_{k,t}^m, X_{k,t}^d} & P_t^{m,x} R_t^{wc,x} X_{k,t}^m + P_t^d R_t^{wc,x} X_{k,t}^d \\
s.t. & \left[\omega_x^{\frac{1}{\eta_x}} \left(X_{k,t}^m\right)^{\frac{\eta_x-1}{\eta_x}} + (1-\omega_x)^{\frac{1}{\eta_x}} \left(X_{k,t}^d\right)^{\frac{\eta_x-1}{\eta_x}}\right]^{\frac{\eta_x}{\eta_x-1}} \geq \left(\frac{P_{k,t}^x}{P_t^x}\right)^{\frac{\lambda_t^x}{1-\lambda_t^x}} X_t
\end{cases},$$

[11] We index the producers of final export goods by k, rather than i, in order to distinguish the demand for the aggregated imported export goods by a certain producer k, $X_{k,t}^m$, from the demand for a differentiated imported good for export production, produced by the intermediate import good producer i, $X_{i,t}^m$.

where

$$R_t^{wc,x} = \nu_t^{wc,x} R_t + 1 - \nu_t^{wc,x}, \tag{3.185}$$

and $\nu_t^{wc,x}$ is the fraction of the export producers' costs that has to be financed in advance. We assume that $\nu_t^{wc,x}$ evolves according to the following process:

$$\log \nu_t^{wc,x} = (1 - \rho_{\nu^{wc,x}}) \log \nu^{wc,x} + \rho_{\nu^{wc,x}} \log \nu_{t-1}^{wc,x} + \sigma_{\nu^{wc,x}} \varepsilon_{\nu^{wc,x},t}. \tag{3.186}$$

Denoting by $MC_{k,t}^x$ the Lagrange multiplier associated with the optimization problem, the FOC writes

$$\begin{aligned} R_t^{wc,x} P_t^{m,x} &= MC_{k,t}^x \left(X_{k,t}^m\right)^{-\frac{1}{\eta_x}} \omega_x^{\frac{1}{\eta_x}} \left(X_{k,t}\right)^{\frac{1}{\eta_x}} \\ R_t^{wc,x} P_t^d &= MC_{k,t}^x \left(X_{k,t}^d\right)^{-\frac{1}{\eta_x}} \left(1 - \omega_x\right)^{\frac{1}{\eta_x}} \left(X_{k,t}\right)^{\frac{1}{\eta_x}}. \end{aligned}$$

Solving for the inputs $X_{k,t}^m$ and $X_{k,t}^d$, we obtain

$$X_{k,t}^m = \frac{\left(MC_{k,t}^x\right)^{\eta_x} \omega_x X_{k,t}}{\left(R_t^{wc,x} P_t^{m,x}\right)^{\eta_x}}, \tag{3.187}$$

$$X_{k,t}^d = \frac{\left(MC_{k,t}^x\right)^{\eta_x} \left(1 - \omega_x\right) X_{k,t}}{\left(R_t^{wc,x} P_t^d\right)^{\eta_x}}, \tag{3.188}$$

which, inserted in the production function (3.184), allows us to obtain the expression of the nominal marginal cost for the producer of the specialized export good (identical across producers and, thus, without the k subscript):

$$MC_t^x = R_t^{wc,x} \left(\omega_x \left(P_t^{m,x}\right)^{1-\eta_x} + \left(1 - \omega_x\right) \left(P_t^d\right)^{1-\eta_x}\right)^{\frac{1}{1-\eta_x}}. \tag{3.189}$$

The firm producing the k^{th} specialized export good is subject to Calvo frictions. At any date, it faces a probability $(1 - \xi_x)$ that it can reoptimize its price, independent on the time that has passed since it was last able to reoptimize. If the firm is not able to reoptimize in period t, the price in period $t + 1$ will be set according to the following indexation rule:

$$\begin{cases} P_{k,t}^x = \tilde{\pi}_t^x P_{k,t-1}^x \\ \tilde{\pi}_t^x \equiv \left(\pi_{t-1}^x\right)^{\kappa_x} \left(\bar{\pi}_t^*\right)^{1-\kappa_x-\varkappa_x} \left(\breve{\pi}\right)^{\varkappa_x}, \end{cases} \tag{3.190}$$

where κ_x and $\varkappa_x$ are parameters such that κ_x, $\varkappa_x$, $\kappa_x + \varkappa_x \in [0, 1]$, π_{t-1}^x is the lagged export gross inflation rate, and $\bar{\pi}_t^*$ is the foreign economy central bank's target inflation rate. Unlike in the earlier Riksbank models, we choose to treat the export firms' indexation equally to the domestic and import firms' to allow for potential medium-term movement in foreign inflation expectations (possibly driven by changes in foreign policy makers' preferences). $\bar{\pi}_t^*$ is assumed to follow the process

$$\log \bar{\pi}_t^* = (1 - \rho_{\bar{\pi}^*}) \log \bar{\pi}^* + \rho_{\bar{\pi}^*} \log \bar{\pi}_{t-1}^* + \sigma_{\bar{\pi}^*} \varepsilon_{\bar{\pi}^*,t}. \tag{3.191}$$

When setting its price at time t, the k^{th} specialized exporting firm will maximize its expected future discounted profits, taking into account that there is a probability ξ_x in each period that it cannot reoptimize. Denoting by $\tilde{P}_{k,t}^x$ the reoptimized price at period t, the firm thus faces the following optimization problem:

$$\begin{cases} \max_{\tilde{P}_{k,t}^x} \quad E_t \sum_{s=0}^{\infty} \left(\beta \xi_x\right)^s \zeta_{t+s}^{\beta} \upsilon_{t+s} \left(S_{t+s} P_{k,t+s}^x \left(\breve{z}_{t+s}^{+,*}\right)^{-\frac{1}{\eta_f}} X_{k,t+s} - mc_{t+s}^x S_{t+s} P_{t+s}^x \left(\breve{z}_{t+s}^{+,*}\right)^{-\frac{1}{\eta_f}} X_{k,t+s}\right) \\ \qquad s.t. \quad X_{k,t} = \left(\frac{P_{k,t}^x}{P_t^x}\right)^{-\frac{\lambda_t^x}{\lambda_t^x - 1}} X_t \end{cases}.$$

Substituting in the demand expression for $X_{k,t}$ along with the following expression

$$P^x_{k,t+s} = \prod_{j=1}^{s} \tilde{\pi}^x_{t+j} \tilde{P}^x_{k,t},$$

the optimization problem becomes

$$\max_{\tilde{P}^x_{k,t}} \quad E_t \sum_{s=0}^{\infty} (\beta \xi_x)^s \zeta^\beta_{t+s} \upsilon_{t+s} \left(\begin{array}{c} S_t P^x_{k,t+s} \left(\tilde{z}^{+,*}_{t+s} \right)^{-\frac{1}{\eta_f}} \left(\left(\frac{\tilde{\pi}^x_{t+1} \cdots \tilde{\pi}^x_{t+s}}{P^x_{t+s}} \tilde{P}^x_{k,t} \right)^{-\frac{\lambda^x_{t+s}}{\lambda^x_{t+s}-1}} X_{t+s} \right) \\ -mc^x_{t+s} S_{t+s} P^x_{t+s} \left(\tilde{z}^{+,*}_{t+s} \right)^{-\frac{1}{\eta_f}} \left(\left(\frac{\tilde{\pi}^x_{t+1} \cdots \tilde{\pi}^x_{t+s}}{P^x_{t+s}} \tilde{P}^x_{k,t} \right)^{-\frac{\lambda^x_{t+s}}{\lambda^x_{t+s}-1}} X_{t+s} \right) \end{array} \right).$$

Rearranging,

$$\begin{aligned} \max_{\tilde{P}^x_{k,t}} \quad & E_t \sum_{s=0}^{\infty} (\beta \xi_x)^s \zeta^\beta_{t+s} \upsilon_{t+s} S_{t+s} P^x_{t+s} \left(\tilde{z}^{+,*}_{t+s} \right)^{-\frac{1}{\eta_f}} X_{t+s} \times \\ & \times \left[\left(\frac{\tilde{\pi}^x_{t+1} \ldots \tilde{\pi}^x_{t+s}}{P^x_{t+s}} \tilde{P}^x_{k,t} \right)^{1-\frac{\lambda^x_{t+s}}{\lambda^x_{t+s}-1}} - mc^x_{t+s} \left(\frac{\tilde{\pi}^x_{t+1} \ldots \tilde{\pi}^x_{t+s}}{P^x_{t+s}} \tilde{P}^x_{k,t} \right)^{-\frac{\lambda^x_{t+s}}{\lambda^x_{t+s}-1}} \right]. \end{aligned}$$

Taking derivatives w.r.t. $\tilde{P}^x_{k,t}$, we get

$$\begin{aligned} & \left(1 - \frac{\lambda^x_{t+s}}{\lambda^x_{t+s}-1}\right) \left(\tilde{P}^x_{k,t}\right)^{-\frac{\lambda^x_{t+s}}{\lambda^x_{t+s}-1}} E_t \sum_{s=0}^{\infty} (\beta \xi_x)^s \zeta^\beta_{t+s} \upsilon_{t+s} S_{t+s} P^x_{t+s} \left(\tilde{z}^{+,*}_{t+s} \right)^{-\frac{1}{\eta_f}} X_{t+s} \left(\frac{\tilde{\pi}^x_{t+1} \ldots \tilde{\pi}^x_{t+s}}{P^x_{t+s}} \right)^{1-\frac{\lambda^x_{t+s}}{\lambda^x_{t+s}-1}} \\ & + \frac{\lambda^x_{t+s}}{\lambda^x_{t+s}-1} \left(\tilde{P}^x_{k,t}\right)^{-\frac{\lambda^d_{t+s}}{\lambda^d_{t+s}-1}-1} E_t \sum_{s=0}^{\infty} (\beta \xi_d)^s \zeta^\beta_{t+s} \upsilon_{t+s} S_{t+s} P^x_{t+s} \left(\tilde{z}^{+,*}_{t+s} \right)^{-\frac{1}{\eta_f}} X_{t+s} mc^x_{t+s} \left(\frac{\tilde{\pi}^x_{t+1} \ldots \tilde{\pi}^x_{t+s}}{P^x_{t+s}} \right)^{-\frac{\lambda^x_{t+s}}{\lambda^x_{t+s}-1}} \\ = \quad & 0. \end{aligned}$$

Multiplying through by $\left(\lambda^x_{t+s} - 1\right) \left(\tilde{P}^x_{k,t}\right)^{\frac{\lambda^x_{t+s}}{\lambda^x_{t+s}-1}+1}$, and simplifying, we get

$$\begin{aligned} -\tilde{P}^x_{k,t} E_t \sum_{s=0}^{\infty} (\beta \xi_x)^s \zeta^\beta_{t+s} \upsilon_{t+s} S_{t+s} P^x_{t+s} \left(\tilde{z}^{+,*}_{t+s} \right)^{-\frac{1}{\eta_f}} X_{t+s} \left(\frac{\tilde{\pi}^x_{t+1} \ldots \tilde{\pi}^x_{t+s}}{P^x_{t+s}} \right)^{1-\frac{\lambda^x_{t+s}}{\lambda^x_{t+s}-1}} & \\ + \lambda^x_{t+s} E_t \sum_{s=0}^{\infty} (\beta \xi_d)^s \zeta^\beta_{t+s} \upsilon_{t+s} S_{t+s} P^x_{t+s} \left(\tilde{z}^{+,*}_{t+s} \right)^{-\frac{1}{\eta_f}} X_{t+s} mc^x_{t+s} \left(\frac{\tilde{\pi}^x_{t+1} \ldots \tilde{\pi}^x_{t+s}}{P^x_{t+s}} \right)^{-\frac{\lambda^x_{t+s}}{\lambda^x_{t+s}-1}} & = 0, \end{aligned}$$

$$\tilde{P}^x_{k,t} = \frac{E_t \sum_{s=0}^{\infty} (\beta \xi_d)^s \zeta^\beta_{t+s} \upsilon_{t+s} S_{t+s} P^x_{t+s} \left(\tilde{z}^{+,*}_{t+s} \right)^{-\frac{1}{\eta_f}} X_{t+s} \lambda^x_{t+s} mc^x_{t+s} \left(\frac{\tilde{\pi}^x_{t+1} \cdots \tilde{\pi}^x_{t+s}}{P^x_{t+s}} \right)^{-\frac{\lambda^x_{t+s}}{\lambda^x_{t+s}-1}}}{E_t \sum_{s=0}^{\infty} (\beta \xi_x)^s \zeta^\beta_{t+s} \upsilon_{t+s} S_{t+s} P^x_{t+s} \left(\tilde{z}^{+,*}_{t+s} \right)^{-\frac{1}{\eta_f}} X_{t+s} \left(\frac{\tilde{\pi}^x_{t+1} \cdots \tilde{\pi}^x_{t+s}}{P^x_{t+s}} \right)^{1-\frac{\lambda^x_{t+s}}{\lambda^x_{t+s}-1}}}$$

$$\tilde{p}^x_t = \frac{E_t \sum\limits_{s=0}^{\infty} (\beta \xi_x)^s \zeta^\beta_{t+s} \upsilon_{t+s} S_{t+s} P^x_{t+s} \left(\tilde{z}^{+,*}_{t+s} \right)^{-\frac{1}{\eta_f}} X_{t+s} \lambda^x_{t+s} mc^x_{t+s} \left(\frac{\tilde{\pi}^x_{t+1} \cdots \tilde{\pi}^x_{t+s}}{\pi^x_{t+1} \cdots \pi^x_{t+s}} \right)^{\frac{\lambda^x_{t+s}}{1-\lambda^x_{t+s}}}}{E_t \sum\limits_{s=0}^{\infty} (\beta \xi_x)^s \zeta^\beta_{t+s} \upsilon_{t+s} S_{t+s} P^x_{t+s} \left(\tilde{z}^{+,*}_{t+s} \right)^{-\frac{1}{\eta_f}} X_{t+s} \left(\frac{\tilde{\pi}^x_{t+1} \cdots \tilde{\pi}^x_{t+s}}{\pi^x_{t+1} \cdots \pi^x_{t+s}} \right)^{\frac{1}{1-\lambda^x_{t+s}}}}, \tag{3.192}$$

where $\tilde{p}^x_t \equiv \frac{\tilde{P}^x_t}{P^x_t}$.

We can derive a second expression for $\tilde{p}_t^x$ from the definition of the aggregate exports price index P_t^x in equation (3.183), in a similar way as for domestic and import intermediate goods producers. We have:

$$\tilde{p}_t^x = \left[\frac{1 - \xi_x \left(\frac{\tilde{\pi}_t^x}{\pi_t^x}\right)^{\frac{1}{1-\lambda_t^x}}}{(1-\xi_x)} \right]^{1-\lambda_t^x}. \tag{3.193}$$

Using the expression of the demand for domestic input in export production (3.188), we can derive the expression of the aggregate export demand for the domestic homogeneous good:

$$X_t^d = \int_0^1 X_{k,t}^d dk = \left(\frac{MC_t^x}{P_t^d R_t^{wc,x}}\right)^{\eta_x} (1-\omega_x) \int_0^1 X_{k,t} dk.$$

Replacing $X_{k,t}$ by its expression in (3.182), we get:

$$X_t^d = \left(\frac{MC_t^x}{P_t^d R_t^{wc,x}}\right)^{\eta_x} (1-\omega_x) (\mathring{p}_t^x)^{\frac{\lambda_t^x}{1-\lambda_t^x}} X_t, \tag{3.194}$$

where $\mathring{p}_t^x \equiv \frac{\mathring{P}_t^x}{P_t^x}$, and $\mathring{P}_t^x$ is a measure of export price dispersion defined as follows:

$$\mathring{P}_t^x = \left[\int_0^1 (P_{k,t}^x)^{\frac{\lambda_t^x}{1-\lambda_t^x}} dk\right]^{\frac{1-\lambda_t^x}{\lambda_t^x}},$$

$$\mathring{p}_t^x = \left[\int_0^1 \left(\frac{P_{k,t}^x}{P_t^x}\right)^{\frac{\lambda_t^x}{1-\lambda_t^x}} dk\right]^{\frac{1-\lambda_t^x}{\lambda_t^x}}. \tag{3.195}$$

We can break the integral, using the Calvo assumption on price setting, and re-express it in terms of relative prices as follows:

$$\mathring{p}_t^x = \left[\xi_x \left(\frac{\tilde{\pi}_t^x}{\pi_t^x} \mathring{p}_{t-1}^x\right)^{\frac{\lambda_t^x}{1-\lambda_t^x}} + (1-\xi_x) (\tilde{p}_t^x)^{\frac{\lambda_t^x}{1-\lambda_t^x}}\right]^{\frac{1-\lambda_t^x}{\lambda_t^x}}.$$

Substituting $\tilde{p}_t^x$ using (3.193) we get:

$$\mathring{p}_t^x = \left[\xi_x \left(\frac{\tilde{\pi}_t^x}{\pi_t^x} \mathring{p}_{t-1}^x\right)^{\frac{\lambda_t^x}{1-\lambda_t^x}} + (1-\xi_x) \left(\frac{1 - \xi_x \left(\frac{\tilde{\pi}_t^x}{\pi_t^x}\right)^{\frac{1}{1-\lambda_t^x}}}{1-\xi_x}\right)^{\lambda_t^x}\right]^{\frac{1-\lambda_t^x}{\lambda_t^x}}. \tag{3.196}$$

We can use (3.189) in (3.194) to obtain the final expression for the aggregate demand for domestic inputs for export goods production:

$$X_t^d = \left(\omega_x (p_t^{m,x})^{1-\eta_x} + (1-\omega_x)\right)^{\frac{\eta_x}{1-\eta_x}} (1-\omega_x) (\mathring{p}_t^x)^{\frac{\lambda_t^x}{1-\lambda_t^x}} X_t. \tag{3.197}$$

Proceeding in the same way as for the domestically produced input good, we can arrive at an expression for the aggregate export demand for the imported input good:

$$X_t^m = \left(\omega_x + (1-\omega_x) (p_t^{m,x})^{\eta_x-1}\right)^{\frac{\eta_x}{1-\eta_x}} \omega_x (\mathring{p}_t^x)^{\frac{\lambda_t^x}{1-\lambda_t^x}} X_t. \tag{3.198}$$

As in the case of imports, we want to distinguish between gross demand for exported goods, denoted above by X_t, and the demand net of fixed costs, which we denote by $\dot{X}_t$. Specifically, we assume that

$$\dot{X}_{k,t} = X_{k,t} - z_t^+ \phi^x,$$

or, in aggregate terms

$$\dot{X}_t = \int_0^1 X_{k,t} dk - z_t^+ \phi^x. \tag{3.199}$$

Inserting demand equation (3.182), we get

$$\dot{X}_t = \int_0^1 \left(\frac{P_{k,t}^x}{P_t^x} \right)^{\frac{\lambda_t^x}{1-\lambda_t^x}} X_t dk - z_t^+ \phi^x.$$

Integrating over i, and using the expression for export price dispersion in equation (3.195), we have

$$\dot{X}_t = (\mathring{p}_t^x)^{\frac{\lambda_t^x}{1-\lambda_t^x}} X_t - z_t^+ \phi^x. \tag{3.200}$$

We also derive the following expression for the exporting firm's profits:

$$\begin{aligned} PROFITS_{k,t}^x &= S_t P_{k,t}^x \left(\tilde{z}_t^{+,*} \right)^{-\frac{1}{\eta_f}} \dot{X}_{k,t} - MC_t^x \left(\dot{X}_{k,t} + z_t^+ \phi^x \right) \\ &= S_t P_{k,t}^x \left(\tilde{z}_t^{+,*} \right)^{-\frac{1}{\eta_f}} \left(X_{k,t} - z_t^+ \phi^x \right) - MC_t^x X_{k,t}. \end{aligned}$$

Using the demand curve faced by the k^{th} specialized exporter in equation (3.182), we have

$$\begin{aligned} PROFITS_{k,t}^x &= S_t P_{k,t}^x \left(\tilde{z}_t^{+,*} \right)^{-\frac{1}{\eta_f}} \left(\left(\frac{P_{k,t}^x}{P_t^x} \right)^{\frac{\lambda_t^x}{1-\lambda_t^x}} X_t - z_t^+ \phi^x \right) - MC_t^x \left(\frac{P_{k,t}^x}{P_t^x} \right)^{\frac{\lambda_t^x}{1-\lambda_t^x}} X_t \\ &= S_t P_t^x \left(\tilde{z}_t^{+,*} \right)^{-\frac{1}{\eta_f}} \left(\left(\frac{P_{k,t}^x}{P_t^x} \right)^{\frac{1}{1-\lambda_t^x}} X_t - \frac{P_{k,t}^x}{P_t^x} z_t^+ \phi^x \right) - MC_t^x \left(\frac{P_{k,t}^x}{P_t^x} \right)^{\frac{\lambda_t^x}{1-\lambda_t^x}} X_t \end{aligned} \tag{3.201}$$

3.5.1 Scaling of the final export goods producers' optimal conditions

We can write the expression for the marginal costs (3.189) in terms of stationary variables as

$$mc_t^x = \frac{R_t^{wc,x} p_t^{c,*}}{q_t p_t^x p_t^c} \left(\omega_x \left(p_t^{m,x} \right)^{1-\eta_x} + (1-\omega_x) \right)^{\frac{1}{1-\eta_x}}, \tag{3.202}$$

where we have used the definitions of relative prices $p_t^{m,x}$, p_t^x, p_t^c and $p_t^{c,*}$, and the real exchange rate q_t, all stated in Section 2.1. Scaling the aggregate demand for domestically produced and imported goods used in export production, (3.197) and (3.198), by z_t^+, we obtain

$$x_t^d = \left(\omega_x \left(p_t^{m,x} \right)^{1-\eta_x} + (1-\omega_x) \right)^{\frac{\eta_x}{1-\eta_x}} (1-\omega_x) \left(\mathring{p}_t^x \right)^{\frac{\lambda_t^x}{1-\lambda_t^x}} x_t, \tag{3.203}$$

$$x_t^m = \left(\omega_x + (1-\omega_x) \left(p_t^{m,x} \right)^{\eta_x - 1} \right)^{\frac{\eta_x}{1-\eta_x}} \omega_x \left(\mathring{p}_t^x \right)^{\frac{\lambda_t^x}{1-\lambda_t^x}} x_t. \tag{3.204}$$

Finally, the expression of the optimal price for domestic exporters (3.192) in terms of stationary variables is given by

$$\tilde{p}_t^x = \frac{E_t \sum_{s=0}^{\infty} (\beta \xi_x)^s \zeta_{t+s}^{\beta} \psi_{z^+,t+s} \frac{q_{t+s} p_{t+s}^c p_{t+s}^x}{p_{t+s}^{c,*}} x_{t+s} \lambda_{t+s}^x mc_{t+s}^x \left(\frac{\tilde{\pi}_{t+1}^x \cdots \tilde{\pi}_{t+s}^x}{\pi_{t+1}^x \cdots \pi_{t+s}^x} \right)^{\frac{\lambda_{t+s}^x}{1-\lambda_{t+s}^x}}}{E_t \sum_{s=0}^{\infty} (\beta \xi_x)^s \zeta_{t+s}^{\beta} \psi_{z^+,t+s} \frac{q_{t+s} p_{t+s}^c p_{t+s}^x}{p_{t+s}^{c,*}} x_{t+s} \left(\frac{\tilde{\pi}_{t+1}^x \cdots \tilde{\pi}_{t+s}^x}{\pi_{t+1}^x \cdots \pi_{t+s}^x} \right)^{\frac{1}{1-\lambda_{t+s}^x}}}, \tag{3.205}$$

where we have used that

$$
\begin{aligned}
\upsilon_{t+s} S_{t+s} P_{t+s}^{x}\left(\tilde{z}_{t}^{+,*}\right)^{-\frac{1}{\eta_f}} X_{t+s} &= \upsilon_{t+s} z_{t+s}^{+} P_{t+s}^{d} \frac{S_{t+s} P_{t+s}^{c,*}}{P_{t+s}^{c}} \frac{P_{t+s}^{*}}{P_{t+s}^{c,*}} \frac{P_{t+s}^{c}}{P_{t+s}^{d}} \frac{P_{t+s}^{x}\left(\tilde{z}_{t}^{+,*}\right)^{-\frac{1}{\eta_f}}}{P_{t+s}^{*}} \frac{X_{t+s}}{z_{t+s}^{+}} \\
&= \psi_{z^{+},t+s} \frac{q_{t+s} p_{t+s}^{c} p_{t+s}^{x}}{p_{t+s}^{c,*}} x_{t+s}.
\end{aligned}
$$

Scaling the expression for $\dot{X}_t$ in equation (3.200), we get

$$
\dot{x}_t = \left(\mathring{p}_t^{x}\right)^{\frac{\lambda_t^x}{1-\lambda_t^x}} x_t - \phi^x. \tag{3.206}
$$

We can also scale the expression for profits in equations (3.201), to obtain

$$
profits_{k,t}^{x} = S_t P_t^{x}\left(\tilde{z}_{t}^{+,*}\right)^{-\frac{1}{\eta_f}} x_t \left[\left(\frac{P_{k,t}^{x}}{P_t^{x}}\right)^{\frac{1}{1-\lambda_t^x}} - \frac{P_{k,t}^{x}}{P_t^{x}} \frac{\phi^x}{x_t} - mc_t^x \left(\frac{P_{k,t}^{x}}{P_t^{x}}\right)^{\frac{\lambda_t^x}{1-\lambda_t^x}}\right].
$$

Integrating this expression over the whole set of specialized exporters, we get:

$$
\begin{aligned}
\int_0^1 profits_{k,t}^{x} dk &= S_t P_t^{x}\left(\tilde{z}_{t}^{+,*}\right)^{-\frac{1}{\eta_f}} x_t \times \\
&\times \left[\left(\frac{1}{P_t^x}\right)^{\frac{1}{1-\lambda_t^x}} \int_0^1 \left(P_{k,t}^{x}\right)^{\frac{1}{1-\lambda_t^x}} dk - \frac{\phi^x}{x_t} \int_0^1 \frac{P_{k,t}^{x}}{P_t^{x}} dk - mc_t^x \int_0^1 \left(\frac{P_{k,t}^{x}}{P_t^{x}}\right)^{\frac{\lambda_t^x}{1-\lambda_t^x}} dk\right] \\
profits_t^x &= S_t P_t^{x}\left(\tilde{z}_{t}^{+,*}\right)^{-\frac{1}{\eta_f}} x_t \left[1 - \frac{\phi^x}{x_t} \int_0^1 \frac{P_{k,t}^{x}}{P_t^{x}} dk - mc_t^x \left(\mathring{p}_t^{x}\right)^{\frac{\lambda_t^x}{1-\lambda_t^x}}\right].
\end{aligned}
$$

We define $\int_0^1 \frac{P_{k,t}^{x}}{P_t^{x}} dk = \mathring{p}_t^{x,temp}$, a price term that will equate one in steady state and under full indexation.[12] Bearing in mind that a fraction ξ_x of firms index their price, while the remaining fraction get to reoptimize, we can derive the following expression for $\mathring{p}_t^{x,temp}$, which will be useful for the steady-state calculations:

$$
\mathring{p}_t^{x,temp} = \xi_x \frac{\tilde{\pi}_t^x}{\pi_t^x} \mathring{p}_{t-1}^{x,temp} + \left(1-\xi_x\right) \tilde{p}_{t-1}^{x}. \tag{3.207}
$$

We thus end up with the following expression for the specialized exporters' real profits:

$$
\overline{profits_t^x} = x_t \left[1 - \frac{\phi^x}{x_t} \mathring{p}_t^{x,temp} - mc_t^x \left(\mathring{p}_t^{x}\right)^{\frac{\lambda_t^x}{1-\lambda_t^x}}\right]. \tag{3.208}
$$

3.5.2 Log-linearization of the final export goods producers' optimal conditions

We first log-linearize the expressions for export goods producers' marginal costs in equation (3.202)

$$
\widehat{mc}_t^x = \hat{R}_t^{wc,x} + \hat{p}_t^{c,*} - \hat{q}_t - \hat{p}_t^x - \hat{p}_t^c + \frac{\omega_x \left(p^{m,x}\right)^{1-\eta_x}}{\omega_x \left(p^{m,x}\right)^{1-\eta_x} + \left(1-\omega_x\right)} \hat{p}_t^{m,x}. \tag{3.209}
$$

Log-linearization of the expression for the working capital interest rate for export goods producers in equation (3.185) yields

$$
\hat{R}_t^{wc,x} = \frac{\nu^{wc,x}\left(R-1\right)}{R^{wc,x}} \hat{\nu}_t^{wc,x} + \frac{\nu^{wc,x} R}{R^{wc,x}} \hat{R}_t. \tag{3.210}
$$

[12] Note that this term will not enter the final set of model equations, but is only needed for the computation of the steady state.

We now turn to the export goods producers' price setting. Starting from equation (3.205), we can proceed with the derivations in the same way as for the for the domestic and import intermediate goods producers. Assuming that $\varkappa_x = 0$, we get the following Phillips curve for the exporting firms:

$$\begin{aligned}\hat{\pi}_t^x - \widehat{\bar{\pi}}_t^* \quad = \quad & \frac{(1-\beta\xi_x)(1-\xi_x)}{\xi_x(1+\beta\kappa_x)}\left(\widehat{mc}_t^x + \hat{\lambda}_t^x\right) + \frac{\kappa_x}{1+\beta\kappa_x}\left(\hat{\pi}_{t-1}^x - \widehat{\bar{\pi}}_t^*\right) \\ & + \frac{\beta}{1+\beta\kappa_x}E_t\left(\hat{\pi}_{t+1}^x - \widehat{\bar{\pi}}_t^*\right) - \frac{\beta\kappa_x}{1+\beta\kappa_x}E_t\left(\widehat{\bar{\pi}}_t^* - \widehat{\bar{\pi}}_{t+1}^*\right),\end{aligned} \tag{3.211}$$

where we have used the log-linearized version of the price indexation rule for the importing firms in equation (3.190):

$$\widehat{\bar{\pi}}_t^x = \kappa_x\hat{\pi}_{t-1}^x + (1-\kappa_x-\varkappa_x)\,\widehat{\bar{\pi}}_t^*. \tag{3.212}$$

For use in later sections, we need also to log-linearize the price dispersion equation (3.196). Rearranging,

$$\left(\mathring{p}_t^x\right)^{\frac{\lambda_t^x}{1-\lambda_t^x}} = \xi_x\left(\frac{\tilde{\pi}_t^x}{\pi_t^x}\mathring{p}_{t-1}^x\right)^{\frac{\lambda_t^x}{1-\lambda_t^x}} + (1-\xi_x)\left(\frac{1-\xi_x\left(\frac{\tilde{\pi}_t^x}{\pi_t^x}\right)^{\frac{1}{1-\lambda_t^x}}}{1-\xi_x}\right)^{\lambda_t^x}.$$

We now log-linearize, proceeding in the same way as for the domestic and imported price dispersion terms before, which yields

$$\begin{aligned}\widehat{\mathring{p}}_t^x = {} & \xi_x\left(\frac{\tilde{\pi}^x}{\pi^x}\right)^{\frac{\lambda^x}{1-\lambda^x}}\left[\widehat{\tilde{\pi}}_t^x - \hat{\pi}_t^x + \widehat{\mathring{p}}_{t-1}^x\right] + \xi_x\ln\left(\frac{\tilde{\pi}^x}{\pi^x}\mathring{p}^x\right)\left(\frac{\tilde{\pi}^x}{\pi^x}\right)^{\frac{\lambda^x}{1-\lambda^x}}\frac{1}{1-\lambda^x}\hat{\lambda}_t^x - \ln\left(\mathring{p}^x\right)\frac{1}{1-\lambda^x}\hat{\lambda}_t^x \\ & - \frac{1}{\left(\mathring{p}^x\right)^{\frac{\lambda^x}{1-\lambda^x}}}\left(\frac{1-\xi_x\left(\frac{\tilde{\pi}^x}{\pi^x}\right)^{\frac{1}{1-\lambda^x}}}{1-\xi_x}\right)^{\lambda^x-1}\xi_x\left(\frac{\tilde{\pi}^x}{\pi^x}\right)^{\frac{1}{1-\lambda^x}}\left[\widehat{\tilde{\pi}}_t^x - \hat{\pi}_t^x\right] \\ & + \frac{1-\xi_x}{\left(\mathring{p}^x\right)^{\frac{\lambda^x}{1-\lambda^x}}}\ln\left(\frac{1-\xi_x\left(\frac{\tilde{\pi}^x}{\pi^x}\right)^{\frac{1}{1-\lambda^x}}}{1-\xi_x}\right)\left(\frac{1-\xi_x\left(\frac{\tilde{\pi}^x}{\pi^x}\right)^{\frac{1}{1-\lambda^x}}}{1-\xi_x}\right)^{\lambda^x}(1-\lambda^x)\,\hat{\lambda}_t^x \\ & - \left(\frac{1-\xi_x\left(\frac{\tilde{\pi}^x}{\pi^x}\right)^{\frac{1}{1-\lambda^x}}}{1-\xi_x}\right)^{\lambda^x-1}\frac{\xi_x}{\left(\mathring{p}^x\right)^{\frac{\lambda^x}{1-\lambda^x}}}\ln\left(\frac{\tilde{\pi}^x}{\pi^x}\right)\left(\frac{\tilde{\pi}^x}{\pi^x}\right)^{\frac{1}{1-\lambda^x}}\frac{\lambda^x}{1-\lambda^x}\hat{\lambda}_t^x.\end{aligned}$$

We use the steady-state relationship $\tilde{\pi}^x = \pi^x$, which implies that $\mathring{p}^x = 1$, to obtain

$$\widehat{\mathring{p}}_t^x = \xi_x\widehat{\mathring{p}}_{t-1}^x. \tag{3.213}$$

Finally, we log-linearize the demand equations for domestically produced and imported goods used in export production. Starting with equation (3.203), we have:

$$\hat{x}_t^d = \frac{\eta_x\omega_x\left(p^{m,x}\right)^{1-\eta_x}}{\omega_x\left(p^{m,x}\right)^{1-\eta_x} + (1-\omega_x)}\hat{p}_t^{m,x} + \frac{\lambda^x}{(1-\lambda^x)^2}\log\mathring{p}^x\hat{\lambda}_t^x + \frac{\lambda^x}{1-\lambda^x}\widehat{\mathring{p}}_t^x + \hat{x}_t.$$

Using that full indexation implies that $\mathring{p}^x = 1$, we arrive at the following expression:

$$\hat{x}_t^d = \frac{\eta_x\omega_x\left(p^{m,x}\right)^{1-\eta_x}}{\omega_x\left(p^{m,x}\right)^{1-\eta_x} + (1-\omega_x)}\hat{p}_t^{m,x} + \frac{\lambda^x}{1-\lambda^x}\widehat{\mathring{p}}_t^x + \hat{x}_t. \tag{3.214}$$

Moving on to equation (3.204), we have:

$$\hat{x}_t^m = -\frac{\eta_x(1-\omega_x)\left(p^{m,x}\right)^{\eta_x-1}}{\omega_x + (1-\omega_x)\left(p^{m,x}\right)^{\eta_x-1}}\hat{p}_t^{m,x} + \frac{\lambda^x}{(1-\lambda^x)^2}\log\mathring{p}^x\hat{\lambda}_t^x + \frac{\lambda^x}{1-\lambda^x}\widehat{\mathring{p}}_t^x + \hat{x}_t.$$

Using that full indexation implies that $\mathring{p}^x = 1$, we finally obtain

$$\hat{x}_t^m = -\frac{\eta_x (1-\omega_x) (p^{m,x})^{\eta_x - 1}}{\omega_x + (1-\omega_x)(p^{m,x})^{\eta_x-1}} \hat{p}_t^{m,x} + \frac{\lambda^x}{1-\lambda^x} \widehat{\mathring{p}}_t^x + \hat{x}_t. \tag{3.215}$$

We note that

$$\begin{aligned}
&\frac{\omega_x (p^{m,x})^{1-\eta_x}}{\omega_x (p^{m,x})^{1-\eta_x} + (1-\omega_x)} + \frac{(1-\omega_x)(p^{m,x})^{\eta_x-1}}{\omega_x + (1-\omega_x)(p^{m,x})^{\eta_x-1}} \\
= &\frac{\omega_x (p^{m,x})^{1-\eta_x}\left(\omega_x + (1-\omega_x)(p^{m,x})^{\eta_x-1}\right) + (1-\omega_x)(p^{m,x})^{\eta_x-1}\left(\omega_x (p^{m,x})^{1-\eta_x} + (1-\omega_x)\right)}{\left(\omega_x (p^{m,x})^{1-\eta_x} + (1-\omega_x)\right)\left(\omega_x + (1-\omega_x)(p^{m,x})^{\eta_x-1}\right)} \\
= &\frac{\omega_x^2 (p^{m,x})^{1-\eta_x} + 2\omega_x (1-\omega_x) + (1-\omega_x)^2 (p^{m,x})^{\eta_x-1}}{\omega_x^2 (p^{m,x})^{1-\eta_x} + 2\omega_x (1-\omega_x) + (1-\omega_x)^2 (p^{m,x})^{\eta_x-1}} \\
= &1,
\end{aligned}$$

and so

$$\frac{(1-\omega_x)(p^{m,x})^{\eta_x-1}}{\omega_x + (1-\omega_x)(p^{m,x})^{\eta_x-1}} = 1 - \frac{\omega_x (p^{m,x})^{1-\eta_x}}{\omega_x (p^{m,x})^{1-\eta_x} + (1-\omega_x)}$$

Thus, defining

$$\tilde{\omega}_x \equiv \frac{\omega_x (p^{m,x})^{1-\eta_x}}{\omega_x (p^{m,x})^{1-\eta_x} + (1-\omega_x)}, \tag{3.216}$$

we can rewrite (3.214) and (3.215) above as

$$\hat{x}_t^d = \eta_x \tilde{\omega}_x \hat{p}_t^{m,x} + \frac{\lambda^x}{1-\lambda^x} \widehat{\mathring{p}}_t^x + \hat{x}_t, \tag{3.217}$$

and

$$\hat{x}_t^m = -\eta_x (1-\tilde{\omega}_x) \hat{p}_t^{m,x} + \frac{\lambda^x}{1-\lambda^x} \widehat{\mathring{p}}_t^x + \hat{x}_t, \tag{3.218}$$

respectively.

3.6 Total export demand

We assume that total demand by foreigners for domestic exports takes the following form:

$$X_t = \left(\frac{P_t^x}{P_t^{d,*}}\right)^{-\eta_f} (Y_t^* - G_t^*), \tag{3.219}$$

where Y_t^* is the foreign GDP, G_t^* is foreign government spending, P_t^x is an index of export prices, and $P_t^{d,*}$ has been defined before as the foreign-currency price of the foreign homogeneous good, and where we have assumed that exports do not enter in the production of foreign government spending. From the foreign economy resource constraint, we know that this is equivalent to

$$X_t = \left(\frac{P_t^x}{P_t^*}\right)^{-\eta_f} \left(C_t^{d,*} + C_t^{e,*} + I_t^{d,*}\right).$$

We can substitute for $C_t^{d,*}$ to obtain

$$X_t = \left(\frac{P_t^x}{P_t^*}\right)^{-\eta_f} \left(C_t^{xe,*} + C_t^{e,*} + I_t^{d,*}\right). \tag{3.220}$$

3.6.1 Scaling of total export demand

Scaling the total demand by foreigners for domestic exports in equation (3.220), we have

$$\frac{X_t}{z_t^+} = \left(\frac{P_t^x}{P_t^{d,*}}\right)^{-\eta_f} \left(\frac{C_t^{xe,*}}{z_t^{+,*}} + \frac{C_t^{e,*}}{z_t^{+,*}} - \frac{I_t^{d,*}}{z_t^{+,*}}\right) \frac{z_t^{+,*}}{z_t^+}.$$

If we assume that z_t^+ and $z_t^{+,*}$, albeit similar, are two different processes, as in Ramses I, and define

$$\tilde{z}_t^{+,*} \equiv \frac{z_t^{+,*}}{z_t^+}$$

as the degree of asymmetry in the technological progess in the domestic economy compared to the rest of the world. We note that the evolution of $\tilde{z}_t^{+,*}$ can be written as follows:

$$\frac{\tilde{z}_t^{+,*}}{\tilde{z}_{t-1}^{+,*}} = \frac{z_t^{+,*}}{z_{t-1}^{+,*}} \frac{z_{t-1}^+}{z_t^+} = \frac{\mu_{z^{+,*},t}}{\mu_{z^+,t}}. \tag{3.221}$$

The relative price of exports is then defined as

$$p_t^x = \frac{P_t^x}{P_t^*} \left(\tilde{z}_t^{+,*}\right)^{-\frac{1}{\eta_f}}. \tag{3.222}$$

Using this definition, the scaled expression for export demand is given by

$$x_t = (p_t^x)^{-\eta_f} \left(c_t^{xe,*} + c_t^{e,*} + i_t^{d,*}\right), \tag{3.223}$$

We note that from equations (11.29), (11.30) and (11.31), we have that

$$c_t^{xe,*} = (1 - \omega_e^*) \left(\frac{1}{p_t^{c,*}}\right)^{-\eta_e^*} c_t^*,$$

$$c_t^{e,*} = \omega_e^* \left(\frac{p_t^{ce,*}}{p_t^{c,*}}\right)^{-\eta_e^*} c_t^*,$$

and

$$i_t^{d,*} = i_t^* + \frac{a\left(u_t^*\right) k_t^{p,*}}{\mu_{z^{+,*},t}\mu_{\Psi^*,t}}.$$

Thus, in a setting where there is only one consumption good and the share of energy is zero so that $\omega_e^* = 0$ and $p_t^{c,*} = 1$, as for example in Smets and Wouters (2003), we get that $c_t^{xe,*} = c_t^*$. Also, if the absence of variable capital utilization or capital utilization costs, we get that $i_t^{d,*} = i_t^*$. (3.223) could then be written as

$$x_t = (p_t^x)^{-\eta_f} \left(c_t^* + i_t^*\right).$$

3.6.2 Log-linearization of total export demand

For the export demand expression given by equation (3.223), we obtain

$$\hat{x}_t = -\eta_f \hat{p}_t^x + \frac{c^{xe,*}}{c^{xe,*} + c^{e,*} + i^{d,*}} \hat{c}_t^{xe,*} + \frac{c^{e,*}}{c^{xe,*} + c^{e,*} + i^{d,*}} \hat{c}_t^{e,*} + \frac{i^{d,*}}{c^{xe,*} + c^{e,*} + i^{d,*}} \widehat{i}_t^{d,*}.$$

Note that it is not clear in practice what the shares of consumption and investment in exports should be. It may not be optimal to assume that those should be equal to their shares in foreign GDP, as the composition of exports from a small open economy may be different than the composition of imports in the rest of the world. Moreover, the import shares of consumption and investment are most likely not equal to one another. We do not model those explicitly as the foreign economy is assumed approximately closed. Instead, in order to take this into account, we allow for more flexibility in the parametrization of export demand. In practice, we thus estimate the parameter ω_c^x in the below specification, where we have grouped together the non-energy and energy consumption into aggregate consumption, and let data decide exactly what it should be rather than assuming that it is equal to the constants specified above.[13,14]

$$\hat{x}_t = -\eta_f \hat{p}_t^x + \omega_c^x \hat{c}_t^* + (1 - \omega_c^x) \widehat{i}_t^{d,*}. \tag{3.224}$$

3.7 Total import demand

Total import demand is given by the sum of demand for the three different types of non-energy good and of energy imports. We then have that

$$M_t = \int_0^1 C_{i,t}^m di + \int_0^1 I_{i,t}^m di + \int_0^1 X_{i,t}^m di + \int_0^1 C_{i,t}^{e,m} di,$$

where we have let M_t denote the sum of all import goods used in the domestic economy. Using demand expressions (3.85), (3.86), (3.87) and (3.88), we get the following expression:[15]

$$M_t = C_t^m \left(\mathring{p}_t^{m,c}\right)^{\frac{\lambda_t^{m,c}}{1-\lambda_t^{m,c}}} + I_t^m \left(\mathring{p}_t^{m,i}\right)^{\frac{\lambda_t^{m,i}}{1-\lambda_t^{m,i}}} + X_t^m \left(\mathring{p}_t^{m,x}\right)^{\frac{\lambda_t^{m,x}}{1-\lambda_t^{m,x}}} + C_t^{e,m} \left(\mathring{p}_t^{m,ce}\right)^{\frac{\lambda_t^{m,ce}}{1-\lambda_t^{m,ce}}}. \tag{3.225}$$

Note that the above expression is the sum of goods *used in*, and not *imported to* the domestic economy. As we have assumed that some of the imported goods are used to cover the fixed costs of the monopolistic importing firms, the gross imports are going to exceed total import demand M_t. This is important for the calculations of the steady state, to ensure consistency between net exports derived from the aggregate resource constraint and net exports derived from the expression for net foreign assets. As the individual importers of non-energy goods are assumed to act under imperfect competition, they set prices as a markup over marginal costs and, in the absence of fixed production costs, make positive profits that generate too high imports in the resource constraint. Denoting gross total imports by $\tilde{M}_t$, we have

$$\begin{aligned}
\tilde{M}_t &= \int_0^1 C_{i,t}^m di + z_t^+ \phi^{m,c} + \int_0^1 I_{i,t}^m di + z_t^+ \phi^{m,i} \\
&\quad + \int_0^1 X_{i,t}^m di + z_t^+ \phi^{m,x} + \int_0^1 C_{i,t}^{e,m} di + z_t^+ \phi^{m,ce} \\
\tilde{M}_t &= C_t^m \left(\mathring{p}_t^{m,c}\right)^{\frac{\lambda_t^{m,c}}{1-\lambda_t^{m,c}}} + I_t^m \left(\mathring{p}_t^{m,i}\right)^{\frac{\lambda_t^{m,i}}{1-\lambda_t^{m,i}}} + X_t^m \left(\mathring{p}_t^{m,x}\right)^{\frac{\lambda_t^{m,x}}{1-\lambda_t^{m,x}}} \\
&\quad + C_t^{e,m} \left(\mathring{p}_t^{m,ce}\right)^{\frac{\lambda_t^{m,ce}}{1-\lambda_t^{m,ce}}} + z_t^+ \left(\phi^{m,c} + \phi^{m,i} + \phi^{m,x} + \phi^{m,ce}\right).
\end{aligned} \tag{3.226}$$

[13] In Ramses I and II, which contained simpler, three-variable models of the foreign economy, export demand was specified as follows:

$$X_t = \left(\frac{P_t^x}{P_t^{d,*}}\right)^{-\eta_f} Y_t^*.$$

It was thus implicitly assumed that exports entered the production of GDP components in a way that made total exports covary one-for-one with foreign total demand, at given relative prices. See Adolfson et al. (2005) and Adolfson et al. (2013).

[14] We note that, for the steady state computations, we use the following expression for export demand:

$$x_t = (p_t^x)^{-\eta_f} (c_t^*)^{\omega_c^x} (i_t^*)^{1-\omega_c^x},$$

in order to take into account that the weights of the export demand shares are different from those implied by the derivation above.

[15] Equations (3.85), (3.86) and (3.87) are also used in the derivation of the evolution of net foreign assets in Section 7, where the scaling and log-linearization of these equations is done.

3.7.1 Scaling of total import demand

We can scale the expression for total import demand (3.225) by z_t^+, to obtain

$$m_t = c_t^m \left(\mathring{p}_t^{m,c}\right)^{\frac{\lambda_t^{m,c}}{1-\lambda_t^{m,c}}} + i_t^m \left(\mathring{p}_t^{m,i}\right)^{\frac{\lambda_t^{m,i}}{1-\lambda_t^{m,i}}} + x_t^m \left(\mathring{p}_t^{m,x}\right)^{\frac{\lambda_t^{m,x}}{1-\lambda_t^{m,x}}} + c_t^{e,m} \left(\mathring{p}_t^{m,ce}\right)^{\frac{\lambda_t^{m,ce}}{1-\lambda_t^{m,ce}}}. \tag{3.227}$$

Scaling (3.226) by z_t^+, we get

$$\begin{aligned} \tilde{m}_t &= c_t^m \left(\mathring{p}_t^{m,c}\right)^{\frac{\lambda_t^{m,c}}{1-\lambda_t^{m,c}}} + i_t^m \left(\mathring{p}_t^{m,i}\right)^{\frac{\lambda_t^{m,i}}{1-\lambda_t^{m,i}}} + x_t^m \left(\mathring{p}_t^{m,x}\right)^{\frac{\lambda_t^{m,x}}{1-\lambda_t^{m,x}}} \\ &\quad + c_t^{e,m} \left(\mathring{p}_t^{m,ce}\right)^{\frac{\lambda_t^{m,ce}}{1-\lambda_t^{m,ce}}} + \left(\phi^{m,c} + \phi^{m,i} + \phi^{m,x} + \phi^{m,ce}\right). \end{aligned} \tag{3.228}$$

3.7.2 Log-linearization of total import demand

Log-linearization of the expression for total import demand (3.227) yields

$$\begin{aligned} \hat{m}_t &= \frac{c^m}{m} \left(\mathring{p}^{m,c}\right)^{\frac{\lambda^{m,c}}{1-\lambda^{m,c}}} \left[\hat{c}_t^m + \frac{\lambda^{m,c}}{1-\lambda^{m,c}} \widehat{\mathring{p}}_t^{m,c} + \ln\left(\mathring{p}^{m,c}\right) \frac{\lambda^{m,c}}{\left(1-\lambda^{m,c}\right)^2} \hat{\lambda}_t^{m,c}\right] \\ &\quad + \frac{i^m}{m} \left(\mathring{p}^{m,i}\right)^{\frac{\lambda^{m,i}}{1-\lambda^{m,i}}} \left[\hat{i}_t^m + \frac{\lambda^{m,i}}{1-\lambda^{m,i}} \widehat{\mathring{p}}_t^{m,i} + \ln\left(\mathring{p}^{m,i}\right) \frac{\lambda^{m,i}}{\left(1-\lambda^{m,i}\right)^2} \hat{\lambda}_t^{m,i}\right] \\ &\quad + \frac{x^m}{m} \left(\mathring{p}^{m,x}\right)^{\frac{\lambda^{m,x}}{1-\lambda^{m,x}}} \left[\hat{x}_t^m + \frac{\lambda^{m,x}}{1-\lambda^{m,x}} \widehat{\mathring{p}}_t^{m,x} + \ln\left(\mathring{p}^{m,x}\right) \frac{\lambda^{m,x}}{\left(1-\lambda^{m,x}\right)^2} \hat{\lambda}_t^{m,x}\right] \\ &\quad + \frac{c^{e,m}}{m} \left(\mathring{p}^{m,ce}\right)^{\frac{\lambda^{m,ce}}{1-\lambda^{m,ce}}} \left[\hat{c}_t^{e,m} + \frac{\lambda^{m,ce}}{1-\lambda^{m,ce}} \widehat{\mathring{p}}_t^{m,ce} + \ln\left(\mathring{p}^{m,ce}\right) \frac{\lambda^{m,ce}}{\left(1-\lambda^{m,ce}\right)^2} \hat{\lambda}_t^{m,ce}\right]. \end{aligned} \tag{3.229}$$

Using that, in steady state $\tilde{\pi}^{m,j} = \pi^{m,j}$ and $\mathring{p}^{m,j} = 1$, this simplifies to

$$\begin{aligned} \hat{m}_t &= \frac{c^m}{m} \left[\hat{c}_t^m + \frac{\lambda^{m,c}}{1-\lambda^{m,c}} \widehat{\mathring{p}}_t^{m,c}\right] + \frac{i^m}{m} \left[\hat{i}_t^m + \frac{\lambda^{m,i}}{1-\lambda^{m,i}} \widehat{\mathring{p}}_t^{m,i}\right] \\ &\quad + \frac{x^m}{m} \left[\hat{x}_t^m + \frac{\lambda^{m,x}}{1-\lambda^{m,x}} \widehat{\mathring{p}}_t^{m,x}\right] + \frac{c^{e,m}}{m} \left[\hat{c}_t^{e,m} + \frac{\lambda^{m,ce}}{1-\lambda^{m,ce}} \widehat{\mathring{p}}_t^{m,ce}\right]. \end{aligned} \tag{3.230}$$

4 Households

4.1 Household preferences

We assume that there is a large representative household, with full risk sharing of consumption among household members as in Merz (1995). There is a continuum of members indexed by $(j,k) \in (0,1) \times (0,1)$.[16] Here, j denotes the type of labour service a household member is specialized in, and k the disutility of work. They attain utility from consumption and disutility from work. The preferences with respect to consumption are unchanged compared to the setup in Christiano, Trabandt, and Walentin (2011) and Adolfson et al. (2013). But while these previous two models assume a specification of the disutility of work as in Erceg, Henderson, and Levin (2000), we here instead rely on the setup in Galí, Smets, and Wouters (2012) as a simple way of introducing unemployment into the model. A household member of type (j,k) has preferences

$$E_0^j \sum_{t=0}^{\infty} \beta^t \zeta_t^\beta \left[\zeta_t^c \log\left(C_{j,k,t} - bC_{j,k,t-1}\right) - 1\left(j,k\right) \zeta_t^n \Theta_t k^\varphi\right], \tag{4.1}$$

[16] In general, the subscripted variables denote household or firm choice variables and the variables without a subscript (other than the time subscript t) denote economy-wide variables.

where β is the household's discount factor, $C_{j,k,t}$ is consumption of household member (j,k), $1(j,k)$ is an indicator function that is equal to one if the household member works and zero otherwise, and Θ_t is an endogenous preference shifter. The parameter $\varphi \geq 0$ determines the shape of the distribution of work disutilities across the individual household members.[17] ζ_t^β, ζ_t^c and ζ_t^n denote shocks to the discount rate, consumption preferences and labour supply, respectively, given by the following AR(1) processes:[18]

$$\log \zeta_t^\beta = \left(1-\rho_{\zeta^\beta}\right)\log\zeta^\beta + \rho_{\zeta^\beta}\log\zeta_{t-1}^\beta + \sigma_{\zeta^\beta}\varepsilon_{\zeta^\beta,t}, \tag{4.2}$$

$$\log \zeta_t^c = \left(1-\rho_{\zeta^c}\right)\log\zeta^c + \rho_{\zeta^c}\log\zeta_{t-1}^c + \sigma_{\zeta^c}\varepsilon_{\zeta^c,t}, \tag{4.3}$$

$$\log \zeta_t^n = \left(1-\rho_{\zeta^n}\right)\log\zeta^n + \rho_{\zeta^n}\log\zeta_{t-1}^n + \sigma_{\zeta^n}\varepsilon_{\zeta^n,t}. \tag{4.4}$$

There is habit persistence in consumption, as indicated by the inclusion of the term $bC_{j,t-1}$. The endogenous preference shifter is defined as[19]

$$\Theta_t = Z_t^C \bar{\Upsilon}_t^N, \tag{4.5}$$

where $\bar{C}_t$ is average aggregate consumption,

$$Z_t^C = \left(Z_{t-1}^C\right)^{1-\nu}\left(\frac{1}{\bar{\Upsilon}_t^N}\right)^{\nu}, \tag{4.6}$$

and $\bar{\Upsilon}_t^N$ is the marginal utility of consumtpion given by

$$\bar{\Upsilon}_t^N = \zeta_t^\beta\zeta_t^c\frac{1}{\bar{C}_t - b\bar{C}_{t-1}} - \beta b E_t \zeta_{t+1}^\beta\zeta_{t+1}^c\frac{1}{\bar{C}_{t+1} - b\bar{C}_t}. \tag{4.7}$$

In a symmetric equilibrium $\bar{C}_t = C_t$. Integrating over all household members' utilities, using that $C_{j,k,t} = C_t$ for all (j,k), gives[20]

$$\begin{aligned} & E_0\sum_{t=0}^{\infty}\beta^t\zeta_t^\beta\left[\zeta_t^c\log\left(C_t - bC_{t-1}\right) - \zeta_t^n\Theta_t\int_0^1\int_0^{n_{j,t}}k^\varphi dk dj\right] \\ = \; & E_0\sum_{t=0}^{\infty}\beta^t\zeta_t^\beta\left[\zeta_t^c\log\left(C_t - bC_{t-1}\right) - \zeta_t^n\Theta_t\int_0^1\frac{n_{j,t}^{1+\varphi}}{1+\varphi}dj\right]. \end{aligned} \tag{4.8}$$

We can derive the household-relevant marginal rate of substitution between consumption and employment for type j workers as follows:

$$MRS_{j,t} = -\frac{\partial\mathcal{U}_t/\partial N_{j,t}}{\partial\mathcal{U}_t/\partial C_t},$$

[17] In this preference specification, the coefficient of relative risk aversion equals one, as implied by the assumption of log utility.

[18] Note that all three shocks cannot be turned on at once in the estimation of the model. They are all included in the derivations to allow for the possibility of choosing any subset of the three.

[19] Note that, when habits are external and $\zeta_t^\beta = \zeta_t^c = 1$, our expressions coincide with Θ_t and Z_t^C in Galí, Smets, and Wouters (2012) (in Galí, Smets, and Wouters (2012), Z_t^C is denoted by Z_t), as given by

$$\begin{aligned} \Theta_t &= \frac{Z_t^C}{\bar{C}_t - b\bar{C}_{t-1}}, \\ Z_t^C &= \left(Z_{t-1}^C\right)^{1-\nu}\left(\bar{C}_t - b\bar{C}_{t-1}\right)^{\nu}. \end{aligned}$$

[20] Note that this expression is very similar to the preference specifications in Christiano, Trabandt, and Walentin (2011) and Adolfson et al. (2013). Instead of ζ_t^h, which was the notation of the shock to the household's labor supply in the model with hours, we now have $\zeta_t^n\Theta_t$ in front of the disutility of work term, and the inverse of the Frisch elasticity of labor supply σ_L is now replaced by the preference parameter φ.

where $\mathcal{U}_t$ denotes the household's utility function. We then have

$$MRS_{j,t} = -\frac{-\zeta_t^{\beta}\zeta_t^{n}\Theta_t n_{j,t}^{\varphi}}{\zeta_t^{\beta}\zeta_t^{c}\frac{1}{C_t - bC_{t-1}} - \beta b E_t \zeta_{t+1}^{\beta}\zeta_{t+1}^{c}\frac{1}{C_{t+1}-bC_t}}.$$

In a symmetric equilibrium where $\bar{C}_t = C_t$, using (4.7) we can rewrite this as

$$MRS_{j,t} = \frac{\zeta_t^{\beta}\zeta_t^{n}\Theta_t n_{j,t}^{\varphi}}{\bar{\Upsilon}_t^N}. \tag{4.9}$$

We thus note that, in a symmetric equilibrium, $\bar{\Upsilon}_t^N$ is the marginal utility of consumption.

4.2 The household's budget constraint

The representative household optimizes its utility subject to the following budget constraint:

$$\begin{aligned} & P_t^c C_t + P_t^i \left(I_t + a\left(u_t\right) K_t^p\right) + P_{k',t}\Delta_t + B_{t+1} + S_t B_{t+1}^F \\ = & \int_0^1 \int_0^{n_{j,t}} W_{j,k,t} dk dj + R_t^k u_t K_t^p + R_{t-1}\chi_{t-1} B_t + R_{t-1}^* \Phi_{t-1}\chi_{t-1} S_t B_t^F + \Pi_t + TR_t, \end{aligned} \tag{4.10}$$

where the left-hand side contains the expenditure terms and the right-hand side the income terms. The budget constraint (4.10) is slightly modified compared to the earlier Riksbank models, due to the assumption that some household members do not work.

The households spend part of their resources on consumption and investment, purchasing aggregate consumption and investment goods at the prices P_t^c and P_t^i, respectively. Moreover, as the households own the economy's physical capital stock, they also pay for the capital utilization costs which we will describe further under 4.2.1 below. Note that we assume that the variable capital utilization involves the use of investment goods, as described in Section 3.4. The term $P_{k',t}\Delta_{j,t}$ is included to allow for the computation of the price of capital in the model, $P_{k',t}$. The term Δ_t reflects the existence of a market for capital, and will be further explained in Section 4.3 below. Finally, the households invest in domestic bonds B_{t+1} (on which they earn interest in period $t+1$), denominated in domestic currency, and foreign bonds B_{t+1}^F, denominated in foreign currency. S_t denotes the nominal exchange rate, defined as the price of a unit of foreign currency expressed in terms of the domestic currency.

The households receive income from wages, with $W_{j,k,t}$ denoting the wage set by the household as specified in Section 4.6 below. Note that $W_{j,k,t} = W_{j,t}$ for all k, as firms pay the same price for the labour of type j irrespectively of which household member that supplies the labour. Note further that, unlike in the setup based on Erceg, Henderson, and Levin (2000) as in Christiano, Trabandt, and Walentin (2011), and Adolfson et al. (2013), the wage in our model is defined as the wage per worker rather than the wage per hour, as reflected by the integral over the household members wages replacing the product of wages and hours worked. The households further receive return on their capital holdings as given by the second term on the right-hand side. There is a distinction between physical and efficient capital in the model, as we allow for a variable capital utilization rate, denoted by $u_{j,t}$. With $K_{j,t}^p$ denoting physical capital, efficient capital is given by

$$K_{j,t} = u_{j,t}K_{j,t}^p, \tag{4.11}$$

and it yields a return of R_t^k. Interest rates are expressed as gross interest rates, so that $R_t = 1 + r_t$. Moreover, households earn interest on their bond holdings. The interest rate they earn on their holdings of domestic bonds are R_{t-1} adjusted by the exogenous process χ_{t-1} — a risk premium shock as in Smets and Wouters (2007), given by the following AR(1) process:

$$\log \chi_t = \left(1 - \rho_\chi\right)\log \chi + \rho_\chi \log \chi_{t-1} + \sigma_\chi \varepsilon_{\chi,t}. \tag{4.12}$$

This shock induces a wedge between the interest rate controlled by the central bank and the return on assets held by the households, and has similar effects as a net-worth shock in Bernanke, Gertler, and Gilchrist (1999).[21] A positive shock to this wedge increases the required return on assets and reduces current consumption. It is different from the discount factor shock (as in Smets and Wouters (2003)) or the shock to consumer preferences (ζ_t^c in our model), however, as it also increases the cost of capital and reduces the value of capital and investment, while the other two affect only the consumption Euler equation. The risk premium shock can thus help generate comovement of consumption and investment.[22] The foreign bonds instead pay a risk-adjusted interest rate of $R_{t-1}^{*}\Phi_{t-1}\chi_{t-1}$, where Φ_{t-1} denotes the premium on foreign bond holdings, discussed further in Section 4.2.2 below. The households own the firms in the economy, from which they recieve profits denoted by Π_t.[23] Finally, TR_t denotes potential lump-sum transfers (or taxes) that the household receives from (or pays to) the government.

4.2.1 The capital utilization costs

The capital utilization cost function, $a(u_t)$, is an increasing, convex ($a'' \geq 0$) function of the utilization rate u_t. It is assumed to satisfy $a(1) = 0$, $u = 1$ and $a' = \bar{r}^k$ in steady state, where $\bar{r}^k$ is the scaled real rental rate of capital further explained in Section 2.1.[24] Under these assumptions, the steady state of the model is independent of $\sigma_a \equiv a''(u)/a'(u)$. The dynamics of the model, however, do depend on σ_a.[25] The specific functional form for $a(u_t)$, assumed also in Christiano, Trabandt, and Walentin (2011) and Adolfson et al. (2013), is given by the following expression:

$$a(u_t) = 0.5\sigma_b\sigma_a u_t^2 + \sigma_b(1-\sigma_a)u_t + \sigma_b((\sigma_a/2)-1), \tag{4.13}$$

where σ_a and σ_b are the parameters of this function. The first- and second-order derivatives are given by

$$a'(u_t) = \sigma_b\sigma_a u_t + \sigma_b(1-\sigma_a), \tag{4.14}$$

$$a''(u_t) = \sigma_b\sigma_a, \tag{4.15}$$

yielding

$$\frac{a''(u)}{a'(u)} = \frac{a''(1)}{a'(1)} = \frac{\sigma_b\sigma_a}{\sigma_b\sigma_a u + \sigma_b - \sigma_b\sigma_a} = \frac{\sigma_b\sigma_a}{\sigma_b} = \sigma_a \tag{4.16}$$

in steady state. Moreover,

$$a(u) = a(1) = 0.5\sigma_b\sigma_a + \sigma_b - \sigma_b\sigma_a + 0.5\sigma_b\sigma_a - \sigma_b = 0. \tag{4.17}$$

4.2.2 Risk adjustment on foreign holdings

The risk adjustment term, Φ_t, depends on the real aggregate net foreign asset position of the domestic economy, $\bar{a}_t$, the anticipated growth rate of the exchange rate, and a time-varying mean-zero shock to the risk premium $\tilde{\phi}_t$ — the country risk premium shock, given by the following AR(1) process:

$$\log\tilde{\phi}_t = \left(1-\rho_{\tilde{\phi}}\right)\log\tilde{\phi} + \rho_{\tilde{\phi}}\log\tilde{\phi}_{t-1} + \sigma_{\tilde{\phi}}\varepsilon_{\tilde{\phi},t}. \tag{4.18}$$

[21] A structural interpretation of this shock is provided in Fisher (2015).

[22] Note that we assume that this risk premium is tied to the household and so it enters the return of both domestic and foreign bonds alike.

[23] Note that the model will be calibrated so that profits equal zero in steady state.

[24] We denote by bar the real version of the corresponding nominal variables and by small letters the scaled version of the corresponding capital-letter variable.

[25] Note that, for estimation purposes, we will introduce the following transformation of σ_a:

$$\sigma_a = \frac{1-\sigma_a^{tr}}{\sigma_a^{tr}},$$

setting a prior on and estimating σ_a^{tr}, instead of σ_a.

Specifically,

$$\Phi_t = \Phi\left(\bar{a}_t, E_t s_{t+1} s_t, \tilde{\phi}_t\right) = \exp\left(-\tilde{\phi}_a\left(\bar{a}_t - \bar{a}\right) - \tilde{\phi}_s\left(E_t s_{t+1} s_t - s^2\right) + \tilde{\phi}_t\right), \tag{4.19}$$

where $\tilde{\phi}_a$ and $\tilde{\phi}_s$ are positive parameters, $s_t \equiv S_t/S_{t-1}$ denotes the growth rate of the exchange rate, and

$$\bar{a}_t \equiv \frac{S_t B^F_{t+1}}{P^d_t z^+_t}. \tag{4.20}$$

Note that variables without time subscript denote the corresponding value in a non-stochastic steady state. The term z^+_t in equation (4.20) is a scaling variable, defined as a combination of investment-specific and neutral technology, which is included in order to ensure the stationarity of the model, as discussed in Section 2.1.

The dependence of Φ_t on $\bar{a}_t$ in equation (4.19) ensures that the steady state of the model is well-defined. Φ_t is assumed to be strictly increasing in $\bar{a}_t$, and it holds that $\Phi = 1$ in a non-stochastic steady state, as shown in Section 12. If the domestic economy is a net borrower (lender), so that $B^F_{t+1} < 0$ ($B^F_{t+1} > 0$), domestic households must pay a premium on the foreign interest rate (recieve lower interest on their savings). This term, along with the stochastic risk premium shock, were included in the Ramses I and Ramses II specifications alike.

The dependence of Φ_t on the exchange rate was instead specific to Ramses II. It is included to allow the model to reproduce two observations regarding the uncovered interest parity (UIP) and the output response to a monetary policy shock, respectively. First, while the standard UIP condition implies that a decrease in R_t relative to R^*_t produces an anticipated appreciation of the domestic currency, acheived by an instantaneous depreciation in the exchange rate on impact, this turns out not to hold empirically.[26] In theory, asset holders respond to the decreased rate of return on domestic assets by attempting to sell these for the purpose of acquiring foreign ones. The implied increased demand for foreign currency puts a pressure on the exchange rate to depreciate, until the anticipated appreciation exactly compensates traders holding domestic assets. An interpretation of why this does not hold in data is that the reduction in the domestic interest rate, say by a monetary policy shock, reduces the general risk in the domestic economy which makes traders happier to hold domestic assets in spite of their lower nominal return. The specification of Φ_t is designed to capture this idea, as an anticipated appreciation in the level of the exchange rate lowers the assessment of risk in the domestic economy. Second, the output response to a monetary policy shock is generally found to be hump-shaped in data, which requires mechanisms that slow down the initial response of demand to the shock. The increase in net exports, specifically, depends on the degree of depreciation when the shock hits, and is reduced by the introduction of the risk adjustment mechanism described above.

The specification of the risk adjustment term in Ramses II differs from that in the published CTW paper, where Φ_t is given by the following expression:

$$\Phi_t = \Phi\left(\bar{a}_t, R^*_t - R_t, \tilde{\phi}_t\right) = \exp\left(-\tilde{\phi}_a\left(\bar{a}_t - \bar{a}\right) - \tilde{\phi}_s\left(R^*_t - R_t - \left(R^* - R\right)\right) + \tilde{\phi}_t\right).$$

This is motivated using the regression interpretation of the uncovered interest parity result, specifically considering the regression coefficient

$$\gamma = \frac{cov\left(\log S_{t+1} - \log S_t, R_t - R^*_t\right)}{var\left(R_t - R^*_t\right)} = \overbrace{1}^{\text{in theory}} \text{ but } \overbrace{< 0}^{\text{in data}}.$$

Log-linearizing the CTW expression for Φ_t, we have that

$$\gamma = \frac{cov\left(\log S_{t+1} - \log S_t, R_t - R^*_t\right)}{var\left(R_t - R^*_t\right)} = \frac{cov\left(R_t - R^*_t - \Phi_t, R_t - R^*_t\right)}{var\left(R_t - R^*_t\right)} = 1 - \frac{cov\left(R_t - R^*_t, \Phi_t\right)}{var\left(R_t - R^*_t\right)}.$$

[26] In earlier/internal versions of the Ramses II documentation (but not in the published occasional paper), there is a nice and intuitive description of the reasoning underlying the exact choice of the specification for the risk adjustment term.

Given that γ is usually found to be negative in the data, in stark contrast to the positive unit value implied by theory, any specification of Φ_t which causes it to have a positive covariance with the interest rate differential will help in accounting for this discrepancy with the data. With the assumption made in CTW that $\tilde{\phi}_s > 1$, γ will be negative as usually found in the data. The reason that this specification was in the end not used in Ramses II has to do with the fact the the motivation for it comes mainly from movements caused by monetary policy shocks. While it is plausible that a negative interest rate differential (i.e. $R_t - R_t^* < 0$) stemming from a monetary policy shock may signal lower risk in the domestic economy, this mechanism is not as intuitive when the interest rate differential is driven by some other underlying shock. As monetary policy shocks account for only a small share of the total variance in the observed variables, this specification was changed in Ramses II to one that more directly addresses the issues related to the observed unoconditional moments of the exchange rate such as the high autocorrelation found in data.

4.3 The law of motion for capital

As mentioned above, the households in the economy own the capital stock, denoted by K_t^p, with the subscript p included to distinguish the physical capital stock from the efficient capital available to firms. We omit the household index j throughout this section. The law of motion for the households physical capital stock is given by

$$K_{t+1}^p = (1-\delta) K_t^p + \Upsilon_t F(I_t, I_{t-1}) + \Delta_t, \tag{4.21}$$

where δ denotes the capital depreciation rate, $F(I_t, I_{t-1})$ summarizes the technology that transforms current and past investment into installed capital for use in the following period, and Υ_t is a stationary investment-specific technology shock that affects the efficiency of transforming investments into capital. It is assumed to evolve according to the following process:

$$\log \Upsilon_t = (1-\rho_\Upsilon) \log \Upsilon + \rho_\Upsilon \log \Upsilon_{t-1} + \sigma_\Upsilon \varepsilon_{\Upsilon,t}. \tag{4.22}$$

Δ_t is included to help define the shadow price of capital $P_{k',t}$. The households have access to a market where they can purchase new capital, K_{t+1}^p. As the market for capital is closed, households wishing to sell K_{t+1}^p are the only source of supply, and households wishing to buy K_{t+1}^p the only source of demand on this market. As all households are identical, the only equilibrium is one where $\Delta_t = 0$. $P_{k',t}$ is the shadow value, in consumption units, of a unit of installed capital (for use in the following period) as of time t when the household makes its investment and capital utilization decision. In other words, the shadow price is what the price of installed capital would be if there were a market for K_{t+1}^p at the beginning of period t.[27] Note that we use the *stock at the beginning of the period* convention, so that the investment during period t determines the capital stock at the beginning of period $t+1$. This is of relevance when setting up the model in Dynare, as Dynare uses the *stock at the end of the period* convention and the the timing of the stock variables therefore must be changed in the code.

The investment technology is assumed to be given by the following function:[28]

$$F(I_t, I_{t-1}) = \left(1 - \tilde{S}\left(\frac{I_t}{I_{t-1}}\right)\right) I_t, \tag{4.23}$$

where $\tilde{S}(x) = \tilde{S}'(x) = 0$, and $\tilde{S}''(x) \equiv \tilde{S}'' > 0$ is assumed to hold in steady state, with $x = \mu_{z^+}\mu_\Psi$ denoting the real investment growth rate in steady state. Note that only the adjustment cost

[27] This is explained in more detail in Christiano, Eichenbaum, and Evans (2005).

[28] This was introduced by Christiano, Eichenbaum, and Evans (2005), and adopted in both Ramses I and Ramses II. Note that the capital adjustment costs are modelled as a function of the change in investment rather than its level. In other words, we introduce investment adjustment costs rather than capital adjustment costs. The reason for this are the additional dynamics in the investment equation, which have proven useful in capturing the humpshaped response of investment following shocks (see, for example, Smets and Wouters (2007)).

parameter, $\tilde{S}''$, needs to be specified, as it affects the dynamics of the model.[29] The steady state of the model does not depend on the adjustment cost parameter. Taking the derivative of $F(I_t, I_{t-1})$ with respect to its arguments, we obtain

$$F_1(I_t, I_{t-1}) \equiv \frac{\partial F(I_t, I_{t-1})}{\partial I_t} = -\tilde{S}'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}} + \left(1 - \tilde{S}\left(\frac{I_t}{I_{t-1}}\right)\right), \quad (4.24)$$

$$F_2(I_t, I_{t-1}) \equiv \frac{\partial F(I_t, I_{t-1})}{\partial I_{t-1}} = \tilde{S}'\left(\frac{I_t}{I_{t-1}}\right)\left(\frac{I_t}{I_{t-1}}\right)^2. \quad (4.25)$$

In steady state, we then have

$$F_1(I, I) = -\tilde{S}'(x)\,x + \left(1 - \tilde{S}(x)\right) = 1, \quad (4.26)$$

$$F_2(I, I) = \tilde{S}'(x)(x)^2 = 0. \quad (4.27)$$

The specific functional form for $\tilde{S}$ used in Christiano, Trabandt, and Walentin (2011) and Adolfson et al. (2013), and its first- and second-order derivatives are given by

$$\begin{aligned}
\tilde{S}(x) &= \frac{1}{2}\underbrace{\left\{\exp\left[\sqrt{\tilde{S}''}\left(x - \mu_{z^+}\mu_\Psi\right)\right] + \exp\left[-\sqrt{\tilde{S}''}\left(x - \mu_{z^+}\mu_\Psi\right)\right] - 2\right\}}_{=1+1-2} \\
&= 0,\ x = \mu_{z^+}\mu_\Psi, \\
\tilde{S}'(x) &= \frac{1}{2}\left\{\sqrt{\tilde{S}''}\exp\left[\sqrt{\tilde{S}''}\left(x - \mu_{z^+}\mu_\Psi\right)\right] + \left(-\sqrt{\tilde{S}''}\right)\exp\left[-\sqrt{\tilde{S}''}\left(x - \mu_{z^+}\mu_\Psi\right)\right]\right\} \\
&= \frac{1}{2}\sqrt{\tilde{S}''}\underbrace{\left\{\exp\left[\sqrt{\tilde{S}''}\left(x - \mu_{z^+}\mu_\Psi\right)\right] - \exp\left[-\sqrt{\tilde{S}''}\left(x - \mu_{z^+}\mu_\Psi\right)\right]\right\}}_{=1-1} \\
&= 0,\ x = \mu_{z^+}\mu_\Psi, \\
\tilde{S}''(x) &= \frac{1}{2}\sqrt{\tilde{S}''}\left\{\sqrt{\tilde{S}''}\exp\left[\sqrt{\tilde{S}''}\left(x - \mu_{z^+}\mu_\Psi\right)\right] - \left(-\sqrt{\tilde{S}''}\right)\exp\left[-\sqrt{\tilde{S}''}\left(x - \mu_{z^+}\mu_\Psi\right)\right]\right\} \\
&= \frac{1}{2}\tilde{S}''\underbrace{\left\{\exp\left[\sqrt{\tilde{S}''}\left(x - \mu_{z^+}\mu_\Psi\right)\right] + \exp\left[-\sqrt{\tilde{S}''}\left(x - \mu_{z^+}\mu_\Psi\right)\right]\right\}}_{=1+1} \\
&= S'',\ x = \mu_{z^+}\mu_\Psi.
\end{aligned}$$

In the Adolfson et al. (2005) documentation of Ramses I, an example is given of a different functional form for $\tilde{S}(x)$ in footnote 11. Both these specifications fulfil the requirements on $\tilde{S}(x)$, why it is of little importance which one we choose. As it may simplify calibration comparisons with Ramses II, we here adopt the more recent specification in Christiano, Trabandt, and Walentin (2011), and Adolfson et al. (2013).

Note also that equation (4.11) yields

$$K_t = u_t K_t^p. \quad (4.28)$$

4.4 The household's optimization problem

In each period, the household chooses its consumption and bond holdings to maximize equation (4.8) subject to the budget constraint given in equation (4.10) and the law of motion for capital in equation

[29] Note that, for estimation purposes, we will introduce the following transformation of S'':

$$S'' = \frac{1 - S'',^{tr}}{S'',^{tr}},$$

setting a prior on and estimating $S'',^{tr}$, instead of S''.

(4.21). Moreover, households decide the level of capital services provided to the firms. They can increase the capital stock by investing in additional physical capital, in which case the newly invested capital becomes operational one period ahead, or by increasing the utilization rate of the existing capital. Specifically, households solve the following Lagrangian problem:

$$\max_{C_t, B_{t+1}, B^F_{t+1}, K^p_{t+1}, \Delta_t, I_t, u_t} E^j_0 \sum_{t=0}^{\infty} \beta^t \zeta^\beta_t \left[\mathcal{L}_t\right], \tag{4.29}$$

$$\mathcal{L}_t = \left\{ \begin{array}{c} \zeta^c_t \log\left(C_t - bC_{t-1}\right) - \zeta^n_t \Theta_t \int_0^1 \frac{n^{1+\varphi}_{j,t}}{1+\varphi} dj \\ +\upsilon_t \left[\begin{array}{c} \int_0^1 \int_0^{n_{j,t}} W_{j,k,t} dk dj + R^k_t u_t K^p_t + R_{t-1}\chi_{t-1} B_t + R^*_{t-1}\Phi_{t-1}\chi_{t-1} S_t B^F_t + \Pi_t + TR_t \\ -\left(P^c_t C_t + P^i_t \left(I_t + a\left(u_t\right) K^p_t\right) + P_{k',t}\Delta_t + B_{t+1} + S_t B^F_{t+1}\right) \end{array} \right] \\ +\omega_t \left[\left(1-\delta\right) K^p_t + \Upsilon_t F\left(I_t, I_{t-1}\right) + \Delta_t - K^p_{t+1}\right] \end{array} \right\},$$

where υ_t is the shadow value in utility terms of domestic currency. We then obtain the following set of first-order conditions:

$$\text{w.r.t. } C_t : \zeta^\beta_t \zeta^c_t \frac{1}{C_t - bC_{t-1}} - \beta b E_t \zeta^\beta_{t+1} \zeta^c_{t+1} \frac{1}{C_{t+1} - bC_t} - \zeta^\beta_t \upsilon_t P^c_t = 0 \tag{4.30}$$

$$\text{w.r.t. } B_{t+1} : -\zeta^\beta_t \upsilon_t + \beta E_t \zeta^\beta_{t+1} \upsilon_{t+1} R_t \chi_t = 0 \tag{4.31}$$

$$\text{w.r.t. } B^F_{t+1} : -\zeta^\beta_t \upsilon_t S_t + \beta E_t \zeta^\beta_{t+1} \upsilon_{t+1} R^*_t \Phi_t \chi_t S_{t+1} = 0 \tag{4.32}$$

$$\text{w.r.t. } K^p_{t+1} : -\zeta^\beta_t \omega_t + \beta E_t \left[\zeta^\beta_{t+1} \upsilon_{t+1} \left(R^k_{t+1} u_{t+1} - P^i_{t+1} a\left(u_{t+1}\right)\right) + \zeta^\beta_{t+1} \omega_{t+1} \left(1-\delta\right)\right] = 0 \tag{4.33}$$

$$\text{w.r.t. } \Delta_t : -\zeta^\beta_t \upsilon_t P_{k',t} + \zeta^\beta_t \omega_t = 0 \tag{4.34}$$

$$\text{w.r.t. } I_t : -\zeta^\beta_t \upsilon_t P^i_t + \zeta^\beta_t \omega_t \Upsilon_t F_1\left(I_t, I_{t-1}\right) + \beta E_t \zeta^\beta_{t+1} \omega_{t+1} \Upsilon_{t+1} F_2\left(I_{t+1}, I_t\right) = 0 \tag{4.35}$$

$$\text{w.r.t. } u_t : \zeta^\beta_t \upsilon_t K^p_t \left(R^k_t - P^i_t a'\left(u_t\right)\right) = 0 \tag{4.36}$$

We note that, using (4.7), we can also write (4.30) as follows:

$$\bar{\Upsilon}^N_t = \zeta^\beta_t \upsilon_t P^c_t. \tag{4.37}$$

4.5 Unemployment and labour supply

As mentioned in Section 4.1 above, modelling the labour market as in Galí, Smets, and Wouters (2012) allows us to introduce unemployment into the model. Just as in Galí, Smets, and Wouters (2012) (following in turn Galí (2011a) and Galí (2011b)) the unemployment rate is defined as

$$U_t = \frac{L_t - N_t}{L_t} = 1 - \frac{N_t}{L_t} \approx \log L_t - \log N_t. \tag{4.38}$$

With this definition, the unemployed include all the individuals who would like to be working but are not currently employed. As argued in Galí, Smets, and Wouters (2012), it can thus be viewed as involuntary. Note that the assumption that the individual members of the household take into account the utility of the household rather than their personal utility is crucial. As the model implies that unemployed individuals will enjoy a higher utility ex post than employed individuals — which follows from the assumption of full consumption risk-sharing — not internalizing the benefits to the household of an individual's unemployment would result in no participation.

An individual specialized in type-j labour and with disutility of work $\zeta^\beta_t \zeta^n_t \Theta_t k^\varphi$ finds it optimal to participate in the labour market whenever

$$\begin{aligned} \left(\zeta^\beta_t \zeta^c_t \frac{1}{C_t - bC_{t-1}} - \beta b \zeta^\beta_{t+1} \zeta^c_{t+1} \frac{1}{C_{t+1} - bC_t}\right) \frac{W_{j,t}}{P^c_t} &\geq \zeta^\beta_t \zeta^n_t \Theta_t k^\varphi \\ \bar{\Upsilon}^N_t \frac{W_{j,t}}{P^c_t} &\geq \zeta^\beta_t \zeta^n_t \Theta_t k^\varphi, \end{aligned}$$

using household welfare as a criterion, and taking as given current labour market conditions as summarized by the type-j prevailing wage. Recall that we in a symmetric equilibrium have $\bar{C}_t = C_t$. The household member that is just willing to supply labour $L_{j,t}$ is then given by the expression above holding with equality. We get, in a symmetric equilibrium,

$$\frac{W_{j,t}}{P_t^c} = \zeta_t^\beta \zeta_t^n Z_t^C L_{j,t}^\varphi. \tag{4.39}$$

4.6 Wage setting

We assume that households are monopolistic suppliers of differentiated labour services hired by the firm. Thus, households can determine their wages. After having set their wages, households inelastically supply the firms' demand for labour at the going wage rate. We suppose that the differentiated labour, $n_{j,t}$, is sold by households to labour contractors who combine it into a homogeneous input good N_t using the following technology:

$$N_t = \left[\int_0^1 (n_{j,t})^{\frac{1}{\lambda_t^w}} dj\right]^{\lambda_t^w}, \quad 1 \leq \lambda_t^w < \infty, \tag{4.40}$$

where λ_t^w is a time-varying wage markup given by the following process:

$$\log \lambda_t^w = (1 - \rho_{\lambda^w}) \log \lambda^w + \rho_{\lambda^w} \log \lambda_{t-1}^w + \sigma_{\lambda^w} \varepsilon_{\lambda^w,t}. \tag{4.41}$$

These labour contractors take the price of the j^{th} differentiated labour input, $W_{j,t}$, and the price of the homogeneous labour service, W_t, as given. Profit maximization writes

$$\max_{n_{j,t}} \quad W_t N_t - \int_0^1 W_{j,t} n_{j,t} dj,$$

and leads to the following first-order condition:

$$n_{j,t} = \left(\frac{W_{j,t}}{W_t}\right)^{\frac{\lambda_t^w}{1-\lambda_t^w}} N_t, \tag{4.42}$$

which is a demand curve for the individual households' labour services. Integrating (4.42) and using the definition of N_t, we obtain the expression for the aggregate wage rate:

$$W_t = \left[\int_0^1 (W_{j,t})^{\frac{1}{1-\lambda_t^w}} dj\right]^{1-\lambda_t^w}. \tag{4.43}$$

We consider that households are subject to Calvo wage setting frictions as in Erceg, Henderson, and Levin (2000). In every period, each labour type (or union representing that labour type) faces a probability $1 - \xi_w$ that it can reoptimize its nominal wage, independent of when it was last allowed to reoptimize. If the union reporesenting the j^{th} labour type is not able to reoptimize in period t, the wage it will charge in period $t + 1$ will be set according to the following indexation rule:

$$\begin{cases} W_{j,t+1} = \tilde{\pi}_{t+1}^w W_{j,t} \\ \tilde{\pi}_{t+1}^w \equiv (\pi_t^c)^{\kappa_w} \left(\bar{\pi}_{t+1}^c\right)^{1-\kappa_w-\varkappa_w} (\breve{\pi})^{\varkappa_w} (\mu_{z^+})^{\vartheta_w}. \end{cases} \tag{4.44}$$

Let us denote by $\tilde{W}_{j,t}$ the reoptimized nominal wage of the union representing the j^{th} labour type that is set in period t, and consider that this union has not been able to reoptimize during s periods ahead. The wage in $t + s$ will be given by

$$W_{j,t+s} = \tilde{\pi}_{t+s}^w \ldots \tilde{\pi}_{t+1}^w \tilde{W}_{j,t}.$$

When reoptimizing their wage in period t, unions representing labour of type j choose the wage in order to maximize the representative households' utility (as opposed to the individuals' utility), subject to the usual sequence of household flow budget constraints and labour demand. In period t, when setting its wage $\tilde{W}_{j,t}$ the unions representing labour of type j will maximize its future discounted utility subject to the budget constraint as in Section 4.4, taking into account that there is a probability ξ_w in each period that it cannot reoptimize. Using (4.8) and ignoring the irrelevant terms (of the utility function) for the wage setting problem, the problem becomes

$$\left\{ \begin{array}{c} \max_{\tilde{W}_{j,t}} \quad E_t \sum_{s=0}^{\infty} (\beta\xi_w)^s \zeta_{t+s}^{\beta} \left[-\zeta_{t+s}^{n} \Theta_{t+s} \frac{n_{j,t+s}^{1+\varphi}}{1+\varphi} + \upsilon_{t+s} W_{j,t+s} n_{j,t+s} \right] \\ s.t. \quad n_{j,t} = \left(\frac{W_{j,t}}{W_t} \right)^{\frac{\lambda_t^w}{1-\lambda_t^w}} N_t \end{array} \right. . \tag{4.45}$$

Replacing both $n_{j,t}$ and the expression for the wage $W_{j,t+s}$, we get[30]

$$\max_{\tilde{W}_{j,t}} \quad E_t \sum_{s=0}^{\infty} (\beta\xi_w)^s \zeta_{t+s}^{\beta} \left[\begin{array}{c} -\zeta_{t+s}^{n} \frac{\Theta_{t+s}}{1+\varphi} \left[\left(\frac{\tilde{\pi}_{t+s}^w \ldots \tilde{\pi}_{t+1}^w}{W_{t+s}} \right)^{\frac{\lambda_{t+s}^w}{1-\lambda_{t+s}^w}} N_{t+s} \right]^{1+\varphi} \tilde{W}_{j,t}^{\frac{\lambda_{t+s}^w (1+\varphi)}{1-\lambda_{t+s}^w}} \\ +\upsilon_{t+s} \left(W_{t+s} \right)^{-\frac{\lambda_{t+s}^w}{1-\lambda_{t+s}^w}} N_{t+s} \left(\tilde{\pi}_{t+s}^w \ldots \tilde{\pi}_{t+1}^w \right)^{\frac{1}{1-\lambda_{t+s}^w}} \left(\tilde{W}_{j,t} \right)^{\frac{1}{1-\lambda_{t+s}^w}} \end{array} \right].$$

The FOC associated with this problem directly yields the expression for the optimal wage $\tilde{W}_t$, which is independent of j, as each union faces the same optimization problem. Taking derivatives w.r.t. $\tilde{W}_{j,t}$ gives

$$\left(\tilde{W}_{j,t} \right)^{1-\frac{\varphi\lambda_{t+s}^w}{1-\lambda_{t+s}^w}} E_t \sum_{s=0}^{\infty} (\beta\xi_w)^s \zeta_{t+s}^{\beta} \upsilon_{t+s} \left(W_{t+s} \right)^{-\frac{\lambda_{t+s}^w}{1-\lambda_{t+s}^w}} N_{t+s} \left(\tilde{\pi}_{t+s}^w \ldots \tilde{\pi}_{t+1}^w \right)^{\frac{1}{1-\lambda_{t+s}^w}}$$

$$= \quad E_t \sum_{s=0}^{\infty} (\beta\xi_w)^s \zeta_{t+s}^{\beta} \zeta_{t+s}^{n} \Theta_{t+s} \lambda_{t+s}^w \left[\left(\frac{\tilde{\pi}_{t+s}^w \ldots \tilde{\pi}_{t+1}^w}{W_{t+s}} \right)^{\frac{\lambda_{t+s}^w}{1-\lambda_{t+s}^w}} N_{t+s} \right]^{1+\varphi} .$$

Rearranging and dropping the index j, we obtain

$$\tilde{W}_t^{\frac{1-\lambda_{t+s}^w(1+\varphi)}{1-\lambda_{t+s}^w}} = \frac{E_t \sum_{s=0}^{\infty} (\beta\xi_w)^s \zeta_{t+s}^{\beta} \zeta_{t+s}^{n} \Theta_{t+s} \lambda_{t+s}^w \left[\left(\frac{\tilde{\pi}_{t+s}^w \ldots \tilde{\pi}_{t+1}^w}{W_{t+s}} \right)^{\frac{\lambda_{t+s}^w}{1-\lambda_{t+s}^w}} N_{t+s} \right]^{1+\varphi}}{E_t \sum_{s=0}^{\infty} (\beta\xi_w)^s \zeta_{t+s}^{\beta} \upsilon_{t+s} W_{t+s} N_{t+s} \left(\frac{\tilde{\pi}_{t+s}^w \ldots \tilde{\pi}_{t+1}^w}{W_{t+s}} \right)^{\frac{1}{1-\lambda_{t+s}^w}}}.$$

To rewrite in terms of relative wages, we divide both sides by $W_t^{\frac{1-\lambda_{t+s}^w(1+\varphi)}{1-\lambda_{t+s}^w}}$ to obtain

$$\tilde{w}_t^{\frac{1-\lambda_{t+s}^w(1+\varphi)}{1-\lambda_{t+s}^w}} = \frac{E_t \sum_{s=0}^{\infty} (\beta\xi_w)^s \zeta_{t+s}^{\beta} \zeta_{t+s}^{n} \Theta_{t+s} \lambda_{t+s}^w \left[\left(\frac{W_t \tilde{\pi}_{t+s}^w \ldots \tilde{\pi}_{t+1}^w}{W_{t+s}} \right)^{\frac{\lambda_{t+s}^w}{1-\lambda_{t+s}^w}} N_{t+s} \right]^{1+\varphi}}{E_t \sum_{s=0}^{\infty} (\beta\xi_w)^s \zeta_{t+s}^{\beta} \upsilon_{t+s} W_{t+s} N_{t+s} \left(\frac{W_t \tilde{\pi}_{t+s}^w \ldots \tilde{\pi}_{t+1}^w}{W_{t+s}} \right)^{\frac{1}{1-\lambda_{t+s}^w}}}, \tag{4.46}$$

where $\tilde{w}_t = \frac{\tilde{W}_t}{W_t}$, and we have used that

$$\frac{\left(W_t \right)^{\frac{\lambda_{t+s}^w}{1-\lambda_{t+s}^w}(1+\varphi)}}{\left(W_t \right)^{\frac{1}{1-\lambda_{t+s}^w}}} = W_t^{-\frac{1-\lambda_{t+s}^w(1+\varphi)}{1-\lambda_{t+s}^w}} .$$

[30] Note that the objective is almost the same as in the Smets and Wouters (2003) model. Instead of $\zeta_{t+s}^{n} \frac{A_L}{1+\sigma_L}$ multiplying the first term in the summation, we instead have $\zeta_{t+s}^{n} \frac{\Theta_{t+s}}{1+\varphi}$ and the term within the square brackets is raised to $1+\varphi$ instead of $1+\sigma_L$. The first-order condition is then almost identical to the Smets and Wouters (2003) model.

Consider again the aggregate wage index (4.43). Due to the Calvo assumption on wage setting frictions, the fraction of unions that are allowed to reoptimize their wages is random, and thus the integral over some subset of the unit interval is equal to the integral over the entire unit interval weighted by the fraction of the unit interval over which the former integral is taken. We can then rewrite the wage index as follows:

$$\begin{aligned} W_t^{\frac{1}{1-\lambda_t^w}} &= \int_0^1 (W_{j,t})^{\frac{1}{1-\lambda_t^w}} dj \\ &= \int_0^{\xi_w} (\tilde{\pi}_t^w W_{j,t-1})^{\frac{1}{1-\lambda_t^w}} dj + \int_{\xi_w}^1 \left(\tilde{W}_t\right)^{\frac{1}{1-\lambda_t^w}} dj \\ &= \xi_w (\tilde{\pi}_t^w W_{t-1})^{\frac{1}{1-\lambda_t^w}} + (1-\xi_w)\left(\tilde{W}_t\right)^{\frac{1}{1-\lambda_t^w}}. \end{aligned}$$

Dividing both sides by $W_t^{\frac{1}{1-\lambda_t^w}}$, we obtain

$$\begin{aligned} 1 &= \xi_w \left(\frac{\tilde{\pi}_t^w}{\pi_t^w}\right)^{\frac{1}{1-\lambda_t^w}} + (1-\xi_w)(\tilde{w}_t)^{\frac{1}{1-\lambda_t^w}} \\ \tilde{w}_t &= \left[\frac{1-\xi_w \left(\frac{\tilde{\pi}_t^w}{\pi_t^w}\right)^{\frac{1}{1-\lambda_t^w}}}{(1-\xi_w)}\right]^{1-\lambda_t^w}, \end{aligned} \tag{4.47}$$

where

$$\pi_t^w = \frac{W_t}{W_{t-1}} = \frac{\bar{w}_t z_t^+ P_t^d}{\bar{w}_{t-1} z_{t-1}^+ P_{t-1}^d} = \frac{\bar{w}_t \mu_{z^+,t} \pi_t^d}{\bar{w}_{t-1}}. \tag{4.48}$$

For use in later sections, we now derive the relationship between aggregate homogeneous labour N_t, and aggregate household labour n_t, defined as

$$n_t \equiv \int_0^1 n_{j,t} dj.$$

Substituting for the demand for $n_{j,t}$, using (4.42), we obtain

$$\begin{aligned} n_t &= N_t \int_0^1 \left(\frac{W_{j,t}}{W_t}\right)^{\frac{\lambda_t^w}{1-\lambda_t^w}} dj \\ n_t &= N_t (\mathring{w}_t)^{\frac{\lambda_t^w}{1-\lambda_t^w}}, \end{aligned} \tag{4.49}$$

where $\mathring{w}_t$ is a measure of wage dispersion defined as

$$\mathring{w}_t \equiv \left[\int_0^1 \left(\frac{W_{j,t}}{W_t}\right)^{\frac{\lambda_t^w}{1-\lambda_t^w}} dj\right]^{\frac{1-\lambda_t^w}{\lambda_t^w}}. \tag{4.50}$$

We can break this integral and re-express it in terms of aggregates using the Calvo assumption on wage setting frictions. The fraction of households that are allowed to reoptimize their wages is random, and thus the integral over some subset of the unit interval is equal to the integral over the entire unit interval weighted by the fraction of the unit interval over which the former integral is taken. Hence,

$$
\begin{aligned}
\mathring{w}_t &= \left[\int_0^{\xi_w}\left(\frac{\tilde{\pi}_t^w W_{j,t-1}}{W_t}\right)^{\frac{\lambda_t^w}{1-\lambda_t^w}} dj + \int_{\xi_w}^1 \left(\frac{\tilde{W}_t}{W_t}\right)^{\frac{\lambda_t^w}{1-\lambda_t^w}} di\right]^{\frac{1-\lambda_t^w}{\lambda_t^w}} \\
&= \left[(\tilde{\pi}_t^w)^{\frac{\lambda_t^w}{1-\lambda_t^w}} \int_0^{\xi_w}\left(\frac{W_{t-1}}{W_t}\frac{W_{j,t-1}}{W_{t-1}}\right)^{\frac{\lambda_t^w}{1-\lambda_t^w}} dj + (1-\xi_w)\left(\tilde{w}_t\right)^{\frac{\lambda_t^w}{1-\lambda_t^w}}\right]^{\frac{1-\lambda_t^w}{\lambda_t^w}} \\
&= \left[\xi_w\left(\frac{\tilde{\pi}_t^w}{\pi_t^w}\mathring{w}_{t-1}\right)^{\frac{\lambda_t^w}{1-\lambda_t^w}} + (1-\xi_w)\left(\tilde{w}_t\right)^{\frac{\lambda_t^w}{1-\lambda_t^w}}\right]^{\frac{1-\lambda_t^w}{\lambda_t^w}}.
\end{aligned}
$$

Using (4.47), we finally obtain

$$
\mathring{w}_t = \left[\xi_w\left(\frac{\tilde{\pi}_t^w}{\pi_t^w}\mathring{w}_{t-1}\right)^{\frac{\lambda_t^w}{1-\lambda_t^w}} + (1-\xi_w)\left(\frac{1-\xi_w\left(\frac{\tilde{\pi}_t^w}{\pi_t^w}\right)^{\frac{1}{1-\lambda_t^w}}}{(1-\xi_w)}\right)^{\lambda_t^w}\right]^{\frac{1-\lambda_t^w}{\lambda_t^w}}. \tag{4.51}
$$

4.7 Scaling of the household equations

To express the model in stationary form, we need to divide the quantities with the trend level of the neutral and, where applicable, investment-specific technologies, as specified in Section 2.1.

4.7.1 Scaling of the preference shifters and MRS

We start by scaling the expression for the endogenous preference shifter Θ_t in equation (4.5). We rewrite (4.5) as

$$
\Theta_t = \frac{Z_t^C}{z_t^+}\bar{\Upsilon}_t^N z_t^+ = \frac{Z_t^C}{z_t^+}\left(\zeta_t^c \frac{1}{\frac{\bar{C}_t}{z_t^+} - b\frac{\bar{C}_{t-1}}{z_{t-1}^+}\frac{z_{t-1}^+}{z_t^+}} - \beta b E_t \zeta_{t+1}^c \frac{1}{\frac{\bar{C}_{t+1}}{z_{t+1}^+}\frac{z_{t+1}^+}{z_t^+} - b\frac{\bar{C}_t}{z_t^+}}\right),
$$

where

$$
\frac{Z_t^C}{z_t^+} = \left(\frac{Z_{t-1}^C}{z_{t-1}^+}\frac{z_{t-1}^+}{z_t^+}\right)^{1-\nu}\left(\frac{1}{z_t^+\bar{\Upsilon}_t^N}\right)^{\nu}.
$$

Defining

$$
\bar{\upsilon}_t^N = \bar{\Upsilon}_t^N z_t^+,
$$

we have that

$$
\bar{\upsilon}_t^N = \bar{\Upsilon}_t^N z_t^+ = \zeta_t^\beta \zeta_t^c \frac{1}{\frac{\bar{C}_t}{z_t^+} - b\frac{\bar{C}_{t-1}}{z_{t-1}^+}\frac{z_{t-1}^+}{z_t^+}} - \beta b E_t \zeta_{t+1}^\beta \zeta_{t+1}^c \frac{1}{\frac{\bar{C}_{t+1}}{z_{t+1}^+}\frac{z_{t+1}^+}{z_t^+} - b\frac{\bar{C}_t}{z_t^+}}.
$$

Then, using that we in a symmetric equilibrium have that $C_t = \bar{C}_t$,

$$
\Theta_t = z_t^C \bar{\upsilon}_t^N = \zeta_t^\beta \zeta_t^c \frac{z_t^C}{c_t - bc_{t-1}\frac{1}{\mu_{z^+,t}}} - \beta b E_t \zeta_{t+1}^\beta \zeta_{t+1}^c \frac{z_t^C}{c_{t+1}\mu_{z^+,t+1} - bc_t}, \tag{4.52}
$$

where

$$
z_t^C = \left(z_{t-1}^C \frac{1}{\mu_{z^+,t}}\right)^{1-\nu}\left(\frac{1}{\bar{\upsilon}_t^N}\right)^{\nu}, \tag{4.53}
$$

and

$$\bar{v}_t^N = \frac{\zeta_t^\beta \zeta_t^c}{c_t - bc_{t-1}\frac{1}{\mu_{z^+,t}}} - \beta b E_t \frac{\zeta_{t+1}^\beta \zeta_{t+1}^c}{c_{t+1}\mu_{z^+,t+1} - bc_t}. \tag{4.54}$$

For future use, we note also that

$$\bar{v}_{t+1}^N = \bar{\Upsilon}_{t+1}^N z_{t+1}^+ = \zeta_{t+1}^\beta \zeta_{t+1}^c \frac{1}{\frac{\bar{C}_{t+1}}{z_{t+1}^+} - b\frac{\bar{C}_t}{z_t^+}\frac{z_t^+}{z_{t+1}^+}} - \beta b E_t \zeta_{t+2}^\beta \zeta_{t+2}^c \frac{1}{\frac{\bar{C}_{t+2}}{z_{t+2}^+}\frac{z_{t+2}^+}{z_{t+1}^+} - b\bar{C}_{t+1}}$$

$$\bar{v}_{t+1}^N = \bar{\Upsilon}_{t+1}^N z_{t+1}^+ = \frac{\zeta_{t+1}^\beta \zeta_{t+1}^c}{c_{t+1} - bc_t\frac{1}{\mu_{z^+,t+1}}} - \beta b E_t \frac{\zeta_{t+2}^\beta \zeta_{t+2}^c}{c_{t+2}\mu_{z^+,t+2} - bc_{t+1}},$$

and

$$\frac{\bar{v}_{t+1}^N}{\mu_{z^+,t+1}} = \frac{\zeta_{t+1}^\beta \zeta_{t+1}^c}{c_{t+1}\mu_{z^+,t+1} - bc_t} - \beta b E_t \frac{\zeta_{t+2}^\beta \zeta_{t+2}^c}{c_{t+2}\mu_{z^+,t+1}\mu_{z^+,t+2} - bc_{t+1}\mu_{z^+,t+1}}. \tag{4.55}$$

We can also scale the marginal rate of substituion in equation (4.9) to obtain

$$\frac{MRS_{j,t}}{z_t^+} = \frac{\zeta_t^\beta \zeta_t^n \Theta_t N_{j,t}^\varphi}{\bar{\Upsilon}_t^N z_t^+}$$

$$mrs_{j,t} = \frac{\zeta_t^\beta \zeta_t^n \Theta_t N_{j,t}^\varphi}{\bar{v}_t^N}. \tag{4.56}$$

4.7.2 Scaling of the household's first-order conditions

Scaling equations (4.30)–(4.36), we obtain

$$\text{w.r.t. } c_t: \frac{\zeta_t^\beta \zeta_t^c}{c_t - bc_{t-1}\frac{1}{\mu_{z^+,t}}} - \beta b E_t \frac{\zeta_{t+1}^\beta \zeta_{t+1}^c}{c_{t+1}\mu_{z^+,t+1} - bc_t} - \zeta_t^\beta \psi_{z^+,t} p_t^c = 0, \tag{4.57}$$

$$\text{w.r.t. } b_{t+1}: -\zeta_t^\beta \psi_{z^+,t} + \beta E_t \frac{\zeta_{t+1}^\beta \psi_{z^+,t+1}}{\mu_{z^+,t+1}} \frac{R_t \chi_t}{\pi_{t+1}^d} = 0, \tag{4.58}$$

$$\text{w.r.t. } b_{t+1}^F: -\zeta_t^\beta \psi_{z^+,t} S_t + \beta E_t \frac{\zeta_{t+1}^\beta \psi_{z^+,t+1}}{\pi_{t+1}^d \mu_{z^+,t+1}} R_t^* \Phi_t \chi_t S_{t+1} = 0, \tag{4.59}$$

$$\text{w.r.t. } k_{t+1}^p: -\zeta_t^\beta \omega_t z_t^+ \Psi_t + \beta E_t \left[\begin{array}{c} \frac{\zeta_{t+1}^\beta \psi_{z^+,t+1}}{\mu_{z^+,t+1}\mu_{\Psi,t+1}} \left(\bar{r}_{t+1}^k u_{t+1} - p_{t+1}^i a\left(u_{t+1}\right)\right) \\ +\zeta_{t+1}^\beta \omega_{t+1} z_{t+1}^+ \Psi_{t+1} \frac{1}{\mu_{z^+,t+1}\mu_{\Psi,t+1}} (1-\delta) \end{array} \right] = 0, \tag{4.60}$$

$$\text{w.r.t. } \tilde{\Delta}_t: -\zeta_t^\beta \psi_{z^+,t} \breve{p}_{k',t} + \zeta_t^\beta \omega_t z_t^+ \Psi_t = 0, \tag{4.61}$$

$$\text{w.r.t. } i_t: \begin{array}{c} -\zeta_t^\beta \psi_{z^+,t} p_t^i + \zeta_t^\beta \omega_t z_t^+ \Psi_t \Upsilon_t F_1\left(i_t, i_{t-1}\right) \\ +\beta E_t \zeta_{t+1}^\beta \omega_{t+1} z_{t+1}^+ \Psi_{t+1} \frac{1}{\mu_{z^+,t+1}\mu_{\Psi,t+1}} \Upsilon_{t+1} F_2\left(i_{t+1}, i_t\right) = 0 \end{array}, \tag{4.62}$$

where $F_1\left(i_t, i_{t-1}\right)$ and $F_2\left(i_{t+1}, i_t\right)$ are specified further below, and

$$\text{w.r.t. } u_t: \zeta_t^\beta \psi_{z^+,t} k_t^p \frac{1}{\mu_{z^+,t}\mu_{\Psi,t}} \left(\bar{r}_t^k - p_t^i a'\left(u_t\right)\right) = 0. \tag{4.63}$$

We can also scale equation (4.37) to get

$$\bar{v}_t^N = \zeta_t^\beta \psi_{z^+,t} p_t^c, \tag{4.64}$$

where we have used the definitions $\bar{v}_t^N = \bar{\Upsilon}_t^N z_t^+$ and $\psi_{z^+,t} = v_t z_t^+$.

We note that equation (4.58) yields the following expression for the rescaled Lagrange multiplier $\psi_{z^+,t}$:

$$\psi_{z^+,t} = \beta E_t \frac{\zeta_{t+1}^\beta}{\zeta_t^\beta} \frac{\psi_{z^+,t+1}}{\mu_{z^+,t+1}} \frac{R_t \chi_t}{\pi_{t+1}^d}. \tag{4.65}$$

We can combine the first-order conditions for domestic and foreign bond holdings, (4.58) and (4.59), to obtain

$$R_t = R_t^* \Phi_t E_t s_{t+1}, \tag{4.66}$$

which is a modified uncovered interest rate parity condition, with the term Φ_t defined as in equation (4.19), and s_{t+1} defined as follows:

$$s_{t+1} = \frac{S_{t+1}}{S_t}.$$

We can rewrite equation (4.66) in real terms as follows:

$$\begin{aligned}
\frac{R_t}{E_t \pi_{t+1}^c} &= \frac{R_t^*}{E_t \pi_{t+1}^{c,*}} \Phi_t E_t \frac{s_{t+1} \pi_{t+1}^{c,*}}{\pi_{t+1}^c} \\
\bar{R}_t &= \bar{R}_t^* \Phi_t E_t \frac{S_{t+1} P_{t+1}^{c,*}}{P_{t+1}^c} \frac{P_t^c}{S_t P_t^{c,*}} \\
\bar{R}_t &= \bar{R}_t^* \Phi_t E_t \frac{q_{t+1}}{q_t},
\end{aligned}$$

where the real interest rates are denoted by a bar, and we have used the definition of the real exchange rate

$$q_t = \frac{S_t P_t^{c,*}}{P_t^c}.$$

Moreover, we can combine equations (4.58) and (4.57) to obtain the following household consumption Euler equation:

$$\begin{aligned}
&\frac{\zeta_t^\beta \zeta_t^c}{c_t - b c_{t-1} \frac{1}{\mu_{z^+,t}}} - \beta b E_t \frac{\zeta_{t+1}^\beta \zeta_{t+1}^c}{c_{t+1} \mu_{z^+,t+1} - b c_t} \\
= \; &\beta E_t \frac{R_t \chi_t}{\pi_{t+1}^d} \frac{p_t^c}{p_{t+1}^c} \left[\frac{\zeta_{t+1}^\beta \zeta_{t+1}^c}{c_{t+1} \mu_{z^+,t+1} - b c_t} - \beta b E_t \frac{\zeta_{t+2}^\beta \zeta_{t+2}^c}{c_{t+2} \mu_{z^+,t+1} \mu_{z^+,t+2} - b c_{t+1} \mu_{z^+,t+1}} \right],
\end{aligned}$$

and noting that

$$\frac{p_t^c}{p_{t+1}^c} = \frac{P_t^c}{P_t^d} \frac{P_{t+1}^d}{P_{t+1}^c} = \frac{\pi_{t+1}^d}{\pi_{t+1}^c},$$

$$\begin{aligned}
&\frac{\zeta_t^\beta \zeta_t^c}{c_t - b c_{t-1} \frac{1}{\mu_{z^+,t}}} - \beta b E_t \frac{\zeta_{t+1}^\beta \zeta_{t+1}^c}{c_{t+1} \mu_{z^+,t+1} - b c_t} \\
= \; &\beta E_t \frac{R_t \chi_t}{\pi_{t+1}^c} \left[\frac{\zeta_{t+1}^\beta \zeta_{t+1}^c}{c_{t+1} \mu_{z^+,t+1} - b c_t} - \beta b E_t \frac{\zeta_{t+2}^\beta \zeta_{t+2}^c}{c_{t+2} \mu_{z^+,t+1} \mu_{z^+,t+2} - b c_{t+1} \mu_{z^+,t+1}} \right].
\end{aligned} \tag{4.67}$$

Using (4.54) and (4.55), we can also rewrite the above equation as

$$\bar{v}_t^N = \beta E_t \frac{R_t \chi_t}{\pi_{t+1}^c \mu_{z^+,t+1}} \bar{v}_{t+1}^N. \tag{4.68}$$

For intuition, note that, in the absence of habit formation, i.e. when $b = 0$, (4.67) collapses to the standard simple Euler equation given by

$$\zeta_t^\beta \zeta_t^c \frac{1}{c_t} = \beta E_t \frac{R_t \chi_t}{\pi_{t+1}^c} \zeta_{t+1}^\beta \zeta_{t+1}^c \frac{1}{c_{t+1} \mu_{z^+,t+1}},$$

which states that the expected utility of one unit of consumption today equals the discounting expected utility of postponing that consumption until the next period. In other words, the consumer chooses her consumption such that she is indifferent between consuming one more unit today, on the one hand, and saving that unit and consuming it in the future, on the other.

Using equation (4.61), we can solve for $\zeta_t^\beta \omega_t z_t^+ \Psi_t$ and substitute into equation (4.60) to obtain

$$E_t \left[\bar{r}_{t+1}^k u_{t+1} - p_{t+1}^i a(u_{t+1}) + (1-\delta) \breve{p}_{k',t+1} \right] = \frac{1}{\beta} E_t \frac{\zeta_t^\beta}{\zeta_{t+1}^\beta} \frac{\psi_{z^+,t}}{\psi_{z^+,t+1}} \mu_{z^+,t+1} \mu_{\Psi,t+1} \breve{p}_{k',t}. \tag{4.69}$$

Combining with equation (4.65), we have

$$E_t \left[\bar{r}_{t+1}^k u_{t+1} - p_{t+1}^i a(u_{t+1}) + (1-\delta) \breve{p}_{k',t+1} \right] = E_t \frac{R_t \chi_t}{\pi_{t+1}^d} \mu_{\Psi,t+1} \breve{p}_{k',t}. \tag{4.70}$$

We can also substitute (4.61) into equation (4.62), yielding

$$p_t^i = \breve{p}_{k',t} \Upsilon_t F_1(i_t, i_{t-1}) + \beta E_t \frac{\zeta_{t+1}^\beta}{\zeta_t^\beta} \frac{\psi_{z^+,t+1}}{\psi_{z^+,t}} \breve{p}_{k',t+1} \frac{1}{\mu_{z^+,t+1} \mu_{\Psi,t+1}} \Upsilon_{t+1} F_2(i_{t+1}, i_t). \tag{4.71}$$

If we combine with equation (4.65), we obtain

$$p_t^i = \breve{p}_{k',t} \Upsilon_t F_1(i_t, i_{t-1}) + E_t \frac{\pi_{t+1}^d}{R_t \chi_t} \breve{p}_{k',t+1} \frac{1}{\mu_{\Psi,t+1}} \Upsilon_{t+1} F_2(i_{t+1}, i_t). \tag{4.72}$$

From this expression it is clear that the risk premium shock, χ_t, affects also the value of investment, in addition to entering in the consumption Euler equation, as discussed in Section 4.2 above. Finally, from equation (4.63), we have that

$$\bar{r}_t^k = p_t^i a'(u_t). \tag{4.73}$$

To make the description of the household problem in terms of scaled variables complete, we need also to scale $F_1(I_t, I_{t-1})$ and $F_2(I_t, I_{t-1})$. We can scale the investment terms in equations (4.24) and (4.25) as follows:

$$F_1(i_t, i_{t-1}) = 1 - \tilde{S}\left(\frac{\mu_{z^+,t}\mu_{\Psi,t} i_t}{i_{t-1}}\right) - \tilde{S}'\left(\frac{\mu_{z^+,t}\mu_{\Psi,t} i_t}{i_{t-1}}\right) \frac{\mu_{z^+,t}\mu_{\Psi,t} i_t}{i_{t-1}} \tag{4.74}$$

$$F_2(i_t, i_{t-1}) = \tilde{S}'\left(\frac{\mu_{z^+,t}\mu_{\Psi,t} i_t}{i_{t-1}}\right) \left(\frac{\mu_{z^+,t}\mu_{\Psi,t} i_t}{i_{t-1}}\right)^2. \tag{4.75}$$

Combining equation (4.71) with expressions (4.74) and (4.75), we have

$$\begin{aligned} p_t^i &= \breve{p}_{k',t} \Upsilon_t \left[1 - \tilde{S}\left(\frac{\mu_{z^+,t}\mu_{\Psi,t} i_t}{i_{t-1}}\right) - \tilde{S}'\left(\frac{\mu_{z^+,t}\mu_{\Psi,t} i_t}{i_{t-1}}\right) \frac{\mu_{z^+,t}\mu_{\Psi,t} i_t}{i_{t-1}} \right] \\ &\quad + \beta E_t \frac{\zeta_{t+1}^\beta}{\zeta_t^\beta} \frac{\psi_{z^+,t+1}}{\psi_{z^+,t}} \breve{p}_{k',t+1} \Upsilon_{t+1} \tilde{S}'\left(\frac{\mu_{z^+,t+1}\mu_{\Psi,t+1} i_{t+1}}{i_t}\right) \left(\frac{i_{t+1}}{i_t}\right)^2 \mu_{z^+,t+1}\mu_{\Psi,t+1}. \end{aligned}$$

4.7.3 Scaling of the law of motion for capital

Starting from equation (4.21) combined with (4.23), we divide the quantities with the trend level of the neutral and investment-specific technologies to obtain

$$k^p_{t+1} = \frac{1-\delta}{\mu_{z^+,t}\mu_{\Psi,t}} k^p_t + \Upsilon_t \left(1 - \tilde{S}\left(\frac{\mu_{z^+,t}\mu_{\Psi,t} i_t}{i_{t-1}}\right)\right) i_t. \tag{4.76}$$

We can also scale equation (4.28) with $z^+_{t-1}\Psi_{t-1}$ to obtain

$$k_t = u_t k^p_t. \tag{4.77}$$

4.7.4 Scaling of labour supply

Using that $\bar{w}_t = \frac{W_t}{z^+_t P^d_t}$ and $p^c_t = \frac{P^c_t}{P^d_t}$, we can scale (4.39) to obtain

$$\int_0^1 \frac{W_{j,t}}{W_t} dj \frac{W_t}{z^+_t P^d_t} \frac{P^d_t}{P^c_t} = \frac{\bar{w}_t}{p^c_t} \int_0^1 \frac{W_{j,t}}{W_t} dj = \zeta^\beta_t \zeta^n_t \frac{Z^C_t}{z^+_t} \int_0^1 L^\varphi_{j,t} dj$$

$$\frac{\bar{w}_t}{p^c_t} \int_0^1 \frac{W_{j,t}}{W_t} dj = \zeta^\beta_t \zeta^n_t z^C_t \int_0^1 L^\varphi_{j,t} dj. \tag{4.78}$$

4.7.5 Scaling of the household's wage setting

We scale equation (4.46), stated here again for convenience:

$$\tilde{w}_t^{\frac{1-\lambda^w_{t+s}(1+\varphi)}{1-\lambda^w_{t+s}}} = \frac{E_t \sum_{s=0}^{\infty} (\beta\xi_w)^s \zeta^\beta_{t+s}\zeta^n_{t+s}\Theta_{t+s}\lambda^w_{t+s} \left[\left(\frac{W_t \tilde{\pi}^w_{t+s}\cdots\tilde{\pi}^w_{t+1}}{W_{t+s}}\right)^{\frac{\lambda^w_{t+s}}{1-\lambda^w_{t+s}}} N_{t+s}\right]^{1+\varphi}}{E_t \sum_{s=0}^{\infty} (\beta\xi_w)^s \zeta^\beta_{t+s}\upsilon_{t+s} W_{t+s} N_{t+s} \left(\frac{W_t \tilde{\pi}^w_{t+s}\cdots\tilde{\pi}^w_{t+1}}{W_{t+s}}\right)^{\frac{1}{1-\lambda^w_{t+s}}}}.$$

First, note that

$$\frac{W_t \tilde{\pi}^w_{t+s} \ldots \tilde{\pi}^w_{t+1}}{W_{t+s}} = \frac{W_t \tilde{\pi}^w_{t+s} \ldots \tilde{\pi}^w_{t+1}}{\bar{w}_{t+s} z^+_{t+s} P^d_{t+s}},$$

where $\bar{w}_t = \frac{W_t}{z^+_t P^d_t}$ is the scaled real wage, and $\mu_{z^+,t+1} \ldots \mu_{z^+,t+s} = \frac{z^+_{t+s}}{z^+_t}$, $\pi^d_{t+1} \ldots \pi^d_{t+s} = \frac{P^d_{t+s}}{P^d_t}$. We can then rewrite the previous ratio as

$$\frac{W_t \tilde{\pi}^w_{t+s} \ldots \tilde{\pi}^w_{t+1}}{\bar{w}_{t+s} z^+_{t+s} P^d_{t+s}} = \frac{W_t}{z^+_t P^d_t} \frac{\tilde{\pi}^w_{t+s} \ldots \tilde{\pi}^w_{t+1}}{\bar{w}_{t+s}\mu_{z^+,t+1} \ldots \mu_{z^+,t+s}\pi^d_{t+1} \ldots \pi^d_{t+s}} = \frac{\bar{w}_t}{\bar{w}_{t+s}} \frac{\tilde{\pi}^w_{t+s} \ldots \tilde{\pi}^w_{t+1}}{\mu_{z^+,t+1} \ldots \mu_{z^+,t+s}\pi^d_{t+1} \ldots \pi^d_{t+s}}.$$

The expression for the optimal wage in terms of stationary variables is then given by

$$\tilde{w}_t^{\frac{1-\lambda^w_{t+s}(1+\varphi)}{1-\lambda^w_{t+s}}} = \frac{\lambda^w_{t+s} E_t \sum_{s=0}^{\infty} (\beta\xi_w)^s \zeta^\beta_{t+s}\zeta^n_{t+s}\Theta_{t+s} \left[\left(\frac{\bar{w}_t}{\bar{w}_{t+s}} \frac{\tilde{\pi}^w_{t+s}\cdots\tilde{\pi}^w_{t+1}}{\mu_{z^+,t+1}\cdots\mu_{z^+,t+s}\pi^d_{t+1}\cdots\pi^d_{t+s}}\right)^{\frac{\lambda^w_{t+s}}{1-\lambda^w_{t+s}}} N_{t+s}\right]^{1+\varphi}}{E_t \sum_{s=0}^{\infty} (\beta\xi_w)^s \zeta^\beta_{t+s}\psi_{z^+,t+s}\bar{w}_{t+s} N_{t+s} \left(\frac{\bar{w}_t}{\bar{w}_{t+s}} \frac{\tilde{\pi}^w_{t+s}\cdots\tilde{\pi}^w_{t+1}}{\mu_{z^+,t+1}\cdots\mu_{z^+,t+s}\pi^d_{t+1}\cdots\pi^d_{t+s}}\right)^{\frac{1}{1-\lambda^w_{t+s}}}}. \tag{4.79}$$

4.8 Log-linearization of the household equations

4.8.1 Log-linearization of the preference shifters and MRS

Log-linearizing the endogenous preference shifter Θ_t in (4.52) gives

$$\hat{\Theta}_t = \hat{z}_t^C + \widehat{\tilde{v}}_t^N. \tag{4.80}$$

Log-linearizing next the smooth trend for the the marginal utility of consumption in equation (4.53), we get[31]

$$\hat{z}_t^C = (1-\nu)\left(\hat{z}_{t-1}^C - \hat{\mu}_{z^+,t}\right) - \nu\widehat{\tilde{v}}_t^N. \tag{4.81}$$

Log-linearizing (4.56), we get

$$\widehat{mrs}_{j,t} = \hat{\zeta}_t^\beta + \hat{\zeta}_t^n + \hat{\Theta}_t + \varphi\hat{N}_{j,t} - \widehat{\tilde{v}}_t^N. \tag{4.82}$$

Integrating over all labour types, we can write the average marginal rate of substitution

$$\widehat{mrs}_t = \int_0^1 \widehat{mrs}_{j,t}dj$$

as follows:

$$\widehat{mrs}_t = \hat{\zeta}_t^\beta + \hat{\zeta}_t^n + \hat{\Theta}_t + \varphi\hat{N}_t - \widehat{\tilde{v}}_t^N, \tag{4.83}$$

where $\hat{N}_t$ denotes the aggregate employment, as given by

$$\hat{N}_t = \int_0^1 \hat{N}_{j,t}dj.$$

4.8.2 Log-linearization of the household's first-order conditions

We start by log-linearizing the UIP condition (4.66), yielding

$$\hat{R}_t = \hat{R}_t^* + \hat{\Phi}_t + E_t\hat{s}_{t+1}. \tag{4.84}$$

We next derive an expression for $\hat{\Phi}_t$, starting from equation (4.19), stated here again for convenience:

$$\Phi_t = \Phi\left(\bar{a}_t, E_t s_{t+1}s_t, \tilde{\phi}_t\right) = \exp\left(-\tilde{\phi}_a\left(\bar{a}_t - \bar{a}\right) - \tilde{\phi}_s\left(E_t s_{t+1}s_t - s^2\right) + \tilde{\phi}_t\right).$$

We linearize the above expression to obtain

$$\hat{\Phi}_t = -\tilde{\phi}_a\breve{a}_t - \tilde{\phi}_s\left(E_t\hat{s}_{t+1} + \hat{s}_t\right) + \widehat{\tilde{\phi}}_t, \tag{4.85}$$

where we have defined $\breve{a}_t \equiv \bar{a}_t - \bar{a}$, since we assume that $\bar{a}$ takes the value of zero in steady state and therefore must use level deviations for deviations of $\bar{a}_t$ from steady state, and used that $s = S/S = 1$ and $\tilde{\phi} = 1$ in steady state. Combining with equation (4.84), we arrive att the following modified uncovered interest rate parity condition:

$$\hat{R}_t = \hat{R}_t^* - \tilde{\phi}_a\breve{a}_t + \left(1 - \tilde{\phi}_s\right)E_t\hat{s}_{t+1} - \tilde{\phi}_s\hat{s}_t + \widehat{\tilde{\phi}}_t. \tag{4.86}$$

[31] If we assume $\nu = 1$ we have

$$\hat{z}_t = -\widehat{\tilde{v}}_t^N$$

and

$$\hat{\Theta}_t = 0,$$

thus obtaining standard KPR (King, Plosser, and Rebelo (1988)) preferences. We note that in estimations, we have typically had ν estimated closer to 1.

Next, we log-linearize the consumption Euler equation. We start from expression (4.57), stated here again for convenience:

$$\frac{\zeta_t^\beta \zeta_t^c}{c_t - bc_{t-1}\frac{1}{\mu_{z^+,t}}} - \beta b E_t \frac{\zeta_{t+1}^\beta \zeta_{t+1}^c}{c_{t+1}\mu_{z^+,t+1} - bc_t} = \zeta_t^\beta \psi_{z^+,t} p_t^c.$$

Using that $\zeta^\beta = \zeta^c = 1$ in steady state, we have the following log-linearized expression of the left-hand side:

$$\begin{aligned}
&\frac{\mu_{z^+}}{c\left(\mu_{z^+} - b\right)}\hat{\zeta}_t^\beta + \frac{\mu_{z^+}}{c\left(\mu_{z^+} - b\right)}\hat{\zeta}_t^c \\
&-\frac{\mu_{z^+}^2}{c^2\left(\mu_{z^+} - b\right)^2}c\hat{c}_t + \frac{b\mu_{z^+}}{c^2\left(\mu_{z^+} - b\right)^2}c\hat{c}_{t-1} - \frac{b}{c\left(\mu_{z^+} - b\right)^2}\mu_{z^+}\hat{\mu}_{z^+,t} \\
&-\beta b\frac{1}{c\left(\mu_{z^+} - b\right)}E_t\hat{\zeta}_{t+1}^\beta - \beta b\frac{1}{c\left(\mu_{z^+} - b\right)}E_t\hat{\zeta}_{t+1}^c \\
&-\beta b\left(-\frac{\mu_{z^+}}{c^2\left(\mu_{z^+} - b\right)^2}\right)cE_t\hat{c}_{t+1} \\
&-\beta b\frac{b}{c^2\left(\mu_{z^+} - b\right)^2}c\hat{c}_t - \beta b\left(-\frac{1}{c\left(\mu_{z^+} - b\right)^2}\right)\mu_{z^+}E_t\hat{\mu}_{z^+,t+1}.
\end{aligned}$$

Multiplying everywhere by $c\left(\mu_{z^+} - b\right)^2$ and simplifying, we have

$$\begin{aligned}
&\mu_{z^+}\left(\mu_{z^+} - b\right)\hat{\zeta}_t^\beta + \mu_{z^+}\left(\mu_{z^+} - b\right)\hat{\zeta}_t^c - \mu_{z^+}^2\hat{c}_t + b\mu_{z^+}\hat{c}_{t-1} - b\mu_{z^+}\hat{\mu}_{z^+,t} \\
&-\beta b\left(\mu_{z^+} - b\right)E_t\hat{\zeta}_{t+1}^\beta - \beta b\left(\mu_{z^+} - b\right)E_t\hat{\zeta}_{t+1}^c \\
&+\beta b\mu_{z^+}E_t\hat{c}_{t+1} - \beta bb\hat{c}_t + \beta b\mu_{z^+}E_t\hat{\mu}_{z^+,t+1}.
\end{aligned}$$

Equalizing this to the right-hand side again (where we have again used that $\zeta^\beta = 1$), multiplied by $c\left(\mu_{z^+} - b\right)^2$, we finally have

$$\begin{aligned}
&\mu_{z^+}\left(\mu_{z^+} - b\right)\hat{\zeta}_t^\beta + \mu_{z^+}\left(\mu_{z^+} - b\right)\hat{\zeta}_t^c - \mu_{z^+}^2\hat{c}_t + b\mu_{z^+}\hat{c}_{t-1} - b\mu_{z^+}\hat{\mu}_{z^+,t} \\
&-\beta b\left[\left(\mu_{z^+} - b\right)E_t\hat{\zeta}_{t+1}^\beta + \left(\mu_{z^+} - b\right)E_t\hat{\zeta}_{t+1}^c - \mu_{z^+}E_t\hat{c}_{t+1} + b\hat{c}_t - \mu_{z^+}E_t\hat{\mu}_{z^+,t+1}\right] \\
= \; & c\left(\mu_{z^+} - b\right)^2\psi_{z^+}p^c\left(\hat{\zeta}_t^\beta + \hat{\psi}_{z^+,t} + \hat{p}_t^c\right). \qquad (4.87)
\end{aligned}$$

Following the calculations above, we can also log-linearize (4.54) to obtain

$$\begin{aligned}
\hat{\bar{v}}_t^N = \; & \mu_{z^+}\left(\mu_{z^+} - b\right)\hat{\zeta}_t^\beta + \mu_{z^+}\left(\mu_{z^+} - b\right)\hat{\zeta}_t^c - \mu_{z^+}^2\hat{c}_t + b\mu_{z^+}\hat{c}_{t-1} - b\mu_{z^+}\hat{\mu}_{z^+,t} \qquad (4.88) \\
&-\beta b\left[\left(\mu_{z^+} - b\right)E_t\hat{\zeta}_{t+1}^\beta + \left(\mu_{z^+} - b\right)E_t\hat{\zeta}_{t+1}^c - \mu_{z^+}E_t\hat{c}_{t+1} + b\hat{c}_t - \mu_{z^+}E_t\hat{\mu}_{z^+,t+1}\right].
\end{aligned}$$

Log-linearization of (4.68) gives

$$\hat{\bar{v}}_t^N = \hat{R}_t + \hat{\chi}_t - E_t\hat{\pi}_{t+1}^c - E_t\hat{\mu}_{z^+,t+1} + E_t\hat{\bar{v}}_{t+1}^N. \qquad (4.89)$$

We note also that the log-linearization of equation (4.64) yields the following log-linear expression for $\bar{v}_t^N$:

$$\hat{\bar{v}}_t^N = \hat{\zeta}_t^\beta + \hat{\psi}_{z^+,t} + \hat{p}_t^c. \qquad (4.90)$$

To complete the log-linearization of the household's consumption decision, we need also to derive an expression for $\hat{\psi}_{z^+,t}$. Log-linearizing equation (4.58), we get

$$\hat{\psi}_{z^+,t} = E_t\hat{\psi}_{z^+,t+1} + E_t\hat{\zeta}_{t+1}^\beta - \hat{\zeta}_t^\beta + \hat{R}_t + \hat{\chi}_t - E_t\hat{\mu}_{z^+,t+1} - E_t\hat{\pi}_{t+1}^d. \qquad (4.91)$$

We note that, in the abscence of habits (i.e. setting $b = 0$), shocks and growth, equation (4.87) boils down to

$$-\frac{1}{cp^c\psi_{z^+}}\hat{c}_t - \hat{p}_t^c = \hat{\psi}_{z^+,t}.$$

Inserting into equation (4.91) and rearranging

$$\begin{aligned} E_t\left(-\frac{1}{cp^c\psi_{z^+}}\hat{c}_{t+1}\right) &= -\frac{1}{cp^c\psi_{z^+}}\hat{c}_t + \left(E_t\hat{p}_{t+1}^c - \hat{p}_t^c\right) - \left(\hat{R}_t - E_t\hat{\pi}_{t+1}^d\right) \\ &= -\frac{1}{cp^c\psi_{z^+}}\hat{c}_t + E_t\left(\hat{\pi}_{t+1}^c - \hat{\pi}_{t+1}^d\right) - \left(\hat{R}_t - E_t\hat{\pi}_{t+1}^d\right) \\ &= -\frac{1}{cp^c\psi_{z^+}}\hat{c}_t - \left(\hat{R}_t - E_t\hat{\pi}_{t+1}^c\right), \\ \hat{c}_t &= E_t\hat{c}_{t+1} - \frac{1}{cp^c\psi_{z^+}}\left(\hat{R}_t - E_t\hat{\pi}_{t+1}^c\right), \end{aligned}$$

and using that under these assumptions $\psi_{z^+} = \frac{1}{cp^c}$ holds in steady state, we arrive at the standard textbook consumption Euler equation given by

$$\hat{c}_t = E_t\hat{c}_{t+1} - \left(\hat{R}_t - E_t\hat{\pi}_{t+1}^c\right).$$

Turning next to the household's optimization with respect to capital, we can log-linearize equation (4.69),

$$E_t\left[\bar{r}_{t+1}^k u_{t+1} - p_{t+1}^i a\left(u_{t+1}\right) + (1-\delta)\breve{p}_{k',t+1}\right] = \frac{1}{\beta}E_t\frac{\zeta_t^\beta}{\zeta_{t+1}^\beta}\frac{\psi_{z^+,t}}{\psi_{z^+,t+1}}\mu_{z^+,t+1}\mu_{\Psi,t+1}\breve{p}_{k',t},$$

to obtain

$$\begin{aligned} &\bar{r}^k u E_t\widehat{\bar{r}}_{t+1}^k + \bar{r}^k u E_t\hat{u}_{t+1} - p^i a\left(u\right)E_t\hat{p}_{t+1}^i - p^i a\left(u\right)E_t\widehat{a\left(u_{t+1}\right)} + (1-\delta)\breve{p}_{k'}\widehat{\breve{p}}_{k',t+1} \\ =\ &\frac{1}{\beta}\mu_{z^+}\mu_\Psi\breve{p}_{k'}E_t\left[\hat{\zeta}_t^\beta - \hat{\zeta}_{t+1}^\beta + \hat{\psi}_{z^+,t} - \hat{\psi}_{z^+,t+1} + \hat{\mu}_{z^+,t+1} + \hat{\mu}_{\Psi,t+1} + \widehat{\breve{p}}_{k',t}\right] \end{aligned}$$

We note that $u = 1$ and $a(u) = 0$ in steady state, and so the above expression simplifies to

$$\begin{aligned} &\bar{r}^k E_t\left(\widehat{\bar{r}}_{t+1}^k + \hat{u}_{t+1}\right) + (1-\delta)\breve{p}_{k'}\widehat{\breve{p}}_{k',t+1} \\ =\ &\frac{1}{\beta}\mu_{z^+}\mu_\Psi\breve{p}_{k'}E_t\left[\hat{\zeta}_t^\beta - \hat{\zeta}_{t+1}^\beta + \hat{\psi}_{z^+,t} - \hat{\psi}_{z^+,t+1} + \hat{\mu}_{z^+,t+1} + \hat{\mu}_{\Psi,t+1} + \widehat{\breve{p}}_{k',t}\right]. \end{aligned}$$

Rearranging and substituing in

$$\hat{\psi}_{z^+,t} - E_t\hat{\psi}_{z^+,t+1} = E_t\hat{\zeta}_{t+1}^\beta - \hat{\zeta}_t^\beta - E_t\hat{\mu}_{z^+,t+1} + \left(\hat{R}_t - E_t\hat{\pi}_{t+1}^d + \hat{\chi}_t\right),$$

from equation (4.91) or, alternatively, log-linearizing equation (4.70), we get the following expression for the real value of capital:

$$\begin{aligned} \widehat{\breve{p}}_{k',t} &= \frac{\beta(1-\delta)}{\mu_{z^+}\mu_\Psi}\widehat{\breve{p}}_{k',t+1} + \frac{\beta\bar{r}^k}{\mu_{z^+}\mu_\Psi\breve{p}_{k'}}E_t\left(\widehat{\bar{r}}_{t+1}^k + \hat{u}_{t+1}\right) \\ &\quad - E_t\hat{\mu}_{\Psi,t+1} - E_t\left(\hat{R}_t - E_t\hat{\pi}_{t+1}^d + \hat{\chi}_t\right). \end{aligned} \tag{4.92}$$

We next consider the investment equation (4.71). Combining with expressions (4.74) and (4.75), we have

$$\begin{aligned} p_t^i &= \breve{p}_{k',t}\Upsilon_t\left[1 - \tilde{S}\left(\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}}\right) - \tilde{S}'\left(\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}}\right)\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}}\right] \\ &\quad + \beta E_t\frac{\zeta_{t+1}^\beta}{\zeta_t^\beta}\frac{\psi_{z^+,t+1}}{\psi_{z^+,t}}\breve{p}_{k',t+1}\Upsilon_{t+1}\tilde{S}'\left(\frac{\mu_{z^+,t+1}\mu_{\Psi,t+1}i_{t+1}}{i_t}\right)\left(\frac{i_{t+1}}{i_t}\right)^2\mu_{z^+,t+1}\mu_{\Psi,t+1}. \end{aligned}$$

Defining

$$\begin{aligned} d_{1,t} &\equiv \tilde{S}\left(\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}}\right), \\ d_{2,t} &\equiv \tilde{S}'\left(\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}}\right), \\ d_{3,t} &\equiv \left[1-d_{1,t}-d_{2,t}\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}}\right], \end{aligned}$$

we can rewrite the above expression as

$$p_t^i = \breve{p}_{k',t}\Upsilon_t d_{3,t} + \beta E_t \frac{\zeta_{t+1}^\beta}{\zeta_t^\beta}\frac{\psi_{z^+,t+1}}{\psi_{z^+,t}}\breve{p}_{k',t+1}\Upsilon_{t+1}d_{2,t+1}\left(\frac{i_{t+1}}{i_t}\right)^2\mu_{z^+,t+1}\mu_{\Psi,t+1}.$$

Log-linearizing yields

$$p^i\hat{p}_t^i = \breve{p}_{k'}\Upsilon\widehat{\breve{p}}_{k',t} + \breve{p}_{k'}\Upsilon\hat{\Upsilon}_t + \breve{p}_{k'}\Upsilon\hat{d}_{3,t} + \beta\breve{p}_{k'}\Upsilon\mu_{z^+}\mu_\Psi\breve{d}_{2,t+1},$$

where $\breve{d}_2 = d_{2,t} - d_2$, and where we have used that $d_1 = d_2 = 0$ and $d_3 = 1$ hold in steady state. Rearranging, we obtain

$$\frac{p^i}{\breve{p}_{k'}\Upsilon}\hat{p}_t^i = \widehat{\breve{p}}_{k',t} + \hat{\Upsilon}_t + \hat{d}_{3,t} + \beta\mu_{z^+}\mu_\Psi\breve{d}_{2,t+1}.$$

We next derive $\hat{d}_{3,t}$, which gives

$$\hat{d}_{3,t} = -\breve{d}_{1,t} - \mu_{z^+}\mu_\Psi\breve{d}_{2,t},$$

where $\breve{d}_{1,t} = d_{1,t} - d_1$ and $\breve{d}_{2,t} = d_{2,t} - d_2$. We then have that

$$\frac{p^i}{\breve{p}_{k'}\Upsilon}\hat{p}_t^i = \widehat{\breve{p}}_{k',t} + \hat{\Upsilon}_t - \breve{d}_{1,t} - \mu_{z^+}\mu_\Psi\breve{d}_{2,t} + \beta\mu_{z^+}\mu_\Psi\breve{d}_{2,t+1}.$$

We now consider $\breve{d}_1$ and $\breve{d}_2$. For d_1, we have

$$\begin{aligned} \breve{d}_{1,t} &= d_{1,t} - d_1 \\ &= \tilde{S}'\mu_{z^+}\mu_\Psi\hat{\mu}_{z^+,t} + \tilde{S}'\mu_{z^+}\mu_\Psi\hat{\mu}_{\Psi,t} + \tilde{S}'\frac{\mu_{z^+}\mu_\Psi}{i}i\hat{i}_t - \tilde{S}'\frac{\mu_{z^+}\mu_\Psi i}{i^2}i\hat{i}_{t-1} = 0, \end{aligned}$$

since $S' = S'(\mu_{z^+}\mu_\Psi) = 0$ in steady state by assumption, as discussed in Section 4.3. For d_2, we have

$$\begin{aligned} \breve{d}_{2,t} &= d_{2,t} - d_2 \\ &= \tilde{S}''\mu_{z^+}\mu_\Psi\hat{\mu}_{z^+,t} + \tilde{S}''\mu_{z^+}\mu_\Psi\hat{\mu}_{\Psi,t} + \tilde{S}''\frac{\mu_{z^+}\mu_\Psi}{i}i\hat{i}_t - \tilde{S}''\frac{\mu_{z^+}\mu_\Psi i}{i^2}i\hat{i}_{t-1} \\ &= \tilde{S}''\mu_{z^+}\mu_\Psi\left[\hat{\mu}_{z^+,t} + \hat{\mu}_{\Psi,t} + \hat{i}_t - \hat{i}_{t-1}\right], \end{aligned}$$

where we have used that

$$\begin{aligned} \tilde{S}''(x) &= \frac{1}{2}\tilde{S}''\left\{\exp\left[\sqrt{\tilde{S}''}(x-\mu_{z^+}\mu_\Psi)\right] + \exp\left[-\sqrt{\tilde{S}''}(x-\mu_{z^+}\mu_\Psi)\right]\right\} \\ &= S'',\ x = \mu_{z^+}\mu_\Psi, \end{aligned}$$

as derived in Section 4.3. We then have that

$$\frac{p^i}{\breve{p}_{k'}\Upsilon}\hat{p}_t^i = \widehat{\breve{p}}_{k',t} + \hat{\Upsilon}_t - (\mu_{z^+}\mu_\Psi)^2\tilde{S}''\left[\hat{i}_t - \hat{i}_{t-1} + \hat{\mu}_{z^+,t} + \hat{\mu}_{\Psi,t} - \beta E_t\left(\hat{i}_{t+1} - \hat{i}_t + \hat{\mu}_{z^+,t+1} + \hat{\mu}_{\Psi,t+1}\right)\right].$$

Rearranging, we finally obtain

$$\widehat{i}_t - \widehat{i}_{t-1} + \hat{\mu}_{z^+,t} + \hat{\mu}_{\Psi,t} = \beta E_t \left(\widehat{i}_{t+1} - \widehat{i}_t + \hat{\mu}_{z^+,t+1} + \hat{\mu}_{\Psi,t+1} \right) + \frac{1}{\left(\mu_{z^+}\mu_\Psi\right)^2 \tilde{S}''} \left[\widehat{\breve{p}}_{k',t} + \hat{\Upsilon}_t - \frac{p^i}{\breve{p}_{k'}\Upsilon} \hat{p}^i_t \right]. \tag{4.93}$$

Finally, log-linearizing equation (4.73), we have

$$\widehat{\bar{r}}^k_t = \hat{p}^i_t + \widehat{a'(u_t)}. \tag{4.94}$$

Remembering from Section 4.2.1 that

$$a'(u_t) = \sigma_b \sigma_a u_t + \sigma_b (1 - \sigma_a),$$

which in stedy state (where $u = 1$) yields

$$\begin{aligned} a'(u) &= \sigma_b \sigma_a u + \sigma_b - \sigma_b \sigma_a \\ &= \sigma_b \sigma_a + \sigma_b - \sigma_b \sigma_a \\ &= \sigma_b, \end{aligned}$$

we can log-linearize to obtain

$$\begin{aligned} a'(u)\, \widehat{a'(u_t)} &= \sigma_b \sigma_a u \hat{u}_t \\ \widehat{a'(u_t)} &= \sigma_a \hat{u}_t. \end{aligned}$$

Inserting into equation (4.94) gives

$$\widehat{\bar{r}}^k_t = \hat{p}^i_t + \sigma_a \hat{u}_t. \tag{4.95}$$

4.8.3 Log-linearization of the law of motion for capital

We log-linearize equation (4.76) to obtain

$$k^p \hat{k}^p_{t+1} = (1 - \delta) \frac{k^p}{\mu_{z^+}\mu_\Psi} \left[\hat{k}^p_t - \hat{\mu}_{z^+,t} - \hat{\mu}_{\Psi,t} \right] + \Upsilon i \left(\hat{\Upsilon}_t + \widehat{i}_t \right),$$

where we have used the steady-state relationship $\tilde{S}(\mu_{z^+}\mu_\Psi) = 0$. Rearranging, we get

$$\hat{k}^p_{t+1} = \frac{1 - \delta}{\mu_{z^+}\mu_\Psi} \left(\hat{k}^p_t - \hat{\mu}_{z^+,t} - \hat{\mu}_{\Psi,t} \right) + \frac{\Upsilon i}{k^p} \left(\hat{\Upsilon}_t + \widehat{i}_t \right). \tag{4.96}$$

We next log-linearize equation (4.77) to obtain

$$\hat{k}_t = \hat{u}_t + \hat{k}^p_t. \tag{4.97}$$

4.8.4 Log-linearization of unemployment and labour supply

As in Galí, Smets, and Wouters (2012), we define

$$\hat{L}_t = \int_0^1 \hat{L}_{j,t} dj. \tag{4.98}$$

Moreover, the unemployment rate is given by

$$\hat{U}_t = \hat{L}_t - \hat{N}_t, \tag{4.99}$$

where $\hat{L}_t$ and $\hat{N}_t$ are log deviations from steady state while $\hat{U}_t$ is the deviation of the unemployment rate from its steady state.[32] We also note that equation (4.43) implies that

$$\begin{aligned}
W_t^{\frac{1}{1-\lambda_t^w}} &= \int_0^1 (W_{j,t})^{\frac{1}{1-\lambda_t^w}} dj \\
W_t^{\frac{1}{1-\lambda_t^w}} &= e^{\frac{1}{1-\lambda_t^w}\log W_t} \\
&\quad \frac{1}{1-\lambda^w}\frac{W^{\frac{1}{1-\lambda^w}}}{W} W\hat{W}_t + W^{\frac{1}{1-\lambda^w}}\log W \frac{1}{(1-\lambda^w)^2}\lambda^w\hat{\lambda}_t^w \\
&= \int_0^1 \left(\frac{1}{1-\lambda^w}\frac{W_j^{\frac{1}{1-\lambda^w}}}{W_j} W_j\hat{W}_{j,t} + W_j^{\frac{1}{1-\lambda^w}}\log W_j \frac{1}{(1-\lambda^w)^2}\lambda^w\hat{\lambda}_t^w\right) dj \\
\frac{1}{1-\lambda^w}\hat{W}_t &= \int_0^1 \frac{1}{1-\lambda^w}\hat{W}_{j,t}dj \\
\hat{W}_t &= \int_0^1 \hat{W}_{j,t}dj,
\end{aligned}$$

where we have used that all unions (denoted by j) choose the same wage in steady state. We can then log-linearize (4.78) to obtain

$$\hat{w}_t - \hat{p}_t^c + \left(\int_0^1 \hat{W}_{j,t}dj - \hat{W}_t\right) = \hat{\zeta}_t^\beta + \hat{\zeta}_t^n + \hat{z}_t^C + \varphi\int_0^1 \hat{L}_{j,t}dj.$$

Using expression (4.98) above, we obtain the following labour market participation equation:

$$\widehat{\bar{w}}_t - \hat{p}_t^c = \hat{\zeta}_t^n + \hat{\zeta}_t^\beta + \hat{z}_t^C + \varphi\hat{L}_t. \tag{4.100}$$

Following Galí, Smets, and Wouters (2012), we define the average wage markup as the (log) deviation between the average real wage and the average marginal rate of substitution, i.e.

$$\hat{\mu}_{w,t} \equiv \left(\widehat{\bar{w}}_t - \hat{p}_t^c\right) - \widehat{mrs}_t.$$

Combining with equations (4.100) and (4.83), we get

$$\begin{aligned}
\hat{\mu}_{w,t} &\equiv \left(\hat{\zeta}_t^n + \hat{\zeta}_t^\beta + \hat{z}_t^C + \varphi\hat{L}_t\right) - \left(\hat{\zeta}_t^\beta + \hat{\zeta}_t^n + \hat{\Theta}_t + \varphi\hat{N}_t - \widehat{v}_t^N\right) \\
&= \hat{z}_t^C - \hat{\Theta}_t + \widehat{v}_t^N + \varphi\left(\hat{L}_t - \hat{N}_t\right).
\end{aligned}$$

Using (4.80), this reduces to the following simple relationship:

$$\hat{\mu}_{w,t} = \varphi\left(\hat{L}_t - \hat{N}_t\right) = \varphi\hat{U}_t. \tag{4.101}$$

4.8.5 Log-linearization of the household's wage setting

To log-linearize equation (4.79), we start by re-expressing it as follows:

$$\begin{aligned}
&\exp\left(\frac{1-\lambda_{t+s}^w(1+\varphi)}{1-\lambda_{t+s}^w}\log\tilde{w}_t\right)\times \\
&\quad E_t\sum_{s=0}^\infty(\beta\xi_w)^s\zeta_{t+s}^\beta\psi_{z^+,t+s}\bar{w}_{t+s}N_{t+s}\exp\left(\frac{1}{1-\lambda_{t+s}^w}\log\left(\frac{\bar{w}_t}{\bar{w}_{t+s}}\frac{\tilde{\pi}_{t+s}^w\cdots\tilde{\pi}_{t+1}^w}{\mu_{z^+,t+1}\cdots\mu_{z^+,t+s}\pi_{t+1}^d\cdots\pi_{t+s}^d}\right)\right) \\
&= \lambda_{t+s}^w E_t\sum_{s=0}^\infty(\beta\xi_w)^s\zeta_{t+s}^\beta\zeta_{t+s}^n\Theta_{t+s}N_{t+s}^{1+\varphi}\exp\left(\frac{\lambda_{t+s}^w(1+\varphi)}{1-\lambda_{t+s}^w}\log\left(\frac{\bar{w}_t}{\bar{w}_{t+s}}\frac{\tilde{\pi}_{t+s}^w\cdots\tilde{\pi}_{t+1}^w}{\mu_{z^+,t+1}\cdots\mu_{z^+,t+s}\pi_{t+1}^d\cdots\pi_{t+s}^d}\right)\right).
\end{aligned}$$

[32] We particularly note that this is the deviation of the unemployment rate (in levels) from its steady state, rather than a log-deviation. Hence,

$$\hat{U}_t = U_t - U.$$

A first-order Taylor expansion of the left-hand side yields:

$$
\begin{aligned}
&\tilde{w}^{\frac{1-\lambda^w(1+\varphi)}{1-\lambda^w}} \sum_{s=0}^{\infty} (\beta\xi_w)^s \zeta^\beta \psi_{z^+} \bar{w} N \left(\frac{\tilde{\pi}^w}{\pi^d \mu_{z^+}}\right)^{\frac{s}{1-\lambda^w}} \\
&+\tilde{w}^{\frac{1-\lambda^w(1+\varphi)}{1-\lambda^w}} \sum_{s=0}^{\infty} (\beta\xi_w)^s \psi_{z^+} \bar{w} N \left(\frac{\tilde{\pi}^w}{\pi^d \mu_{z^+}}\right)^{\frac{s}{1-\lambda^w}} \left(\zeta^\beta_{t+s} - \zeta^\beta\right) \\
&+\tilde{w}^{\frac{1-\lambda^w(1+\varphi)}{1-\lambda^w}} E_t \sum_{s=0}^{\infty} (\beta\xi_w)^s \zeta^\beta \bar{w} N \left(\frac{\tilde{\pi}^w}{\pi^d \mu_{z^+}}\right)^{\frac{s}{1-\lambda^w}} \left(\psi_{z^+,t+s} - \psi_{z^+}\right) \\
&+\tilde{w}^{\frac{1-\lambda^w(1+\varphi)}{1-\lambda^w}} E_t \sum_{s=0}^{\infty} (\beta\xi_w)^s \zeta^\beta \psi_{z^+} \bar{w} \left(\frac{\tilde{\pi}^w}{\pi^d \mu_{z^+}}\right)^{\frac{s}{1-\lambda^w}} \left(N_{t+s} - N\right) \\
&+\frac{1-\lambda^w(1+\varphi)}{1-\lambda^w} \tilde{w}^{\frac{-\lambda^w \varphi}{1-\lambda^w}} E_t \sum_{s=0}^{\infty} (\beta\xi_w)^s \zeta^\beta \psi_{z^+} \bar{w} N \left(\frac{\tilde{\pi}^w}{\pi^d \mu_{z^+}}\right)^{\frac{s}{1-\lambda^w}} \left(\tilde{w}_t - \tilde{w}\right) \\
&-\tilde{w}^{\frac{1-\lambda^w(1+\varphi)}{1-\lambda^w}} E_t \sum_{s=0}^{\infty} (\beta\xi_w)^s \zeta^\beta \psi_{z^+} \bar{w} N \left(\frac{\tilde{\pi}^w}{\pi^d \mu_{z^+}}\right)^{\frac{s}{1-\lambda^w}} \left[\frac{1}{\bar{w}} \frac{\lambda^w}{1-\lambda^w}\right] \left(\bar{w}_{t+s} - \bar{w}\right) \\
&+\tilde{w}^{\frac{1-\lambda^w(1+\varphi)}{1-\lambda^w}} E_t \sum_{s=0}^{\infty} (\beta\xi_w)^s \zeta^\beta \psi_{z^+} \bar{w} N \left(\frac{\tilde{\pi}^w}{\pi^d \mu_{z^+}}\right)^{\frac{s}{1-\lambda^w}} \left[\frac{1}{\bar{w}} \frac{1}{1-\lambda^w}\right] \left(\bar{w}_t - \bar{w}\right) \\
&+\tilde{w}^{\frac{1-\lambda^w(1+\varphi)}{1-\lambda^w}} E_t \sum_{s=0}^{\infty} (\beta\xi_w)^s \zeta^\beta \psi_{z^+} \bar{w} N \left(\frac{\tilde{\pi}^w}{\pi^d \mu_{z^+}}\right)^{\frac{s}{1-\lambda^w}} \times \\
&\times \left[\begin{array}{c} -\frac{1}{(1-\lambda^w)^2} \times \\ \left(\begin{array}{c} \log\left(\frac{\tilde{\pi}^w}{\pi^d \mu_{z^+}}\right)^s \\ +\left(1-\lambda^w(1+\varphi)\right)\log \tilde{w} \end{array}\right) \\ -(1+\varphi)\log \tilde{w} \frac{1}{1-\lambda^w} \end{array}\right] \left(\lambda^w_{t+s} - \lambda^w\right) \\
&+\tilde{w}^{\frac{1-\lambda^w(1+\varphi)}{1-\lambda^w}} E_t \sum_{s=0}^{\infty} (\beta\xi_w)^s \zeta^\beta \psi_{z^+} \bar{w} N \left(\frac{\tilde{\pi}^w}{\pi^d \mu_{z^+}}\right)^{\frac{s}{1-\lambda^w}} \frac{1}{1-\lambda^w} \times \\
&\times \left[\begin{array}{c} \frac{1}{\tilde{\pi}^w}\left(\tilde{\pi}^w_{t+1} + \ldots + \tilde{\pi}^w_{t+s} - s\tilde{\pi}^w\right) \\ -\frac{1}{\pi^d}\left(\pi^d_{t+1} + \ldots + \pi^d_{t+s} - s\pi^d\right) \\ -\frac{1}{\mu_{z^+}}\left(\mu_{z^+,t+1} + \ldots + \mu_{z^+,t+s} - s\mu_{z^+}\right) \end{array}\right].
\end{aligned}
$$

A first-order Taylor expansion of the right-hand side yields:

$$\sum_{s=0}^{\infty} (\beta\xi_w)^s \zeta^\beta \zeta^n \Theta \lambda^w N^{1+\varphi} \left(\frac{\tilde{\pi}^w}{\pi^d \mu_{z^+}}\right)^{s\frac{\lambda^w(1+\varphi)}{1-\lambda^w}}$$

$$+E_t \sum_{s=0}^{\infty} (\beta\xi_w)^s \zeta^\beta \Theta \lambda^w N^{1+\varphi} \left(\frac{\tilde{\pi}^w}{\pi^d \mu_{z^+}}\right)^{s\frac{\lambda^w(1+\varphi)}{1-\lambda^w}} \left(\zeta^n_{t+s} - \zeta^n\right)$$

$$+E_t \sum_{s=0}^{\infty} (\beta\xi_w)^s \zeta^n \Theta \lambda^w N^{1+\varphi} \left(\frac{\tilde{\pi}^w}{\pi^d \mu_{z^+}}\right)^{s\frac{\lambda^w(1+\varphi)}{1-\lambda^w}} \left(\zeta^\beta_{t+s} - \zeta^\beta\right)$$

$$+E_t \sum_{s=0}^{\infty} (\beta\xi_w)^s \zeta^\beta \zeta^n \lambda^w N^{1+\varphi} \left(\frac{\tilde{\pi}^w}{\pi^d \mu_{z^+}}\right)^{s\frac{\lambda^w(1+\varphi)}{1-\lambda^w}} \left(\Theta_{t+s} - \Theta\right)$$

$$+E_t \sum_{s=0}^{\infty} (\beta\xi_w)^s \zeta^\beta \zeta^n \Theta \lambda^w (1+\varphi) N^{\varphi} \left(\frac{\tilde{\pi}^w}{\pi^d \mu_{z^+}}\right)^{s\frac{\lambda^w(1+\varphi)}{1-\lambda^w}} \left(N_{t+s} - N\right)$$

$$+E_t \sum_{s=0}^{\infty} (\beta\xi_w)^s \zeta^\beta \zeta^n \Theta N^{1+\varphi} \left(\frac{\tilde{\pi}^w}{\pi^d \mu_{z^+}}\right)^{s\frac{\lambda^w(1+\varphi)}{1-\lambda^w}} \times$$

$$\times \left[\begin{array}{c} 1 + \frac{1+\varphi}{(1-\lambda^w)^2} \times \\ \log\left(\frac{\tilde{\pi}^w}{\pi^d \mu_{z^+}}\right)^s \left(\frac{\tilde{\pi}^w}{\pi^d \mu_{z^+}}\right)^{s\frac{\lambda^w(1+\varphi)}{1-\lambda^w}} \end{array} \right] \left(\lambda^w_{t+s} - \lambda^w\right)$$

$$+E_t \sum_{s=0}^{\infty} (\beta\xi_w)^s \zeta^\beta \zeta^n \Theta \lambda^w N^{1+\varphi} \left(\frac{\tilde{\pi}^w}{\pi^d \mu_{z^+}}\right)^{s\frac{\lambda^w(1+\varphi)}{1-\lambda^w}} \frac{\lambda^w(1+\varphi)}{1-\lambda^w} \times$$

$$\times \left[\begin{array}{c} \frac{1}{\tilde{\pi}^w}\left(\tilde{\pi}^w_{t+1} + \ldots + \tilde{\pi}^w_{t+s}\right) \\ -\frac{1}{\pi^d}\left(\pi^d_{t+1} + \ldots + \pi^d_{t+s} - s\pi^d\right) \\ -\frac{1}{\mu_{z^+}}\left(\mu_{z^+,t+1} + \ldots + \mu_{z^+,t+s} - s\mu_{z^+}\right) \end{array} \right].$$

Now recall that $\tilde{\pi}^w_{t+1} = (\pi^c_t)^{\kappa_w} \left(\bar{\pi}^c_{t+1}\right)^{1-\kappa_w-\varkappa_w} (\breve{\pi})^{\varkappa_w} \left(\mu_{z^+}\right)^{\vartheta_w}$. We know from Section 12 that $\pi^c = \bar{\pi}^c = \pi^d$, which implies that $\tilde{\pi}^w = \left(\pi^d\right)^{1-\varkappa_w} (\breve{\pi})^{\varkappa_w} \left(\mu_{z^+}\right)^{\vartheta_w}$. Under the additional assumptions that $\varkappa_w = 0$ and $\vartheta_w = 1$, we have that

$$\tilde{\pi}^w = \pi^d \mu_{z^+}.$$

From equation (4.48), we moreover have that $\pi^w = \pi^d \mu_{z^+}$. Under the previous assumptions, we thus have that $\frac{\tilde{\pi}^w}{\pi^w_t} = 1$, which yields

$$\tilde{w} = 1.$$

Using these steady-state relationships, we equate both sides of the optimal-wage equation derived above and, after simpliflying, we obtain

$$\frac{1-\lambda^w(1+\varphi)}{1-\lambda^w}\widehat{\tilde{w}}_t\zeta^\beta\psi_{z^+}\bar{w}N\sum_{s=0}^{\infty}(\beta\xi_w)^s$$
$$+E_t\sum_{s=0}^{\infty}(\beta\xi_w)^s\zeta^\beta\psi_{z^+}\bar{w}N\times$$
$$\times\left[\hat{\zeta}^\beta_{t+s}+\hat{\psi}_{z^+,t+s}+\hat{N}_{t+s}+\frac{1}{1-\lambda^w}\left(\widehat{\tilde{w}}_t-\lambda^w\widehat{\bar{w}}_{t+s}+\widehat{\tilde{\pi}}^w_{t+1}+\ldots+\widehat{\tilde{\pi}}^w_{t+s}\right.\right.$$
$$\left.\left.-\hat{\pi}^d_{t+1}-\ldots-\hat{\pi}^d_{t+s}-\hat{\mu}_{z^+,t+1}-\ldots-\hat{\mu}_{z^+,t+s}\right)\right]$$
$$=E_t\sum_{s=0}^{\infty}(\beta\xi_w)^s\zeta^\beta\zeta^n\lambda^w\Theta N^{1+\varphi}\left[\hat{\zeta}^\beta_{t+s}+\hat{\zeta}^n_{t+s}+\hat{\Theta}_{t+s}+(1+\varphi)\hat{N}_{t+s}+\hat{\lambda}^w_{t+s}\right.$$
$$\left.+\frac{\lambda^w(1+\varphi)}{1-\lambda^w}\left(\widehat{\tilde{w}}_t-\widehat{\bar{w}}_{t+s}\widehat{\tilde{\pi}}^w_{t+1}+\ldots+\widehat{\tilde{\pi}}^w_{t+s}-\hat{\pi}^d_{t+1}-\ldots-\hat{\pi}^d_{t+s}-\hat{\mu}_{z^+,t+1}-\ldots-\hat{\mu}_{z^+,t+s}\right)\right].$$

Solving for steady state in equation (4.79) implies $\psi_{z^+}\bar{w}N=\lambda^w\Theta\zeta^nN^{1+\varphi}$. Simplifying the previous expression, we get

$$\frac{1-\lambda^w(1+\varphi)}{1-\lambda^w}\left(\widehat{\tilde{w}}_t+\widehat{\bar{w}}_t\right)$$
$$=(1-\beta\xi_w)E_t\sum_{s=0}^{\infty}(\beta\xi_w)^s\left[\hat{\zeta}^n_{t+s}+\hat{\Theta}_{t+s}+\varphi\hat{N}_{t+s}-\hat{\psi}_{z^+,t+s}+\hat{\lambda}^w_{t+s}-\frac{\lambda^w\varphi}{1-\lambda^w}\widehat{\bar{w}}_{t+s}\right.$$
$$\left..+\frac{\lambda^w(1+\varphi)-1}{1-\lambda^w}\left(\widehat{\tilde{\pi}}^w_{t+1}+\ldots+\widehat{\tilde{\pi}}^w_{t+s}-\hat{\pi}^d_{t+1}-\ldots-\hat{\pi}^d_{t+s}-\hat{\mu}_{z^+,t+1}-\ldots-\hat{\mu}_{z^+,t+s}\right)\right]$$
$$=(1-\beta\xi_w)E_t\left\{\hat{\zeta}^n_t+\hat{\Theta}_t+\varphi\hat{N}_t-\hat{\psi}_{z^+,t}+\hat{\lambda}^w_t-\frac{\lambda^w\varphi}{1-\lambda^w}\widehat{\bar{w}}_t\right. \tag{4.102}$$
$$+(\beta\xi_w)\left[\hat{\zeta}^n_{t+1}+\hat{\Theta}_{t+1}+\varphi\hat{N}_{t+1}-\hat{\psi}_{z^+,t+1}\right.$$
$$\left.+\hat{\lambda}^w_{t+1}-\frac{\lambda^w\varphi}{1-\lambda^w}\widehat{\bar{w}}_{t+1}+\frac{\lambda^w(1+\varphi)-1}{1-\lambda^w}\left(\widehat{\tilde{\pi}}^w_{t+1}-\hat{\pi}^d_{t+1}-\hat{\mu}_{z^+,t+1}\right)\right]$$
$$+(\beta\xi_w)^2\left[\hat{\zeta}^n_{t+2}+\hat{\Theta}_{t+2}+\varphi\hat{N}_{t+2}-\hat{\psi}_{z^+,t+2}+\hat{\lambda}^w_{t+2}-\frac{\lambda^w\varphi}{1-\lambda^w}\widehat{\bar{w}}_{t+2}\right.$$
$$\left.+\frac{\lambda^w(1+\varphi)-1}{1-\lambda^w}\left(\widehat{\tilde{\pi}}^w_{t+1}+\widehat{\tilde{\pi}}^w_{t+2}-\hat{\pi}^d_{t+1}-\hat{\pi}^d_{t+2}-\hat{\mu}_{z^+,t+1}-\hat{\mu}_{z^+,t+2}\right)\right]$$
$$\left.+\ldots\right\}.$$

Leading one period forward, we can rewrite expression (4.102) in the following recursive form:

$$\begin{aligned}\frac{1-\lambda^w(1+\varphi)}{1-\lambda^w}\left(\widehat{\tilde{w}}_t+\widehat{\bar{w}}_t\right) &= (1-\beta\xi_w)\left(\hat{\zeta}^n_t+\hat{\Theta}_t+\varphi\hat{N}_t-\hat{\psi}_{z^+,t}+\hat{\lambda}^w_t-\frac{\lambda^w\varphi}{1-\lambda^w}\widehat{\bar{w}}_t\right)\\ &\quad+\beta\xi_w\frac{1-\lambda^w(1+\varphi)}{1-\lambda^w}E_t\left(\widehat{\tilde{w}}_{t+1}+\widehat{\bar{w}}_{t+1}\right)\\ &\quad+(1-\beta\xi_w)\sum_{s=0}^{\infty}(\beta\xi_w)^s\frac{\lambda^w(1+\varphi)-1}{1-\lambda^w}E_t\left(\widehat{\tilde{\pi}}^w_{t+1}-\hat{\pi}^d_{t+1}-\hat{\mu}_{z^+,t+1}\right).\end{aligned} \tag{4.103}$$

Now, going back and log-linearizing the expression for $\tilde{w}_t$ in equation (4.47), that we derived from the aggregate wage index:

$$\begin{aligned}
1 &= \xi_w \left(\frac{\tilde{\pi}_t^w}{\pi_t^w}\right)^{\frac{1}{1-\lambda_t^w}} + (1-\xi_w)(\tilde{w}_t)^{\frac{1}{1-\lambda_t^w}} \\
&= \xi_w \exp\left(\frac{1}{1-\lambda_t^w}\log\frac{\tilde{\pi}_t^w}{\pi_t^w}\right) + (1-\xi_w)\exp\left(\frac{1}{1-\lambda_t^w}\log\tilde{w}_t\right) \\
&= \xi_w \left(\frac{\tilde{\pi}^w}{\pi_t^w}\right)^{\frac{1}{1-\lambda^w}} + (1-\xi_w)(\tilde{w})^{\frac{1}{1-\lambda^w}} + \xi_w \frac{-1}{(1-\lambda^w)^2}\log\frac{\tilde{\pi}^w}{\pi_t^w}\left(\frac{\tilde{\pi}^w}{\pi_t^w}\right)^{\frac{1}{1-\lambda^w}}(\lambda_t^w-\lambda^w) \\
&\quad + \xi_w \frac{1}{1-\lambda^w}\left(\frac{\tilde{\pi}^w}{\pi_t^w}\right)^{\frac{1}{1-\lambda^w}}\left[\frac{1}{\tilde{\pi}^w}(\tilde{\pi}_t^w-\tilde{\pi}^w) - \frac{1}{\pi_t^w}(\pi_t^w-\pi_t^w)\right] \\
&\quad + (1-\xi_w)\left[\frac{-1}{(1-\lambda^w)^2}\log\tilde{w}(\tilde{w})^{\frac{1}{1-\lambda^w}}(\lambda_t^w-\lambda^w) + \frac{1}{1-\lambda^w}(\tilde{w})^{\frac{1}{1-\lambda^w}}\frac{1}{\tilde{w}}(\tilde{w}_t-\tilde{w})\right].
\end{aligned}$$

Using the steady-state relationships $\tilde{w}=1$, $\frac{\tilde{\pi}^w}{\pi_t^w}=1$, and

$$1 = \xi_w \left(\frac{\tilde{\pi}^w}{\pi_t^w}\right)^{\frac{1}{1-\lambda^w}} + (1-\xi_w)(\tilde{w})^{\frac{1}{1-\lambda^w}},$$

and rearranging, we obtain,

$$\hat{\tilde{w}}_t = \frac{\xi_w}{1-\xi_w}\left(\hat{\pi}_t^w - \hat{\tilde{\pi}}_t^w\right). \tag{4.104}$$

Log-linearizing the expressions for π_t^w and $\tilde{\pi}_t^w$, under the assumption that $\varkappa_w = 0$, we get

$$\hat{\tilde{\pi}}_t^w = \kappa_w \hat{\pi}_{t-1}^c + (1-\kappa_w)\hat{\bar{\pi}}_t^c, \tag{4.105}$$

and

$$\hat{\pi}_t^w = \hat{\bar{w}}_t - \hat{\bar{w}}_{t-1} + \hat{\pi}_t^d + \hat{\mu}_{z^+,t}. \tag{4.106}$$

Substituting in expressions (4.104)–(4.106) into the recursive equation for the optimal wage, (4.103), yields

$$\begin{aligned}
&\frac{1-\lambda^w(1+\varphi)}{(1-\lambda^w)(1-\xi_w)}\left(1+\beta\xi_w^2\right)\hat{\bar{w}}_t + (1-\beta\xi_w)\frac{\lambda^w\varphi}{1-\lambda^w}\hat{\bar{w}}_t \\
&+\frac{1-\lambda^w(1+\varphi)}{(1-\lambda^w)(1-\xi_w)}\xi_w\left(-\hat{\bar{w}}_{t-1} + \hat{\pi}_t^d + \hat{\mu}_{z^+,t} - \kappa_w\hat{\pi}_{t-1}^c + (1-\kappa_w)\hat{\bar{\pi}}_t^c\right) \\
=\;& (1-\beta\xi_w)\left(\hat{\zeta}_t^n + \hat{\Theta}_t + \varphi\hat{N}_t - \hat{\psi}_{z^+,t} + \hat{\lambda}_t^w\right) \\
&+\beta\xi_w\frac{1-\lambda^w(1+\varphi)}{1-\lambda^w}E_t\left(\hat{\bar{w}}_{t+1} + \hat{\pi}_{t+1}^d + \hat{\mu}_{z^+,t+1} - \kappa_w\hat{\pi}_t^c + (1-\kappa_w)\hat{\bar{\pi}}_{t+1}^c\right).
\end{aligned}$$

Multiplying by $\frac{1-\lambda^w}{1-\beta\xi_w}$, we get

$$\begin{aligned}
&\xi_w b_w \hat{\bar{w}}_{t-1} + \left(\lambda^w\varphi - b_w\left(1+\beta\xi_w^2\right)\right)\hat{\bar{w}}_t + \beta\xi_w b_w E_t\hat{\bar{w}}_{t+1} - \xi_w b_w\left(\hat{\pi}_t^d - \hat{\bar{\pi}}_t^c\right) \\
&+\beta\xi_w b_w E_t\left(\hat{\pi}_{t+1}^d - \hat{\bar{\pi}}_{t+1}^c\right) + \kappa_w\xi_w b_w\left(\hat{\pi}_{t-1}^c - \hat{\bar{\pi}}_t^c\right) - \beta\xi_w\kappa_w b_w E_t\left(\hat{\pi}_t^c - \hat{\bar{\pi}}_{t+1}^c\right) \\
&+(1-\lambda^w)\left(\hat{\psi}_{z^+,t} - \hat{\zeta}_t^n - \hat{\Theta}_t - \varphi\hat{N}_t - \hat{\lambda}_t^w\right) - \xi_w b_w\hat{\mu}_{z^+,t} + \beta\xi_w b_w E_t\hat{\mu}_{z^+,t+1} \\
=\;& 0,
\end{aligned}$$

where

$$b_w = \frac{\lambda^w(1+\varphi)-1}{(1-\beta\xi_w)(1-\xi_w)}.$$

We can rearrange this equation in order to simplify it further. Dividing through by $\xi_w b_w$, we have

$$\begin{aligned}
& \widehat{\bar{w}}_{t-1}+\frac{\left(\lambda^{w}\varphi-b_{w}\left(1+\beta\xi_{w}^{2}\right)\right)}{\xi_{w}b_{w}}\widehat{\bar{w}}_{t}+\beta E_{t}\widehat{\bar{w}}_{t+1}-\left(\hat{\pi}_{t}^{d}-\widehat{\bar{\pi}}_{t}^{c}\right) \\
& +\beta E_{t}\left(\hat{\pi}_{t+1}^{d}-\widehat{\bar{\pi}}_{t+1}^{c}\right)+\kappa_{w}\left(\hat{\pi}_{t-1}^{c}-\widehat{\bar{\pi}}_{t}^{c}\right)-\beta\kappa_{w}E_{t}\left(\hat{\pi}_{t}^{c}-\widehat{\bar{\pi}}_{t+1}^{c}\right) \\
& +\frac{\left(1-\lambda^{w}\right)}{\xi_{w}b_{w}}\left(\hat{\psi}_{z^{+},t}-\hat{\zeta}_{t}^{n}-\hat{\Theta}_{t}-\varphi\hat{N}_{t}-\hat{\lambda}_{t}^{w}\right)-\hat{\mu}_{z^{+},t}+\beta E_{t}\hat{\mu}_{z^{+},t+1} \\
= \; & 0. \qquad (4.107)
\end{aligned}$$

We next define the parameter

$$d_{w}\equiv\frac{\left(1-\beta\xi_{w}\right)\left(1-\xi_{w}\right)}{\xi_{w}}\frac{\lambda^{w}-1}{\lambda^{w}\left(1+\varphi\right)-1}=\frac{\lambda^{w}-1}{\xi_{w}b_{w}}. \qquad (4.108)$$

Now, adding and subtracting d_w, the coefficient multiplying $\widehat{\bar{w}}_t$ above can be written as

$$\begin{aligned}
\frac{\left(b_{w}\left(1+\beta\xi_{w}^{2}\right)-\lambda^{w}\varphi\right)}{\xi_{w}b_{w}} &= d_{w}+\frac{\left(b_{w}\left(1+\beta\xi_{w}^{2}\right)-\lambda^{w}\varphi-\left(\lambda^{w}-1\right)\right)}{\xi_{w}b_{w}} \\
&= d_{w}+\frac{\left(b_{w}\left(1+\beta\xi_{w}^{2}\right)-\lambda^{w}\left(\varphi+1\right)+1\right)}{\xi_{w}b_{w}} \\
&= d_{w}+\frac{\left(b_{w}\left(1+\beta\xi_{w}^{2}\right)-b_{w}\left(1-\beta\xi_{w}\right)\left(1-\xi_{w}\right)\right)}{\xi_{w}b_{w}} \\
&= d_{w}+\frac{b_{w}\xi_{w}\left(1+\beta\right)}{\xi_{w}b_{w}} \\
&= d_{w}+\left(1+\beta\right).
\end{aligned}$$

The wage Philips curve in equation (4.107) can then be written as

$$\begin{aligned}
& \widehat{\bar{w}}_{t-1}+\beta E_{t}\widehat{\bar{w}}_{t+1}-\left(\hat{\pi}_{t}^{d}-\widehat{\bar{\pi}}_{t}^{c}\right) \\
& +\beta E_{t}\left(\hat{\pi}_{t+1}^{d}-\widehat{\bar{\pi}}_{t+1}^{c}\right)+\kappa_{w}\left(\hat{\pi}_{t-1}^{c}-\widehat{\bar{\pi}}_{t}^{c}\right)-\beta\kappa_{w}E_{t}\left(\hat{\pi}_{t}^{c}-\widehat{\bar{\pi}}_{t+1}^{c}\right) \\
& -d_{w}\left(\hat{\psi}_{z^{+},t}-\hat{\zeta}_{t}^{n}-\hat{\Theta}_{t}-\varphi\hat{N}_{t}-\hat{\lambda}_{t}^{w}\right)-\hat{\mu}_{z^{+},t}+\beta E_{t}\hat{\mu}_{z^{+},t+1} \\
= \; & \left(d_{w}+\left(1+\beta\right)\right)\widehat{\bar{w}}_{t}.
\end{aligned}$$

Re-grouping terms, we get

$$\begin{aligned}
& -\left(\widehat{\bar{w}}_{t}-\widehat{\bar{w}}_{t-1}+\hat{\pi}_{t}^{d}+\hat{\mu}_{z^{+},t}\right)+\beta E_{t}\left(\widehat{\bar{w}}_{t+1}-\widehat{\bar{w}}_{t}+\hat{\pi}_{t+1}^{d}+\hat{\mu}_{z^{+},t+1}\right) \\
& +\widehat{\bar{\pi}}_{t}^{c}-\beta E_{t}\widehat{\bar{\pi}}_{t+1}^{c}+\kappa_{w}\left(\hat{\pi}_{t-1}^{c}-\widehat{\bar{\pi}}_{t}^{c}\right)-\beta\kappa_{w}E_{t}\left(\hat{\pi}_{t}^{c}-\widehat{\bar{\pi}}_{t+1}^{c}\right) \\
& -d_{w}\left(\hat{\psi}_{z^{+},t}-\hat{\zeta}_{t}^{n}-\hat{\Theta}_{t}-\varphi\hat{N}_{t}-\hat{\lambda}_{t}^{w}\right) \\
= \; & d_{w}\widehat{\bar{w}}_{t}.
\end{aligned}$$

Using that nominal wage inflation is

$$\hat{\pi}_{t}^{w}=\widehat{\bar{w}}_{t}-\widehat{\bar{w}}_{t-1}+\hat{\pi}_{t}^{d}+\hat{\mu}_{z^{+},t},$$

we have

$$\begin{aligned}
& -\hat{\pi}_{t}^{w}+\beta E_{t}\hat{\pi}_{t+1}^{w}+ \\
& +\widehat{\bar{\pi}}_{t}^{c}-\beta E_{t}\widehat{\bar{\pi}}_{t+1}^{c}+\kappa_{w}\left(\hat{\pi}_{t-1}^{c}-\widehat{\bar{\pi}}_{t}^{c}\right)-\beta\kappa_{w}E_{t}\left(\hat{\pi}_{t}^{c}-\widehat{\bar{\pi}}_{t+1}^{c}\right) \\
& -d_{w}\left(\hat{\psi}_{z^{+},t}-\hat{\zeta}_{t}^{n}-\hat{\Theta}_{t}-\varphi\hat{N}_{t}-\hat{\lambda}_{t}^{w}\right) \\
= \; & d_{w}\widehat{\bar{w}}_{t}. \qquad (4.109)
\end{aligned}$$

Now, using equations (4.100), (4.80), (4.81) and (4.90), together with (4.109) we have

$$\begin{aligned}
& \widehat{\mathring{w}}_t + \left(\hat{\psi}_{z^+,t} - \hat{\zeta}_t^n - \hat{\Theta}_t - \varphi \hat{N}_t - \hat{\lambda}_t^w\right) \\
= & \widehat{\mathring{w}}_t + \left(\hat{\psi}_{z^+,t} - \hat{\zeta}_t^n - \left(\hat{z}_t^C + \widehat{\mathring{v}}_t^N\right) - \varphi \hat{N}_t - \hat{\lambda}_t^w\right) \\
= & \widehat{\mathring{w}}_t + \left(\hat{\psi}_{z^+,t} - \hat{\zeta}_t^n - \left(\hat{z}_t^C + \hat{\zeta}_t^\beta + \hat{\psi}_{z^+,t} + \hat{p}_t^c\right) - \varphi \hat{N}_t - \hat{\lambda}_t^w\right) \\
= & \widehat{\mathring{w}}_t - \hat{p}_t^c - \left(\hat{\zeta}_t^n + \hat{\zeta}_t^\beta + \hat{z}_t^C + \varphi \hat{L}_t - \varphi \hat{L}_t + \varphi \hat{N}_t + \hat{\lambda}_t^w\right) \\
= & \widehat{\mathring{w}}_t - \hat{p}_t^c - \left(\hat{\zeta}_t^n + \hat{\zeta}_t^\beta + \hat{z}_t^C + \varphi \hat{L}_t\right) + \varphi \hat{L}_t - \varphi \hat{N}_t - \hat{\lambda}_t^w \\
= & \varphi\left(\hat{L}_t - \hat{N}_t\right) - \hat{\lambda}_t^w \\
= & \varphi \hat{U}_t - \hat{\lambda}_t^w,
\end{aligned}$$

where $\hat{U}_t$ is the deviation of the unemployment rate from its steady state. The wage Phillips curve then becomes

$$\hat{\pi}_t^w - \widehat{\bar{\pi}}_t^c = \beta E_t\left(\hat{\pi}_{t+1}^w - \widehat{\bar{\pi}}_{t+1}^c\right) + \kappa_w\left(\hat{\pi}_{t-1}^c - \widehat{\bar{\pi}}_t^c\right) - \beta\kappa_w E_t\left(\hat{\pi}_t^c - \widehat{\bar{\pi}}_{t+1}^c\right) - d_w\left(\varphi \hat{U}_t - \hat{\lambda}_t^w\right).$$

We note that, in our model (and in Galí, Smets, and Wouters (2012)), unlike in Christiano, Trabandt, and Walentin (2011), and Adolfson et al. (2013) (and in Erceg, Henderson, and Levin (2000)), the wage Phillips curve includes only shocks to the wage markup and not preference (labour supply) shocks, which allows us to separately identify both of those shocks.[33] As in Galí, Smets, and Wouters (2012), we define the natural rate of unemployment as

$$\hat{U}_t^n = \frac{1}{\varphi}\hat{\lambda}_t^w, \tag{4.110}$$

i.e. the unemployment rate that would prevail in the absence of nominal wage rigidities. The wage Philips curve can then be expressed as

$$\hat{\pi}_t^w - \widehat{\bar{\pi}}_t^c = \beta E_t\left(\hat{\pi}_{t+1}^w - \widehat{\bar{\pi}}_{t+1}^c\right) + \kappa_w\left(\hat{\pi}_{t-1}^c - \widehat{\bar{\pi}}_t^c\right) - \beta\kappa_w E_t\left(\hat{\pi}_t^c - \widehat{\bar{\pi}}_{t+1}^c\right) - d_w\varphi\left(\hat{U}_t - \hat{U}_t^n\right) \tag{4.111}$$

We note that, if the inflation trend is constant, so that $\widehat{\bar{\pi}}_t^c = 0$, this reduces to

$$\hat{\pi}_t^w = \beta E_t \hat{\pi}_{t+1}^w + \kappa_w \hat{\pi}_{t-1}^c - \beta\kappa_w E_t \hat{\pi}_t^c - d_w\varphi\left(\hat{U}_t - \hat{U}_t^n\right). \tag{4.112}$$

Moreover, if unions fully index to the constant inflation target, i.e. if $\kappa_w = 0$, we obtain a purely forward-looking wage Philips curve

$$\hat{\pi}_t^w = \beta E_t \hat{\pi}_{t+1}^w - d_w\varphi\left(\hat{U}_t - \hat{U}_t^n\right). \tag{4.113}$$

We complete the wage setting block by log-linearizing equation (4.49), which yields

$$\hat{n}_t = \hat{N}_t + \frac{\lambda^w}{1-\lambda^w}\widehat{\mathring{w}}_t + \frac{\lambda^w \log\left(\mathring{w}\right)}{\left(1-\lambda^w\right)^2}\hat{\lambda}_t^w.$$

Assuming that there is full indexation, which implies that $\mathring{w} = 1$, we can simplify this further to get the following expression:

$$\hat{n}_t = \hat{N}_t + \frac{\lambda^w}{1-\lambda^w}\widehat{\mathring{w}}_t. \tag{4.114}$$

[33] See also discussion at the end of Section 15.4.

For use in later sections, we need also to log-linearize the wage dispersion equation (4.51). Rearranging,

$$\mathring{w}_t^{\frac{\lambda_t^w}{1-\lambda_t^w}} = \xi_w \left(\frac{\tilde{\pi}_t^w}{\pi_t^w} \mathring{w}_{t-1}\right)^{\frac{\lambda_t^w}{1-\lambda_t^w}} + (1-\xi_w) \left(\frac{1-\xi_w \left(\frac{\tilde{\pi}_t^w}{\pi_t^w}\right)^{\frac{1}{1-\lambda_t^w}}}{(1-\xi_w)}\right)^{\lambda_t^w}.$$

We can now log-linearize in the same way as for price dispersion terms in Section 3 earlier, which yields

$$\begin{aligned}
\widehat{\mathring{w}}_t &= \xi_w \left(\frac{\tilde{\pi}^w}{\pi^w}\right)^{\frac{\lambda^w}{1-\lambda^w}} \left[\widehat{\tilde{\pi}}_t^w - \hat{\pi}_t^w + \widehat{\mathring{w}}_{t-1}\right] + \xi_w \ln\left(\frac{\tilde{\pi}^w}{\pi^w}\mathring{w}\right)\left(\frac{\tilde{\pi}^w}{\pi^w}\right)^{\frac{\lambda^w}{1-\lambda^w}} \frac{1}{1-\lambda^w}\hat{\lambda}_t^w - \ln(\mathring{w})\frac{1}{1-\lambda^w}\hat{\lambda}_t^w \\
&- \frac{1}{(\mathring{w})^{\frac{\lambda^w}{1-\lambda^w}}} \left(\frac{1-\xi_w\left(\frac{\tilde{\pi}^w}{\pi^w}\right)^{\frac{1}{1-\lambda^w}}}{1-\xi_w}\right)^{\lambda^w-1} \xi_w \left(\frac{\tilde{\pi}^w}{\pi^w}\right)^{\frac{1}{1-\lambda^w}} \left[\widehat{\tilde{\pi}}_t^w - \hat{\pi}_t^w\right] \\
&+ \frac{1-\xi_w}{(\mathring{w})^{\frac{\lambda^w}{1-\lambda^w}}} \ln\left(\frac{1-\xi_w\left(\frac{\tilde{\pi}^w}{\pi^w}\right)^{\frac{1}{1-\lambda^w}}}{1-\xi_w}\right)\left(\frac{1-\xi_w\left(\frac{\tilde{\pi}^w}{\pi^w}\right)^{\frac{1}{1-\lambda^w}}}{1-\xi_w}\right)^{\lambda^w} (1-\lambda^w)\hat{\lambda}_t^w \\
&- \left(\frac{1-\xi_w\left(\frac{\tilde{\pi}^w}{\pi^w}\right)^{\frac{1}{1-\lambda^w}}}{1-\xi_w}\right)^{\lambda^w-1} \frac{\xi_w}{(\mathring{w})^{\frac{\lambda^w}{1-\lambda^w}}} \ln\left(\frac{\tilde{\pi}^w}{\pi^w}\right)\left(\frac{\tilde{\pi}^w}{\pi^w}\right)^{\frac{1}{1-\lambda^w}} \frac{\lambda^w}{1-\lambda^w}\hat{\lambda}_t^w.
\end{aligned}$$

Using the steady-state relationship $\tilde{\pi}^w = \pi^w$, which implies that $\mathring{w} = 1$, we finally arrive at the following expression:

$$\widehat{\mathring{w}}_t = \xi_w \widehat{\mathring{w}}_{t-1}. \tag{4.115}$$

5 Monetary and fiscal authorities

5.1 The central bank

We assume that monetary policy is conducted according to an instrument rule. Following the specification in Ramses II, we assume that the policy maker can adjust the short-run interest rate in response to deviations of CPI inflation from the inflation target, some measure of a resource utilization gap, and the real exchange rate gap. Moreover, the policy maker can also take into account the rate of change in inflation and in resource utilization. We allow for interest rate smoothing, assuming that the policy maker places some weight on the lagged interest rate. Monetary policy is thus approximated with the following rule:

$$\begin{aligned}
\log\left(\frac{R_t}{R}\right) &= \rho_R \log\left(\frac{R_{t-1}}{R}\right) + (1-\rho_R)\left[\log\left(\frac{\bar{\pi}_t^c}{\bar{\pi}^c}\right) + r_\pi \log\left(\frac{\pi_{t-1}^c}{\bar{\pi}_t^c}\right)\right. \qquad (5.1) \\
&\left. + r_{RU}(U_{t-1} - U) + r_q \log\left(\frac{q_{t-1}}{q}\right)\right] + r_{\Delta\pi}\Delta\log\left(\frac{\pi_t^c}{\pi^c}\right) + r_{\Delta RU}\Delta U_t + \log\varepsilon_{R,t},
\end{aligned}$$

where R_t is the short-term interest rate, π_t^c the CPI inflation rate, U_t the unemployment rate, $\bar{\pi}_t^c$ an exogenous process that characterizes the consumer price inflation target, with a steady-state value that corresponds to the steady state of actual inflation, and $\varepsilon_{R,t}$ an interest rate shock. Just as in Adolfson et al. (2013), in this document the first will be referred to as an inflation target shock and the second as a monetary policy shock. The monetary policy shock is assumed to follow the process

$$\log\varepsilon_{R,t} = \left(1-\rho_{\varepsilon_R}\right)\log\varepsilon_R + \rho_{\varepsilon_R}\log\varepsilon_{R,t-1} + \sigma_{\varepsilon_R}\varepsilon_{\varepsilon_R,t}. \tag{5.2}$$

Note that, given our labour market modelling and unlike in Ramses I and II, we have here chosen unemployment as the measure of resource utilization, rather than hours or some measure of the GDP gap.[34]

5.2 Government consumption

We model government consumption expenditures as follows:

$$G_t = g_t z_t^+, \tag{5.3}$$

where g_t is an exogenous stochastic process given by

$$\log g_t = \left(1 - \rho_g\right) \log g + \rho_g \log g_{t-1} + \sigma_g \varepsilon_{g,t}. \tag{5.4}$$

Here, $g = \eta_g Y$, where η_g denotes the steady-state government consumption as a fraction of GDP.

5.3 Log-linearization of the monetary policy rule and government consumption

Recalling the definition $\hat{X}_t \equiv \log X_t - \log X = \log\left(\frac{X_t}{X}\right)$, we can write the policy rule in terms of log-deviations from steady state as follows:[35]

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1 - \rho_R)\left[\hat{\bar{\pi}}_t^c + r_\pi\left(\hat{\pi}_{t-1}^c - \hat{\bar{\pi}}_t^c\right) + r_y \hat{U}_{t-1} + r_q \hat{q}_{t-1}\right] + r_{\Delta\pi}\Delta\hat{\pi}_t^c + r_{\Delta y}\Delta\hat{U}_t + \hat{\varepsilon}_{R,t}, \tag{5.5}$$

where

$$\hat{\bar{\pi}}_t^c = \rho_{\bar{\pi}^c}\hat{\bar{\pi}}_{t-1}^c + \sigma_{\bar{\pi}^c}\varepsilon_{\bar{\pi}^c,t}. \tag{5.6}$$

Government consumption in scaled form, defined as $g_t = G_t/z_t^+$, is given by the following exogenous stochastic process in terms of log-deviations from steady state:

$$\hat{g}_t = \rho_g \hat{g}_{t-1} + \sigma_g \varepsilon_{g,t}. \tag{5.7}$$

6 The aggregate resource constraint

We begin by deriving a relationship between total output of the domestic homogeneous good, Y_t, and aggregate factors of production. We then proceed with the aggregate resource constraint.

[34] In Christiano, Trabandt, and Walentin (2011), as well as in Ramses I, the central bank is assumed to take into account the output gap as the measure of resource utilization. In Ramses II, the output gap was replaced by hours worked. The motivation for introducing hours in Ramses II was twofold. First, filtered hours worked is an observed variable (filtered with an HP or a KAMEL-trend) which enables judgment to directly influence monetary policy, and ii) the specification with hours was preferred by the data (although only with a slight advantage). In our model, hours are no longer observed, which is why our baseline specification has the unemployment rate as the measure of resource utilization.

We note also that, in Ramses II, r_q is set to zero, and so the real exchange rate term is excluded. In preliminary versions of the Christiano, Trabandt, and Walentin (2011) model and in Ramses I, the term is included in the rule. The final version of Christiano, Trabandt, and Walentin (2011) assumes that $r_q = 0$ as well as $r_{\Delta\pi} = r_{\Delta y} = 0$. The final version of Christiano, Trabandt, and Walentin (2011) also has π_t entering the rule instead of π_{t-1} as in Ramses I and II.

[35] Recall that U_t denotes the deviation in levels, rather than the log-deviation, of the unemployment rate from its steady state, and thus

$$\hat{U}_t = U_t - U.$$

The unweighted average of the domestic intermediate goods is given by

$$\begin{aligned} Y_t^{sum} &= \int_0^1 Y_{i,t} di \\ &= \int_0^1 \left[(z_t N_{i,t})^{1-\alpha} \epsilon_t K_{i,t}^{\alpha} - z_t^+ \phi^d \right] di \\ &= \int_0^1 \left[z_t^{1-\alpha} \epsilon_t \left(\frac{K_{i,t}}{N_{i,t}} \right)^{\alpha} N_{i,t} - z_t^+ \phi^d \right] di \\ &= z_t^{1-\alpha} \epsilon_t \left(\frac{K_t}{N_t} \right)^{\alpha} \int_0^1 N_{i,t} di - z_t^+ \phi^d, \end{aligned}$$

where K_t and N_t are the economy-wide averages of capital services and homogeneous labour, respectively. The last step above makes use of the fact that all intermediate good firms face the same factor prices, regardless of wheter or not they have the opportunity to reoptimize, and so they adopt the same capital services to homogeneous labour ratio.[36] We then have that

$$Y_t^{sum} = z_t^{1-\alpha} \epsilon_t K_t^{\alpha} N_t^{1-\alpha} - z_t^+ \phi^d. \tag{6.1}$$

We consider next the demand for homogeneous goods. Using demand equation (3.3), stated here again for convenience,

$$Y_{i,t} = \left(\frac{P_{i,t}^d}{P_t^d} \right)^{\frac{\lambda_t^d}{1-\lambda_t^d}} Y_t,$$

we have

$$\begin{aligned} \mathring{Y}_t &\equiv \int_0^1 Y_{i,t} di \\ &= \int_0^1 \left(\frac{P_{i,t}^d}{P_t^d} \right)^{\frac{\lambda_t^d}{1-\lambda_t^d}} Y_t di \\ &= Y_t \left(P_t^d \right)^{\frac{\lambda_t^d}{\lambda_t^d - 1}} \left(\mathring{P}_t^d \right)^{\frac{\lambda_t^d}{1-\lambda_t^d}}, \end{aligned} \tag{6.2}$$

where

$$\mathring{P}_t^d = \left[\int_0^1 \left(P_{i,t}^d \right)^{\frac{\lambda_t^d}{1-\lambda_t^d}} di \right]^{\frac{1-\lambda_t^d}{\lambda_t^d}}.$$

Dividing by P_t^d,

$$\mathring{p}_t^d = \left[\int_0^1 \left(\frac{P_{i,t}^d}{P_t^d} \right)^{\frac{\lambda_t^d}{1-\lambda_t^d}} di \right]^{\frac{1-\lambda_t^d}{\lambda_t^d}}. \tag{6.3}$$

We can break this integral and re-express it in terms of aggregate prices using the Calvo assumption

[36] This follows from the firms' cost minimization problem.

on price setting:

$$
\begin{aligned}
\mathring{p}_t^d &= \left[\int_0^{\xi_d}\left(\frac{\tilde{\pi}_t^d P_{i,t-1}^d}{P_t^d}\right)^{\frac{\lambda_t^d}{1-\lambda_t^d}} di + \int_{\xi_d}^1 \left(\frac{\tilde{P}_t^d}{P_t^d}\right)^{\frac{\lambda_t^d}{1-\lambda_t^d}} di\right]^{\frac{1-\lambda_t^d}{\lambda_t^d}} \\
&= \left[\left(\tilde{\pi}_t^d\right)^{\frac{\lambda_t^d}{1-\lambda_t^d}} \int_0^{\xi_d}\left(\frac{P_{t-1}^d}{P_t^d}\frac{P_{i,t-1}^d}{P_{t-1}^d}\right)^{\frac{\lambda_t^d}{1-\lambda_t^d}} di + (1-\xi_d)\left(\tilde{p}_t^d\right)^{\frac{\lambda_t^d}{1-\lambda_t^d}}\right]^{\frac{1-\lambda_t^d}{\lambda_t^d}} \\
&= \left[\xi_d\left(\frac{\tilde{\pi}_t^d}{\pi_t^d}\mathring{p}_{t-1}^d\right)^{\frac{\lambda_t^d}{1-\lambda_t^d}} + (1-\xi_d)\left(\tilde{p}_t^d\right)^{\frac{\lambda_t^d}{1-\lambda_t^d}}\right]^{\frac{1-\lambda_t^d}{\lambda_t^d}}.
\end{aligned}
$$

Substituting $\tilde{p}_t^d$ using (3.23) we get:

$$
\mathring{p}_t^d = \left[\xi_d\left(\frac{\tilde{\pi}_t^d}{\pi_t^d}\mathring{p}_{t-1}^d\right)^{\frac{\lambda_t^d}{1-\lambda_t^d}} + (1-\xi_d)\left(\frac{1-\xi_d\left(\frac{\tilde{\pi}_t^d}{\pi_t^d}\right)^{\frac{1}{1-\lambda_t^d}}}{1-\xi_d}\right)^{\lambda_t^d}\right]^{\frac{1-\lambda_t^d}{\lambda_t^d}}. \tag{6.4}
$$

Combining (6.3) and (6.2) with (6.1), we have the following expression for GDP in terms of aggregate factors of production:

$$
Y_t = \left(\mathring{p}_t^d\right)^{\frac{\lambda_t^d}{\lambda_t^d-1}} \mathring{Y}_t = \left(\mathring{p}_t^d\right)^{\frac{\lambda_t^d}{\lambda_t^d-1}} \left[z_t^{1-\alpha}\epsilon_t K_t^\alpha N_t^{1-\alpha} - z_t^+\phi^d\right]. \tag{6.5}
$$

The aggregate resource constraint, equalizing the uses of domestic homogeneous goods to the above expression for GDP from the production side, is given by the following equation:

$$
G_t + C_t^d + C_t^{e,d} + I_t^d + X_t^d \leq \left(\mathring{p}_t^d\right)^{\frac{\lambda_t^d}{\lambda_t^d-1}} \left[z_t^{1-\alpha}\epsilon_t K_t^\alpha N_t^{1-\alpha} - z_t^+\phi^d\right]. \tag{6.6}
$$

Substituting in the demand equations for C_t^d, $C_t^{e,d}$, I_t^d, and X_t^d, given by (3.121) together with (3.127), (3.124) together with (3.128), (3.165), and (3.197), respectively, we have[37]

$$
\begin{aligned}
& G_t + (1-\omega_c)(1-\omega_e)\left[\frac{P_t^d}{P_t^{cxe}}\right]^{-\eta_c}\left[\frac{P_t^{cxe}}{P_t^c}\right]^{-\eta_e} C_t \\
& + (1-\omega_{em})\,\omega_e\left[\frac{P_t^{d,ce}}{P_t^{ce}}\right]^{-\eta_{em}}\left[\frac{P_t^{ce}}{P_t^c}\right]^{-\eta_e} C_t \\
& + (1-\omega_i)\left[\frac{P_t^d}{P_t^i}\right]^{-\eta_i}\Psi_t^{\eta_i-1}\left(I_t + a(u_t)K_t^p\right) \\
& + \left(\omega_x\left[\frac{P_t^{m,x}}{P_t^d}\right]^{1-\eta_x} + (1-\omega_x)\right)^{\frac{\eta_x}{1-\eta_x}}(1-\omega_x)\left(\mathring{p}_t^x\right)^{\frac{\lambda_t^x}{1-\lambda_t^x}} X_t \\
\leq\ & \left(\mathring{p}_t^d\right)^{\frac{\lambda_t^d}{\lambda_t^d-1}}\left[z_t^{1-\alpha}\epsilon_t K_t^\alpha N_t^{1-\alpha} - z_t^+\phi^d\right].
\end{aligned} \tag{6.7}
$$

[37] Note that there is a price dispersion term in the export demand equation, while there are none in the consumption and investment demand equation. This has to do with the fact that the Calvo frictions are placed on the aggregate level for exports, i.e. export firms face staggered price setting when pricing the final export good, and only on the intermediate level – staggered price setting for the domestic and imported intermediate good – for consumption and investment.

6.1 Scaling of the aggregate resource constraint

Scaling by z_t^+, remembering that $z_t^+ = \Psi_t^{\frac{\alpha}{1-\alpha}} z_t$,

$$\frac{Y_t}{z_t^+} = \left(\mathring{p}_t^d\right)^{\frac{\lambda_t^d}{\lambda_t^d - 1}} \left[\frac{z_t^{1-\alpha} \epsilon_t K_t^\alpha N_t^{1-\alpha}}{\Psi_t^{\frac{\alpha}{1-\alpha}} z_t} - \frac{z_t^+ \phi^d}{z_t^+}\right]$$

$$y_t = \left(\mathring{p}_t^d\right)^{\frac{\lambda_t^d}{\lambda_t^d - 1}} \left[\epsilon_t \left(\frac{k_t}{\mu_{\Psi,t}\mu_{z^+,t}}\right)^\alpha N_t^{1-\alpha} - \phi^d\right]. \tag{6.8}$$

Finally, using equation (4.49) we replace aggregate homogeneous labour, N_t, with aggregate household labour, n_t, to obtain

$$y_t = \left(\mathring{p}_t^d\right)^{\frac{\lambda_t^d}{\lambda_t^d - 1}} \left[\epsilon_t \left(\frac{k_t}{\mu_{\Psi,t}\mu_{z^+,t}}\right)^\alpha \left(\mathring{w}_t^{-\frac{\lambda_t^w}{1-\lambda_t^w}} n_t\right)^{1-\alpha} - \phi^d\right]. \tag{6.9}$$

Scaling (6.6) by z_t^+, we have

$$\begin{aligned} & g_t + c_t^d + c_t^{e,d} + i_t^d + x_t^d \\ \leq \; & \left(\mathring{p}_t^d\right)^{\frac{\lambda_t^d}{\lambda_t^d - 1}} \left[\epsilon_t \left(\frac{k_t}{\mu_{\Psi,t}\mu_{z^+,t}}\right)^\alpha \left(\mathring{w}_t^{-\frac{\lambda_t^w}{1-\lambda_t^w}} n_t\right)^{1-\alpha} - \phi^d\right]. \end{aligned} \tag{6.10}$$

6.2 Log-linearization of the aggregate resource constraint

We begin by log-linearizing the left-hand side of equation (6.10):

$$y_t = g_t + c_t^d + c_t^{e,d} + i_t^d + x_t^d. \tag{6.11}$$

Log-linearizing, we get

$$\hat{y}_t = \frac{g}{y}\hat{g}_t + \frac{c^d}{y}\hat{c}_t^d + \frac{c^{e,d}}{y}\hat{c}_t^{e,d} + \frac{i^d}{y}\widehat{i}_t^d + \frac{x^d}{y}\hat{x}_t^d. \tag{6.12}$$

We now focus on the right-hand side of equation (6.10), i.e.

$$y_t = \left(\mathring{p}_t^d\right)^{\frac{\lambda_t^d}{\lambda_t^d - 1}} \left[\epsilon_t \left(\frac{k_t}{\mu_{\Psi,t}\mu_{z^+,t}}\right)^\alpha \left(\mathring{w}_t^{-\frac{\lambda_t^w}{1-\lambda_t^w}} n_t\right)^{1-\alpha} - \phi^d\right].$$

Rearranging,

$$y_t \left(\mathring{p}_t^d\right)^{\frac{\lambda_t^d}{1-\lambda_t^d}} = \epsilon_t \left(\frac{k_t}{\mu_{\Psi,t}\mu_{z^+,t}}\right)^\alpha \left(\mathring{w}_t^{-\frac{\lambda_t^w}{1-\lambda_t^w}} n_t\right)^{1-\alpha} - \phi^d.$$

Log-linearizing, we get

$$\begin{aligned} & y\left(\mathring{p}^d\right)^{\frac{\lambda^d}{1-\lambda^d}} \left[\hat{y}_t + \frac{\lambda^d}{1-\lambda^d}\widehat{\mathring{p}}_t^d + \frac{\lambda^d \log\left(\mathring{p}^d\right)}{\left(1-\lambda^d\right)^2}\hat{\lambda}_t^d\right] \\ = \; & \epsilon\left(\frac{k}{\mu_\Psi \mu_{z^+}}\right)^\alpha \left(\mathring{w}^{-\frac{\lambda^w}{1-\lambda^w}} n\right)^{1-\alpha} \times \\ & \times \left[\hat{\epsilon}_t + \alpha\left(\hat{k}_t - \hat{\mu}_{\Psi,t} - \hat{\mu}_{z^+,t}\right) - \frac{\lambda^w(1-\alpha)}{1-\lambda^w}\widehat{\mathring{w}}_t - \frac{\lambda^w (1-\alpha)\log\left(\mathring{w}\right)}{\left(1-\lambda^w\right)^2}\hat{\lambda}_t^w + (1-\alpha)\hat{n}_t\right], \end{aligned}$$

Using that $\epsilon = 1$ and that full indexation implies that $\mathring{p}^d = \mathring{w} = 1$, and rearranging,

$$\begin{aligned}\hat{y}_t &= \frac{1}{y}\left(\frac{k}{\mu_\Psi \mu_{z^+}}\right)^\alpha n^{1-\alpha} \times \\ &\times \left[\hat{\epsilon}_t + \alpha\left(\hat{k}_t - \hat{\mu}_{\Psi,t} - \hat{\mu}_{z^+,t}\right) - \frac{\lambda^w (1-\alpha)}{1-\lambda^w}\widehat{\mathring{w}}_t + (1-\alpha)\hat{n}_t\right] \\ &- \frac{\lambda^d}{\lambda^d - 1}\widehat{\mathring{p}}^d_t .\end{aligned} \tag{6.13}$$

The log-linear version of equation (6.10) is then given by equations (6.12) and (6.13) combined.

7 Evolution of net foreign assets

In this section, we derive the expression linking net exports and the current account. Expenses on imports and net new purchases of foreign assets must equal income from exports and from previously purchased net foreign assets. Hence, the evolution of net foreign assets at the aggregate level must satisfy the following equation

$$S_t B^F_{t+1} + \text{expenses on imports}_t = \text{receipts from exports}_t + R^*_{t-1}\Phi_{t-1}\chi_{t-1}S_t B^F_t .$$

We focus first on the expenses on imports. We begin by noting that the relevant measure here is the total value of all imports that cross the border, that is the marginal cost times the gross imports as given by equation (3.226), and so

$$\begin{aligned}\text{expenses on imports}_t &= S_t P^{d,*}_t R^{wc,m}_t \left(\int_0^1 C^m_{i,t} di + \int_0^1 I^m_{i,t} di + \int_0^1 X^m_{i,t} di \right. \\ &\quad \left. + z^+_t \left(\phi^{m,c} + \phi^{m,i} + \phi^{m,x}\right)\right) \\ &\quad + S_t P^{ce,*}_t R^{wc,m}_t \left(\int_0^1 C^{e,m}_{i,t} di + z^+_t \phi^{m,ce}\right).\end{aligned}$$

This is an important distinction as the value of the imports used in the domestic economy, i.e. the imports entering the production of final goods, is higher than the value of imports net of fixed costs above due to positive markups from the monopolistic importing firms. In earlier models, this distinction was not made, why the implications for net exports from the resource constraint and from the evolution of net foreign assets were not consistent.[38] This, in turn, implied that the consumption-to-output ratios were not in line with the data. Moving on to the receipts from exports, given by

$$\text{receipts from exports}_t = S_t P^x_t \left(\tilde{z}^{+,*}_t\right)^{-\frac{1}{\eta_f}} \left(\left(\mathring{p}^x_t\right)^{\frac{\lambda^x_t}{1-\lambda^x_t}} X_t - z^+_t \phi^x\right).$$

As for imports, the relevant measure is the value of all exports that cross the border, that is what remains of exports once the fixed costs of production are covered times the aggregate export price.

[38] This issue is discussed in more detail in internal Riksbank memos by Malin Adolfsson, where the inconsistency between the expressions for net exports from the resource constraint and the evolution of net foreign assets is derived. The treatment of imports and exports is problematic as the two economies in the model are not treated symmetrically. Instead, it is assumed that there is monopolistic power in both the domestic import and export sector and that they are both meeting a single foreign good price. Moreover, the assumption of advance financing of firms' costs introduces an additional friction that drives a wedge between the resources used in the domestic economy and the trande balance. No solutions are proposed in the memos by Adolfsson.

As the biggest wedges are induced by the markups, we here handle the problem of monopolistic competition and positive profits through the introduction of fixed costs in production of imports and exports. These are set so as to ensure that profits in steady state are zero. We do not handle the wedge stemming from the advance financing assumption, as it is likely to have much more limited quantitative implications.

We can substitute for $C_{i,t}^m$, $I_{i,t}^m$, $X_{i,t}^m$ and $C_{i,t}^{e,m}$ in the expenses on imports using (3.60), (3.67), (3.72) and (3.77). We then have:

$$\begin{aligned}
\text{expenses on imports}_t &= S_t P_t^{d,*} R_t^{wc,m} \left(C_t^m \int_0^1 \left(\frac{P_{i,t}^{m,c}}{P_t^{m,c}} \right)^{\frac{\lambda_t^{m,c}}{1-\lambda_t^{m,c}}} di + I_t^m \int_0^1 \left(\frac{P_{i,t}^{m,i}}{P_t^{m,i}} \right)^{\frac{\lambda_t^{m,i}}{1-\lambda_t^{m,i}}} di \right. \\
&\qquad \left. + X_t^m \int_0^1 \left(\frac{P_{i,t}^{m,x}}{P_t^{m,x}} \right)^{\frac{\lambda_t^{m,x}}{1-\lambda_t^{m,x}}} di + z_t^+ \left(\phi^{m,c} + \phi^{m,i} + \phi^{m,x} \right) \right) \\
&\quad + S_t P_t^{ce,*} R_t^{wc,m} \left(C_t^{e,m} \int_0^1 \left(\frac{P_{i,t}^{m,ce}}{P_t^{m,ce}} \right)^{\frac{\lambda_t^{m,ce}}{1-\lambda_t^{m,ce}}} di + z_t^+ \phi^{m,ce} \right) \\
&= S_t P_t^{d,*} R_t^{wc,m} \left(C_t^m \left(\mathring{p}_t^{m,c} \right)^{\frac{\lambda_t^{m,c}}{1-\lambda_t^{m,c}}} + I_t^m \left(\mathring{p}_t^{m,i} \right)^{\frac{\lambda_t^{m,i}}{1-\lambda_t^{m,i}}} + X_t^m \left(\mathring{p}_t^{m,x} \right)^{\frac{\lambda_t^{m,x}}{1-\lambda_t^{m,x}}} \right. \\
&\qquad \left. + z_t^+ \left(\phi^{m,c} + \phi^{m,i} + \phi^{m,x} \right) \right) \\
&\quad + S_t P_t^{ce,*} R_t^{wc,m} \left(C_t^{e,m} \left(\mathring{p}_t^{m,ce} \right)^{\frac{\lambda_t^{m,ce}}{1-\lambda_t^{m,ce}}} + z_t^+ \phi^{m,ce} \right),
\end{aligned}$$

where for $j = c, i, x, ce$, we define $\mathring{p}_t^{m,j}$, a measure of price dispersion, as follows:

$$\mathring{p}_t^{m,j} = \left[\int_0^1 \left(\frac{P_{i,t}^{m,j}}{P_t^{m,j}} \right)^{\frac{\lambda_t^{m,j}}{1-\lambda_t^{m,j}}} \right]^{\frac{1-\lambda_t^{m,j}}{\lambda_t^{m,j}}}. \tag{7.1}$$

Hence,

$$\begin{aligned}
& S_t B_{t+1}^F + S_t P_t^{d,*} R_t^{wc,m} \left(C_t^m \left(\mathring{p}_t^{m,c} \right)^{\frac{\lambda_t^{m,c}}{1-\lambda_t^{m,c}}} + I_t^m \left(\mathring{p}_t^{m,i} \right)^{\frac{\lambda_t^{m,i}}{1-\lambda_t^{m,i}}} + X_t^m \left(\mathring{p}_t^{m,x} \right)^{\frac{\lambda_t^{m,x}}{1-\lambda_t^{m,x}}} \right. \\
&\qquad \left. + z_t^+ \left(\phi^{m,c} + \phi^{m,i} + \phi^{m,x} \right) \right) \\
& + S_t P_t^{ce,*} R_t^{wc,m} \left(C_t^{e,m} \left(\mathring{p}_t^{m,ce} \right)^{\frac{\lambda_t^{m,ce}}{1-\lambda_t^{m,ce}}} + z_t^+ \phi^{m,ce} \right) \\
= \; & S_t P_t^x \left(\left(\mathring{p}_t^x \right)^{\frac{\lambda_t^x}{1-\lambda_t^x}} X_t - z_t^+ \phi^x \right) + R_{t-1}^* \Phi_{t-1} \chi_{t-1} S_t B_t^F,
\end{aligned} \tag{7.2}$$

where we have used equations (3.85), (3.86), (3.87) and (3.88), and where Φ_t is defined in Section 4.2.2.

7.1 Scaling of the evolution of net foreign assets

We scale equation (7.2) by z_t^+ to obtain

$$\bar{a}_t P_t^d z_t^+ + S_t P_t^{d,*} z_t^+ R_t^{wc,m} \left(c_t^m \left(\mathring{p}_t^{m,c}\right)^{\frac{\lambda_t^{m,c}}{1-\lambda_t^{m,c}}} + i_t^m \left(\mathring{p}_t^{m,i}\right)^{\frac{\lambda_t^{m,i}}{1-\lambda_t^{m,i}}} + x_t^m \left(\mathring{p}_t^{m,x}\right)^{\frac{\lambda_t^{m,x}}{1-\lambda_t^{m,x}}} \right.$$

$$\left. + \left(\phi^{m,c} + \phi^{m,i} + \phi^{m,x}\right)\right) + S_t P_t^{ce,*} z_t^+ R_t^{wc,m} \left(c_t^{e,m} \left(\mathring{p}_t^{m,ce}\right)^{\frac{\lambda_t^{m,ce}}{1-\lambda_t^{m,ce}}} + \phi^{m,ce} \right)$$

$$= \quad S_t P_t^x \left(\tilde{z}_t^{+,*}\right)^{-\frac{1}{\eta_f}} z_t^+ \left(\left(\mathring{p}_t^x\right)^{\frac{\lambda_t^x}{1-\lambda_t^x}} x_t - \phi^x \right) + R_{t-1}^* \Phi_{t-1} \chi_{t-1} \frac{S_t}{S_{t-1}} \bar{a}_{t-1} P_{t-1}^d z_{t-1}^+,$$

where we have used the definition of the real aggregate net foreign asset position

$$\bar{a}_t \equiv \frac{\bar{A}_t}{z_t^+} = \frac{S_t B_{t+1}^F}{P_t^d z_t^+}.$$

Dividing by $P_t^d z_t^+$, we have

$$\bar{a}_t + \frac{q_t p_t^c}{p_t^{c,*}} R_t^{wc,m} \left(c_t^m \left(\mathring{p}_t^{m,c}\right)^{\frac{\lambda_t^{m,c}}{1-\lambda_t^{m,c}}} + i_t^m \left(\mathring{p}_t^{m,i}\right)^{\frac{\lambda_t^{m,i}}{1-\lambda_t^{m,i}}} \right.$$

$$\left. + x_t^m \left(\mathring{p}_t^{m,x}\right)^{\frac{\lambda_t^{m,x}}{1-\lambda_t^{m,x}}} + \phi^{m,c} + \phi^{m,i} + \phi^{m,x} \right)$$

$$+ \frac{q_t p_t^c p_t^{ce,*}}{p_t^{c,*}} R_t^{wc,m} \left(c_t^{e,m} \left(\mathring{p}_t^{m,ce}\right)^{\frac{\lambda_t^{m,ce}}{1-\lambda_t^{m,ce}}} + \phi^{m,ce} \right)$$

$$= \quad \frac{q_t p_t^x p_t^c}{p_t^{c,*}} \left(\left(\mathring{p}_t^x\right)^{\frac{\lambda_t^x}{1-\lambda_t^x}} x_t - \phi^x \right) + R_{t-1}^* \Phi_{t-1} \chi_{t-1} s_t \frac{\bar{a}_{t-1}}{\pi_t^d \mu_{z^+,t}}, \tag{7.3}$$

where we have used definitions of relative prices p_t^c, $p_t^{c,*}$, $p_t^{ce,*}$ and p_t^x, the definition of the real exchange rate

$$q_t = \frac{S_t P_t^{c,*}}{P_t^c},$$

and the growth of the nominal exchange rate

$$s_t = \frac{S_t}{S_{t-1}}.$$

7.2 Log-linearization of the evolution of net foreign assets

We log-linearize equation (7.3). To do so, we apply a first-order Taylor expansion of the expression around the steady-state of each variable. Note that $\bar{a}$ takes the value of zero in steady state, so level deviations must be used for deviations of $\bar{a}_t$ from steady state. We therefore define $\breve{a}_t = \bar{a}_t - \bar{a}$. Note also that the steady-state version of (7.3) writes

$$\frac{qp^c}{p^{c,*}} R^{wc,m} \left(c^m \left(\mathring{p}^{m,c}\right)^{\frac{\lambda^{m,c}}{1-\lambda^{m,c}}} + i^m \left(\mathring{p}^{m,i}\right)^{\frac{\lambda^{m,i}}{1-\lambda^{m,i}}} + x^m \left(\mathring{p}^{m,x}\right)^{\frac{\lambda^{m,x}}{1-\lambda^{m,x}}} \right.$$

$$\left. + \phi^{m,c} + \phi^{m,i} + \phi^{m,x} \right) + \frac{qp^c p^{ce,*}}{p^{c,*}} R^{wc,m} \left(c^{e,m} \left(\mathring{p}^{m,ce}\right)^{\frac{\lambda^{m,ce}}{1-\lambda^{m,ce}}} + \phi^{m,ce} \right)$$

$$= \quad \frac{qp^c p^x}{p^{c,*}} \left(\left(\mathring{p}^x\right)^{\frac{\lambda^x}{1-\lambda^x}} x - \phi^x \right).$$

In what follows, we directly state the Taylor expansion substracting the previous steady-state expression for the sake of conciseness. We then have (dropping terms involving $\bar{a}=0$):

$$
\begin{aligned}
& \breve{a}_t + \frac{qp^c}{p^{c,*}} R^{wc,m} \left(c^m \left(\mathring{p}^{m,c}\right)^{\frac{\lambda^{m,c}}{1-\lambda^{m,c}}} + i^m \left(\mathring{p}^{m,i}\right)^{\frac{\lambda^{m,i}}{1-\lambda^{m,i}}} + x^m \left(\mathring{p}^{m,x}\right)^{\frac{\lambda^{m,x}}{1-\lambda^{m,x}}} + \phi^{m,c} + \phi^{m,i} + \phi^{m,x} \right) \times \\
& \times \left(\hat{q}_t + \hat{p}_t^c - \hat{p}_t^{c,*} + \hat{R}_t^{wc,m} \right) \\
& + \frac{qp^c}{p^{c,*}} R^{wc,m} c^m \left(\mathring{p}^{m,c}\right)^{\frac{\lambda^{m,c}}{1-\lambda^{m,c}}} \left(\hat{c}_t^m + \frac{\lambda^{m,c}}{1-\lambda^{m,c}} \widehat{\mathring{p}}_t^{m,c} + \frac{\lambda^{m,c}}{\left(1-\lambda^{m,c}\right)^2} \log\left(\mathring{p}^{m,c}\right) \hat{\lambda}_t^{m,c} \right) \\
& + \frac{qp^c}{p^{c,*}} R^{wc,m} i^m \left(\mathring{p}^{m,i}\right)^{\frac{\lambda^{m,i}}{1-\lambda^{m,i}}} \left(\hat{i}_t^m + \frac{\lambda^{m,i}}{1-\lambda^{m,i}} \widehat{\mathring{p}}_t^{m,i} + \frac{\lambda^{m,i}}{\left(1-\lambda^{m,i}\right)^2} \log\left(\mathring{p}^{m,i}\right) \hat{\lambda}_t^{m,i} \right) \\
& + \frac{qp^c}{p^{c,*}} R^{wc,m} x^m \left(\mathring{p}^{m,x}\right)^{\frac{\lambda^{m,x}}{1-\lambda^{m,x}}} \left(\hat{x}_t^m + \frac{\lambda^{m,x}}{1-\lambda^{m,x}} \widehat{\mathring{p}}_t^{m,x} + \frac{\lambda^{m,x}}{\left(1-\lambda^{m,x}\right)^2} \log\left(\mathring{p}^{m,x}\right) \hat{\lambda}_t^{m,x} \right) \\
& + \frac{qp^c p^{ce,*}}{p^{c,*}} R^{wc,m} \left(c^{e,m} \left(\mathring{p}^{m,ce}\right)^{\frac{\lambda_t^{m,ce}}{1-\lambda_t^{m,ce}}} + \phi^{m,ce} \right) \left(\hat{q}_t + \hat{p}_t^c + \hat{p}_t^{ce,*} - \hat{p}_t^{c,*} + \hat{R}_t^{wc,m} \right) \\
& + \frac{qp^c p^{ce,*}}{p^{c,*}} R^{wc,m} c^{e,m} \left(\mathring{p}^{m,ce}\right)^{\frac{\lambda^{m,ce}}{1-\lambda^{m,ce}}} \left(\hat{c}_t^{e,m} + \frac{\lambda^{m,ce}}{1-\lambda^{m,ce}} \widehat{\mathring{p}}_t^{m,ce} + \frac{\lambda^{m,ce}}{\left(1-\lambda^{m,ce}\right)^2} \log\left(\mathring{p}^{m,ce}\right) \hat{\lambda}_t^{m,ce} \right) \\
= \; & \frac{qp^x p^c}{p^{c,*}} \left(\left(\mathring{p}^x\right)^{\frac{\lambda^x}{1-\lambda^x}} x - \phi^x \right) \left(\hat{q}_t + \hat{p}_t^x + \hat{p}_t^c - \hat{p}_t^{c,*} \right) + \frac{qp^x p^c}{p^{c,*}} x \left(\mathring{p}^x\right)^{\frac{\lambda^x}{1-\lambda^x}} \times \\
& \times \left(\hat{x}_t + \frac{\lambda^x}{\left(1-\lambda^x\right)^2} \log(\mathring{p}^x) \hat{\lambda}_t^x + \frac{\lambda^x}{1-\lambda^x} \widehat{\mathring{p}}_t^x \right) \\
& + \frac{R^* \Phi \chi s}{\pi^d \mu_{z^+}} \breve{a}_{t-1}.
\end{aligned}
$$

Assuming full indexation, that is $\mathring{p}^{m,c} = \mathring{p}^{m,i} = \mathring{p}^{m,x} = \mathring{p}^{m,ce} = \mathring{p}^x = 1$, the expression simplifies to

$$
\begin{aligned}
& \frac{p^{c,*}}{qp^c} \breve{a}_t + R^{wc,m} \left(c^m + i^m + x^m + \phi^{m,c} + \phi^{m,i} + \phi^{m,x} \right) \left(\hat{q}_t + \hat{p}_t^c - \hat{p}_t^{c,*} + \hat{R}_t^{wc,m} \right) \\
& + R^{wc,m} c^m \left(\hat{c}_t^m + \frac{\lambda^{m,c}}{1-\lambda^{m,c}} \widehat{\mathring{p}}_t^{m,c} \right) \\
& + R^{wc,m} i^m \left(\hat{i}_t^m + \frac{\lambda^{m,i}}{1-\lambda^{m,i}} \widehat{\mathring{p}}_t^{m,i} \right) \\
& + R^{wc,m} x^m \left(\hat{x}_t^m + \frac{\lambda^{m,x}}{1-\lambda^{m,x}} \widehat{\mathring{p}}_t^{m,x} \right) \\
& + p^{ce,*} R^{wc,m} \left(c^{e,m} + \phi^{m,ce} \right) \left(\hat{q}_t + \hat{p}_t^c + \hat{p}_t^{ce,*} - \hat{p}_t^{c,*} + \hat{R}_t^{wc,m} \right) \\
& + p^{ce,*} R^{wc,m} c^{e,m} \left(\hat{c}_t^{e,m} + \frac{\lambda^{m,ce}}{1-\lambda^{m,ce}} \widehat{\mathring{p}}_t^{m,ce} \right) \\
= \; & p^x \left(x - \phi^x \right) \left(\hat{q}_t + \hat{p}_t^x + \hat{p}_t^c - \hat{p}_t^{c,*} \right) + p^x x \left(\hat{x}_t + \frac{\lambda^x}{1-\lambda^x} \widehat{\mathring{p}}_t^x \right) + \frac{p^{c,*}}{qp^c} \frac{R^* \Phi \chi s}{\pi^d \mu_{z^+}} \breve{a}_{t-1}.
\end{aligned}
$$

Now note that the steady-state relationship under full indexation and given that $\bar{a}$ takes the value of zero in steady state yields

$$
R^{wc,m} \left(c^m + i^m + x^m + \phi^{m,c} + \phi^{m,i} + \phi^{m,x} \right) + p^{ce,*} R^{wc,m} \left(c^{e,m} + \phi^{m,ce} \right) = p^x \left(x - \phi^x \right),
$$

which allows to further simplify the log-linearized expression to obtain

$$\frac{p^{c,*}}{qp^c}\breve{a}_t = p^x\left(x-\phi^x\right)\hat{p}_t^x + p^x x\left(\hat{x}_t + \frac{\lambda^x}{1-\lambda^x}\widehat{\mathring{p}}_t^x\right) + \frac{p^{c,*}}{qp^c}\frac{R^*\Phi\chi s}{\pi^d\mu_{z^+}}\breve{a}_{t-1} \tag{7.4}$$
$$-p^x\left(x-\phi^x\right)\hat{R}_t^{wc,m} - p^{ce,*}R^{wc,m}\left(c^{e,m}+\phi^{m,ce}\right)\hat{p}_t^{ce,*}$$
$$-R^{wc,m}c^m\left(\hat{c}_t^m + \frac{\lambda^{m,c}}{1-\lambda^{m,c}}\widehat{\mathring{p}}_t^{m,c}\right) - R^{wc,m}\imath^m\left(\hat{\imath}_t^m + \frac{\lambda^{m,i}}{1-\lambda^{m,i}}\widehat{\mathring{p}}_t^{m,i}\right)$$
$$-R^{wc,m}x^m\left(\hat{x}_t^m + \frac{\lambda^{m,x}}{1-\lambda^{m,x}}\widehat{\mathring{p}}_t^{m,x}\right) - p^{ce,*}R^{wc,m}c^{e,m}\left(\hat{c}_t^{e,m} + \frac{\lambda^{m,ce}}{1-\lambda^{m,ce}}\widehat{\mathring{p}}_t^{m,ce}\right).$$

8 Inflation rates and relative price formulas

We have defined the following relative prices in Section 2.2, stated here again for convenience:

$$\begin{aligned}
p_t^c &= \frac{P_t^c}{P_t^d},\ p_t^{cxe} = \frac{P_t^{cxe}}{P_t^d},\ p_t^{ce} = \frac{P_t^{ce}}{P_t^d},\ p_t^{d,ce} = \frac{P_t^{d,ce}}{P_t^d},\\
p_t^i &= \frac{\Psi_t P_t^i}{P_t^d},\ p_t^x = \frac{P_t^x}{P_t^{d,*}}\left(\tilde{z}_t^{+,*}\right)^{-\frac{1}{\eta_f}},\\
p_t^{m,c} &= \frac{P_t^{m,c}}{P_t^d},\ p_t^{m,i} = \frac{P_t^{m,i}}{P_t^d},\ p_t^{m,x} = \frac{P_t^{m,x}}{P_t^d},\ p_t^{m,ce} = \frac{P_t^{m,ce}}{P_t^d}.
\end{aligned}$$

These definitions imply the following ten restrictions across inflation rates for the relative prices of total consumption, aggregate non-energy consumption, energy consumption, domestically produced energy consumption, investment, exports, imported consumption goods, imported investment goods, goods imported for the production of exports, and imported energy consumption, respectively:

$$p_t^c = \frac{P_t^c}{P_t^d}\frac{P_{t-1}^d}{P_{t-1}^c}p_{t-1}^c = \frac{\pi_t^c}{\pi_t^d}p_{t-1}^c, \tag{8.1}$$

$$p_t^{cxe} = \frac{P_t^{cxe}}{P_t^d}\frac{P_{t-1}^d}{P_{t-1}^{cxe}}p_{t-1}^{cxe} = \frac{\pi_t^{cxe}}{\pi_t^d}p_{t-1}^{cxe}, \tag{8.2}$$

$$p_t^{ce} = \frac{P_t^{ce}}{P_t^d}\frac{P_{t-1}^d}{P_{t-1}^{ce}}p_{t-1}^{ce} = \frac{\pi_t^{ce}}{\pi_t^d}p_{t-1}^{ce}, \tag{8.3}$$

$$p_t^{d,ce} = \frac{P_t^{d,ce}}{P_t^d}\frac{P_{t-1}^d}{P_{t-1}^{d,ce}}p_{t-1}^{d,ce} = \frac{\pi_t^{d,ce}}{\pi_t^d}p_{t-1}^{d,ce}, \tag{8.4}$$

$$p_t^i = \frac{\Psi_t P_t^i}{P_t^d}\frac{P_{t-1}^d}{\Psi_{t-1}P_{t-1}^i}p_{t-1}^i = \frac{\mu_{\Psi,t}\pi_t^i}{\pi_t^d}p_{t-1}^i, \tag{8.5}$$

$$p_t^x = \frac{P_t^x}{P_t^{d,*}}\frac{P_{t-1}^{d,*}}{P_{t-1}^x}\left(\frac{\tilde{z}_t^{+,*}}{\tilde{z}_{t-1}^{+,*}}\right)^{-\frac{1}{\eta_f}}p_{t-1}^x = \frac{\pi_t^x}{\pi_t^{d,*}}\left(\frac{\mu_{z^{+,*},t}}{\mu_{z^+,t}}\right)^{-\frac{1}{\eta_f}}p_{t-1}^x, \tag{8.6}$$

where we have used equation (3.221),

$$p_t^{m,c} = \frac{P_t^{m,c}}{P_t^d}\frac{P_{t-1}^{m,c}}{P_{t-1}^d}p_{t-1}^{m,c} = \frac{\pi_t^{m,c}}{\pi_t^d}p_{t-1}^{m,c}, \tag{8.7}$$

$$p_t^{m,i} = \frac{P_t^{m,i}}{P_t^d}\frac{P_{t-1}^{m,i}}{P_{t-1}^d}p_{t-1}^{m,i} = \frac{\pi_t^{m,i}}{\pi_t^d}p_{t-1}^{m,i}, \tag{8.8}$$

$$p_t^{m,x} = \frac{P_t^{m,x}}{P_t^d}\frac{P_{t-1}^{m,x}}{P_{t-1}^d}p_{t-1}^{m,x} = \frac{\pi_t^{m,x}}{\pi_t^d}p_{t-1}^{m,x}, \tag{8.9}$$

and

$$p_t^{m,ce} = \frac{P_t^{m,ce}}{P_t^d}\frac{P_{t-1}^{m,ce}}{P_{t-1}^d}p_{t-1}^{m,ce} = \frac{\pi_t^{m,ce}}{\pi_t^d}p_{t-1}^{m,ce}. \tag{8.10}$$

Note that the first three and the fifth relative price equations (for aggregate and non-energy consumption, and investment) are not needed for the solution of the model since we already derived expressions (3.162), (3.160), (3.161), and (3.179).[39]

8.1 Log-linearization of the relative price restrictions

Log-linearizing equations (8.4) and (8.6)–(8.10) yields the following five expressions (ordered as above):

$$\hat{p}_t^{d,ce} = \hat{\pi}_t^{d,ce} - \hat{\pi}_t^d + \hat{p}_{t-1}^{d,ce}, \tag{8.11}$$

$$\hat{p}_t^x = \hat{\pi}_t^x - \hat{\pi}_t^* - \frac{1}{\eta_f}\left(\hat{\mu}_{z^+,*,t} - \hat{\mu}_{z^+,t}\right) + \hat{p}_{t-1}^x, \tag{8.12}$$

$$\hat{p}_t^{m,c} = \hat{\pi}_t^{m,c} - \hat{\pi}_t^d + \hat{p}_{t-1}^{m,c}, \tag{8.13}$$

$$\hat{p}_t^{m,i} = \hat{\pi}_t^{m,i} - \hat{\pi}_t^d + \hat{p}_{t-1}^{m,i}, \tag{8.14}$$

$$\hat{p}_t^{m,x} = \hat{\pi}_t^{m,x} - \hat{\pi}_t^d + \hat{p}_{t-1}^{m,x}. \tag{8.15}$$

$$\hat{p}_t^{m,ce} = \hat{\pi}_t^{m,ce} - \hat{\pi}_t^d + \hat{p}_{t-1}^{m,ce}. \tag{8.16}$$

The log-linear expression for the relative prices of the three different types of aggregate consumption and for investment have already been derived in Sections 3.3.2 and 3.4.2.

9 Real exchange rate and the terms of trade

We define the real exchange rate as

$$q_t = \frac{S_t P_t^{c,*}}{P_t^c},$$

and note that we can write the definition of the real exchange rate in terms of inflation rates and changes in the nominal exchange rate as follows:

$$q_t = \frac{S_t P_t^{c,*}}{P_t^c}\frac{P_{t-1}^c}{S_{t-1}P_{t-1}^{c,*}}q_{t-1} = \frac{s_t \pi_t^{c,*}}{\pi_t^c}q_{t-1}. \tag{9.1}$$

We define the terms of trade as

$$ToT_t = \frac{S_t P_t^x \left(\tilde{z}_t^{+,*}\right)^{-\frac{1}{\eta_f}}}{P_t^M},$$

where P_t^M is the price at time t of the total bundle of imports.

We next derive a price index for total imports, to be used in the definition of terms of trade. From equation (3.225), we have that

$$M_t = C_t^m \left(\mathring{p}_t^{m,c}\right)^{\frac{\lambda_t^{m,c}}{1-\lambda_t^{m,c}}} + I_t^m \left(\mathring{p}_t^{m,i}\right)^{\frac{\lambda_t^{m,i}}{1-\lambda_t^{m,i}}} + X_t^m \left(\mathring{p}_t^{m,x}\right)^{\frac{\lambda_t^{m,x}}{1-\lambda_t^{m,x}}} + C_t^{e,m} \left(\mathring{p}_t^{m,ce}\right)^{\frac{\lambda_t^{m,ce}}{1-\lambda_t^{m,ce}}}.$$

[39] The inclusion of these four equations instead of (3.162), (3.160), (3.161) and (3.179) causes a collinearity issue when solving the model.

We then define the price index for total imports, P_t^M, in the following way:

$$P_t^M M_t = P_t^{m,c} C_t^m \left(\mathring{p}_t^{m,c}\right)^{\frac{\lambda_t^{m,c}}{1-\lambda_t^{m,c}}} + P_t^{m,i} I_t^m \left(\mathring{p}_t^{m,i}\right)^{\frac{\lambda_t^{m,i}}{1-\lambda_t^{m,i}}} + P_t^{m,x} X_t^m \left(\mathring{p}_t^{m,x}\right)^{\frac{\lambda_t^{m,x}}{1-\lambda_t^{m,x}}} + P_t^{m,ce} C_t^{e,m} \left(\mathring{p}_t^{m,ce}\right)^{\frac{\lambda_t^{m,ce}}{1-\lambda_t^{m,ce}}}.$$

Given that full indexation implies that $\mathring{p}_t^{m,c} = \mathring{p}_t^{m,i} = \mathring{p}_t^{m,x} = 1$, we ignore the price dispersion terms to simplify calculations. Then,

$$P_t^M M_t = P_t^{m,c} C_t^m + P_t^{m,i} I_t^m + P_t^{m,x} X_t^m + P_t^{m,ce} C_t^{e,m}.$$

We divide through by $S_t P_t^x \left(\tilde{z}_t^{+,*}\right)^{-\frac{1}{\eta_f}}$ and rewrite in terms of relative prices[40]

$$\begin{aligned}
\frac{P_t^M}{S_t P_t^x \left(\tilde{z}_t^{+,*}\right)^{-\frac{1}{\eta_f}}} M_t &= \frac{P_t^{m,c}}{P_t^d} \frac{P_t^d}{P_t^c} \frac{P_t^c}{S_t P_t^{c,*}} \frac{P_t^{c,*}}{P_t^{d,*}} \frac{P_t^{d,*}}{P_t^x \left(\tilde{z}_t^{+,*}\right)^{-\frac{1}{\eta_f}}} C_t^m \\
&+ \frac{P_t^{m,i}}{P_t^d} \frac{P_t^d}{P_t^c} \frac{P_t^c}{S_t P_t^{c,*}} \frac{P_t^{c,*}}{P_t^{d,*}} \frac{P_t^{d,*}}{P_t^x \left(\tilde{z}_t^{+,*}\right)^{-\frac{1}{\eta_f}}} I_t^m \\
&+ \frac{P_t^{m,x}}{P_t^d} \frac{P_t^d}{P_t^c} \frac{P_t^c}{S_t P_t^{c,*}} \frac{P_t^{c,*}}{P_t^{d,*}} \frac{P_t^{d,*}}{P_t^x \left(\tilde{z}_t^{+,*}\right)^{-\frac{1}{\eta_f}}} X_t^m \\
&+ \frac{P_t^{m,ce}}{P_t^d} \frac{P_t^d}{P_t^c} \frac{P_t^c}{S_t P_t^{c,*}} \frac{P_t^{c,*}}{P_t^{d,*}} \frac{P_t^{d,*}}{P_t^x \left(\tilde{z}_t^{+,*}\right)^{-\frac{1}{\eta_f}}} C_t^{e,m}
\end{aligned}$$

$$\frac{1}{ToT_t} M_t = \frac{p_t^{m,c} p_t^{c,*}}{p_t^c q_t p_t^x} C_t^m + \frac{p_t^{m,i} p_t^{c,*}}{p_t^c q_t p_t^x} I_t^m + \frac{p_t^{m,x} p_t^{c,*}}{p_t^c q_t p_t^x} X_t^m + \frac{p_t^{m,ce} p_t^{c,*}}{p_t^c q_t p_t^x} C_t^{e,m}.$$

Rearranging,

$$\frac{p_t^c q_t p_t^x}{p_t^{c,*} ToT_t} M_t = p_t^{m,c} C_t^m + p_t^{m,i} I_t^m + p_t^{m,x} X_t^m + p_t^{m,ce} C_t^{e,m}. \tag{9.2}$$

Alternatively, we could define

$$ToT_t^c = \frac{S_t P_t^x}{P_t^{m,c}},$$

as well as corresponding measures for investment and exports. In terms of relative prices, we would then have

$$\begin{aligned}
ToT_t^c &= \frac{P_t^d}{P_t^{m,c}} \frac{P_t^c}{P_t^d} \frac{S_t P_t^{c,*}}{P_t^c} \frac{P_t^*}{P_t^{c,*}} \frac{P_t^x}{P_t^*} \\
&= \frac{p_t^c q_t p_t^x}{p_t^{c,*} p_t^{m,c}}.
\end{aligned}$$

[40] Note that

$$p_t^x = \frac{P_t^x}{P_t^{d,*}} \left(\tilde{z}_t^{+,*}\right)^{-\frac{1}{\eta_f}}$$

simplifies to

$$p_t^x = \frac{P_t^x}{P_t^{d,*}}$$

if the technological growth rates in the two economies coincide.

9.1 Scaling of the terms of trade

Scaling,

$$\frac{p_t^c q_t p_t^x}{p_t^{c,*} ToT_t} \frac{M_t}{z_t^+} = p_t^{m,c} \frac{C_t^m}{z_t^+} + p_t^{m,i} \frac{I_t^m}{z_t^+} + p_t^{m,x} \frac{X_t^m}{z_t^+} + p_t^{m,ce} \frac{C_t^{e,m}}{z_t^+}$$

$$\frac{p_t^c q_t p_t^x}{p_t^{c,*} ToT_t} m_t = p_t^{m,c} c_t^m + p_t^{m,i} i_t^m + p_t^{m,x} x_t^m + p_t^{m,ce} c_t^{e,m}. \tag{9.3}$$

We note that, in steady state,

$$ToT = \frac{p^c q p^x m}{p^{c,*} \left(p^{m,c} c^m + p^{m,i} i^m + p^{m,x} x^m + p^{m,ce} c^{e,m} \right)}.$$

9.2 Log-linearization of the real exchange rate and the terms of trade

Log-linearizing (9.1), we have

$$\hat{q}_t = \hat{s}_t + \hat{\pi}_t^{c,*} - \hat{\pi}_t^c + \hat{q}_{t-1}. \tag{9.4}$$

Log-linearizing the expression for the terms of trade as given by equation (9.3),

$$\begin{aligned} & \frac{p^c q p^x m}{p_t^{c,*} ToT} \left(\hat{p}_t^c + \hat{q}_t + \hat{p}_t^x + \hat{m}_t - \hat{p}_t^{c,*} - \widehat{ToT}_t \right) \\ = \quad & p^{m,c} c^m \left(\hat{p}_t^{m,c} + \hat{c}_t^m \right) + p^{m,i} i^m \left(\hat{p}_t^{m,i} + \hat{i}_t^m \right) \\ & + p^{m,x} x^m \left(\hat{p}_t^{m,x} + \hat{x}_t^m \right) + p^{m,ce} c^{e,m} \left(\hat{p}_t^{m,ce} + \hat{c}_t^{e,m} \right). \end{aligned}$$

We can rewrite this as

$$\begin{aligned} & \hat{p}_t^c + \hat{q}_t + \hat{p}_t^x + \hat{m}_t - \hat{p}_t^{c,*} - \widehat{ToT}_t \\ = \quad & \frac{p_t^{c,*} ToT}{p^c q p^x} \left[p^{m,c} \frac{c^m}{m} \left(\hat{p}_t^{m,c} + \hat{c}_t^m \right) + p^{m,i} \frac{i^m}{m} \left(\hat{p}_t^{m,i} + \hat{i}_t^m \right) \right. \\ & \left. + p^{m,x} \frac{x^m}{m} \left(\hat{p}_t^{m,x} + \hat{x}_t^m \right) + p^{m,ce} \frac{c^{e,m}}{m} \left(\hat{p}_t^{m,ce} + \hat{c}_t^{e,m} \right) \right] \end{aligned}$$

Rearranging, we finally obtain

$$\begin{aligned} \widehat{ToT}_t \quad = \quad & -\frac{p_t^{c,*} ToT}{p^c q p^x} \left[p^{m,c} \frac{c^m}{m} \left(\hat{p}_t^{m,c} + \hat{c}_t^m \right) + p^{m,i} \frac{i^m}{m} \left(\hat{p}_t^{m,i} + \hat{i}_t^m \right) \right. \\ & \left. + p^{m,x} \frac{x^m}{m} \left(\hat{p}_t^{m,x} + \hat{x}_t^m \right) + p^{m,ce} \frac{c^{e,m}}{m} \left(\hat{p}_t^{m,ce} + \hat{c}_t^{e,m} \right) \right] \\ & + \hat{p}_t^c + \hat{q}_t + \hat{p}_t^x - \hat{p}_t^{c,*} + \hat{m}_t. \end{aligned} \tag{9.5}$$

Log-linearizing the alternative definition would yield

$$\widehat{ToT}_t^c = \hat{p}_t^c + \hat{q}_t + \hat{p}_t^x - \hat{p}_t^{c,*} - \hat{p}_t^{m,c}.$$

10 Exogenous processes

The domestic side of our model contains a total of 23 exogenous processes, most of which are given by AR(1) processes

$$\log e_t = (1 - \rho_e) \log e + \rho_e \log e_{t-1} + \sigma_e \varepsilon_{e,t},$$

where $e = \epsilon, \Upsilon, \mu_z, \mu_\Psi, \lambda^d, \lambda^{m,c}, \lambda^{m,i}, \lambda^{m,x}, \lambda^{m,ce}, \lambda^x, \lambda^w, \nu_t^{wc,d}, \nu_t^{wc,m}, \nu_t^{wc,x}, \zeta^\beta, \zeta^c, \zeta^n, \chi, \tilde{\phi}, \varepsilon_R, \bar{\pi}^c, p^{d,ce}, g$, and $\varepsilon_{e,t} \sim N(0,1)$.[41] In practice, it may be empirically motivated to allow some of the shock processes to be correlated with their foreign counterparts, and to model some as ARMA processes. Moreover, some of the shock processes are turned off in the estimation. In this document, we restrict the discussion to the theoretical model which was used as the starting point for the estimations, and leave empirical considerations aside.

In terms of log-deviations from steady state, we thus have: the neutral stationary technology shock in equation (3.6)

$$\hat{\epsilon}_t = \rho_\epsilon \hat{\epsilon}_{t-1} + \sigma_\epsilon \varepsilon_{\epsilon,t}, \tag{10.1}$$

the investment-specific stationary technology shock in equation (4.22)

$$\hat{\Upsilon}_t = \rho_\Upsilon \hat{\Upsilon}_{t-1} + \sigma_\Upsilon \varepsilon_{\Upsilon,t}, \tag{10.2}$$

the shock to neutral technology growth in equation (3.7)

$$\hat{\mu}_{z,t} = \rho_{\mu_z} \hat{\mu}_{z,t-1} + \sigma_{\mu_z} \varepsilon_{\mu_z,t}, \tag{10.3}$$

the shock to investment-specific technology growth in equation (3.8)

$$\hat{\mu}_{\Psi,t} = \rho_{\mu_\Psi} \hat{\mu}_{\Psi,t-1} + \sigma_{\mu_\Psi} \varepsilon_{\mu_\Psi,t}, \tag{10.4}$$

the domestic price markup shock in equation (3.2)

$$\hat{\lambda}_t^d = \rho_{\lambda^d} \hat{\lambda}_{t-1}^d + \sigma_{\lambda^d} \varepsilon_{\lambda^d,t}, \tag{10.5}$$

the import (non-energy) consumption price markup shock

$$\hat{\lambda}_t^{m,c} = \rho_{\lambda^{m,c}} \hat{\lambda}_{t-1}^{m,c} + \sigma_{\lambda^{m,c}} \varepsilon_{\lambda^{m,c},t}, \tag{10.6}$$

the import investment price markup shock

$$\hat{\lambda}_t^{m,i} = \rho_{\lambda^{m,i}} \hat{\lambda}_{t-1}^{m,i} + \sigma_{\lambda^{m,i}} \varepsilon_{\lambda^{m,i},t}, \tag{10.7}$$

the import-to-export price markup shock

$$\hat{\lambda}_t^{m,x} = \rho_{\lambda^{m,x}} \hat{\lambda}_{t-1}^{m,x} + \sigma_{\lambda^{m,x}} \varepsilon_{\lambda^{m,x},t}, \tag{10.8}$$

tha import energy consumption markup shock

$$\hat{\lambda}_t^{m,ce} = \rho_{\lambda^{m,ce}} \hat{\lambda}_{t-1}^{m,ce} + \sigma_{\lambda^{m,ce}} \varepsilon_{\lambda^{m,ce},t}, \tag{10.9}$$

all four in equation (3.59), the export price markup shock in equation (3.181)

$$\hat{\lambda}_t^x = \rho_{\lambda^x} \hat{\lambda}_{t-1}^x + \sigma_{\lambda^x} \varepsilon_{\lambda^x,t}, \tag{10.10}$$

the wage markup shock in equation (4.41)

$$\hat{\lambda}_t^w = \rho_{\lambda^w} \hat{\lambda}_{t-1}^w + \sigma_{\lambda^w} \varepsilon_{\lambda^w,t}, \tag{10.11}$$

the fraction of the domestic firms' wage bill that has to be financed in advance in equation (3.11)

$$\hat{\nu}_t^{wc,d} = \rho_{\nu^{wc,d}} \hat{\nu}_{t-1}^{wc,d} + \sigma_{\nu^{wc,d}} \varepsilon_{\nu^{wc,d},t}, \tag{10.12}$$

[41] In Ramses I and II, the processes for the fractions of the firms' costs that have to be financed in advance, ν_t^f, ν_t^*, and ν_t^x, are assumed to be constants. Here, we define them as AR(1) processes, that can be switched on and off, depending on the calibration.

the fraction of the import retailers' costs that has to be financed in advance in equation (3.63)

$$\hat{\nu}_t^{wc,m} = \rho_{\nu^{wc,m}} \hat{\nu}_{t-1}^{wc,m} + \sigma_{\nu^{wc,m}} \varepsilon_{\nu^{wc,m},t}, \tag{10.13}$$

he fraction of the export producers' costs that has to be financed in advance in equation (3.186)

$$\hat{\nu}_t^{wc,x} = \rho_{\nu^{wc,x}} \hat{\nu}_{t-1}^{wc,x} + \sigma_{\nu^{wc,x}} \varepsilon_{\nu^{wc,x},t}, \tag{10.14}$$

the shock to the houshold's discount rate in equation (4.2)

$$\hat{\zeta}_t^{\beta} = \rho_{\zeta^{\beta}} \hat{\zeta}_{t-1}^{\beta} + \sigma_{\zeta^{\beta}} \varepsilon_{\zeta^{\beta},t}, \tag{10.15}$$

the shock to consumption preferences in equation (4.3)

$$\hat{\zeta}_t^{c} = \rho_{\zeta^{c}} \hat{\zeta}_{t-1}^{c} + \sigma_{\zeta^{c}} \varepsilon_{\zeta^{c},t}, \tag{10.16}$$

the labour supply shock in equation (4.4)[42]

$$\hat{\zeta}_t^{n} = \rho_{\zeta^{n}} \hat{\zeta}_{t-1}^{n} + \sigma_{\zeta^{n}} \varepsilon_{\zeta^{n},t}, \tag{10.17}$$

the household risk premium shock in equation (4.12)

$$\hat{\chi}_t = \rho_{\chi} \hat{\chi}_{t-1} + \sigma_{\chi} \varepsilon_{\chi,t}, \tag{10.18}$$

the country risk premium shock in equation (4.18)

$$\hat{\tilde{\phi}}_t = \rho_{\tilde{\phi}} \hat{\tilde{\phi}}_{t-1} + \sigma_{\tilde{\phi}} \varepsilon_{\tilde{\phi},t}, \tag{10.19}$$

the monetary policy shock in equation (5.2)

$$\hat{\varepsilon}_{R,t} = \rho_{\varepsilon_R} \hat{\varepsilon}_{R,t-1} + \sigma_{\varepsilon_R} \varepsilon_{\varepsilon_R,t}, \tag{10.20}$$

the inflation target shock in equation (3.19)

$$\hat{\bar{\pi}}_t^{c} = \rho_{\bar{\pi}^{c}} \hat{\bar{\pi}}_{t-1}^{c} + \sigma_{\bar{\pi}^{c}} \varepsilon_{\bar{\pi}^{c},t}, \tag{10.21}$$

the evolution of the relative price of energyin equation (3.163)

$$\hat{p}_t^{d,ce} = \rho_{p^{d,ce}} \hat{p}_{t-1}^{d,ce} + \sigma_{p^{d,ce}} \varepsilon_{p^{d,ce},t}. \tag{10.22}$$

and government consumption expenditures in equation (5.4)

$$\hat{g}_t = \rho_g \hat{g}_{t-1} + \sigma_g \varepsilon_{g,t}. \tag{10.23}$$

[42] We note that the labour supply shock in theory is modified to a random walk (allowing for drift in the labour supply), i.e.

$$\hat{\zeta}_t^{n} = \hat{\zeta}_{t-1}^{n} + \sigma_{\zeta^{n}} \varepsilon_{\zeta^{n},t}.$$

In practice, however, this is implemented by setting $\rho_{\zeta^n} = 0.999$ in the AR process listed in the main text.

Also, as a shortcut to allow for composition effects from variations in the labour force (in a boom workers of lower productivity than average tend to enter the labour force; see the code from Lindé, Maih, and Wouters (2017)) the stationary technology shock is modified to

$$\hat{\epsilon}_t = \rho_{\epsilon} \hat{\epsilon}_{t-1} + \sigma_{\epsilon} \varepsilon_{\epsilon,t} + \sigma_{\epsilon,\zeta^h} \varepsilon_{\zeta^h,t}.$$

Potentially one could, following Lindé, Maih, and Wouters (2017), modify the labour supply shock to

$$\hat{\zeta}_t^{n} = \rho_{\zeta^n} \hat{\zeta}_{t-1}^{n} + \sigma_{\zeta^{n}} \varepsilon_{\zeta^{n},t} + \sigma_{\zeta^{n},\epsilon} \varepsilon_{\epsilon,t},$$

i.e., allow the technology shock to affect labour supply.

11 Foreign economy

11.1 Structural model of the foreign economy

The structure of the foreign economy is analogous to that of the domestic economy. All functional forms (utility function, production technologies, various costs) are unchanged. Nonetheless, given our small open-economy assumption, the influence of the domestic economy is negligible, and the foreign economy is thus approximately closed. Most of this block's optimal conditions are identical to those of the domestic block, with variables and parameters assigned an additional superscript "$*$" where appropriate. We won't derive the full set of foreign's optimal conditions in this section (the derivations can be found in the corresponding sections for the home economy), but we will rather focus on the equations that differ with respect to what has been previously derived. The entire set of equilibrium conditions is presented in Section 13.2.[43]

11.1.1 Firms

The structure of the foreign economy's corporate sector is analogous to that of the domestic economy's homogeneous good sector. Foreign intermediate good firms produce the differentiated goods using the same technology as in (3.5), that is:

$$Y_{i,t}^{*} = \left(z_t^{*} N_{i,t}^{*}\right)^{1-\alpha^{*}} \epsilon_t^{*} \left(K_{i,t}^{*}\right)^{\alpha^{*}} - z_t^{+,*}\phi^{*}, \tag{11.1}$$

where all variables are defined as in the domestic economy. As for the domestic economy, we assume that the foreign economy has two sources of growth: a positive drift in the neutral technology, z_t^{*}, and a positive drift in the investment-specific technology, Ψ_t^{*}. The stochastic processes of ϵ_t^{*} and the growth rates of z_t^{*} and $z_t^{+,*}$ are defined as in (3.6), (3.7) and (3.9). The goods are sold monopolistically to foreign retailers who produce the homogeneous foreign good, Y_t^{*}, by aggregating the intermediate foreign goods $Y_{i,t}^{*}$ as in (3.1):

$$Y_t^{*} = \left[\int_0^1 \left(Y_{i,t}^{*}\right)^{\frac{1}{\lambda_t^{*}}} di\right]^{\lambda_t^{*}}, \qquad 1 \leq \lambda_t^{*} \leq \infty,$$

where λ_t^{*} is a stochastic process determining the time-varying price markup in the foreign goods market defined as in (3.2). From the cost-minimization problem of the foreign intermediate goods producers, we obtain expressions for the marginal products of labour and capital as in (3.35) and (3.36), for marginal costs as in (3.37) and (3.38), and for the nominal rental rate of capital services as in (3.17). We moreover have a corresponding expression to the gross effective nominal interest rate faced by firms in (3.10), the price setting equations (3.22) and (3.23), and the price dispersion equation in (3.27). After log-linearization, we end up with the starred version of the following eight equations: the two marginal cost expressions for foreign intermediate goods producers (3.46) and (3.47), the marginal product of labour (3.48), the marginal product of capital (3.49), the capital-to-labour ratio (3.50), the gross effective nominal interest rate (3.42), the Phillips curve (3.56), and the price dispersion expression (3.57). We also need to include the starred version of the log-linearized expression of the technological growth rate $\mu_{z^{+},t}$ in (3.51).

The foreign homogeneous good is allocated among the alternatives uses as follows:

$$Y_t^{*} = G_t^{*} + C_t^{d,*} + C_t^{e,*} + I_t^{d,*}. \tag{11.2}$$

The fact that X_t^{*} does not appear in the previous equation arises from the small open-economy assumption – foreign exports to the home economy represent a negligible share of total output and the foreign export sector can thus be omitted from the analysis. By the same reasoning, although the

[43] The foreign economy model is very close to the model in Smets and Wouters (2003), on which the first Ramses model (Adolfson et al. (2005) and Adolfson et al. (2007)) also was based.

foreign economy does include an import sector, we can omit the production of imported intermediate goods in the foreign block. Again, this is because imports from the small open economy form a negligible part of the foreign consumption and investment bundles, which implies that variations in import prices have an insignificant impact on the evolution of the foreign price index $P_t^{d,*}$.

We next turn to the production of final goods. We start by assuming that final consumption and investment goods are produced by foreign, competitive, representative firms using the same production technologies as in (3.115), (3.116) and (3.164), and thus

$$C_t^* = \left[(1-\omega_e^*)^{\frac{1}{\eta_e^*}} \left(C_t^{xe,*}\right)^{\frac{\eta_e^*-1}{\eta_e^*}} + (\omega_e^*)^{\frac{1}{\eta_c^*}} \left(C_t^{e,*}\right)^{\frac{\eta_e^*-1}{\eta_e^*}} \right]^{\frac{\eta_e^*}{\eta_e^*-1}}, \tag{11.3}$$

$$C_t^{xe,*} = \left[(1-\omega_c^*)^{\frac{1}{\eta_c^*}} \left(C_t^{d,*}\right)^{\frac{\eta_c^*-1}{\eta_c^*}} + (\omega_c^*)^{\frac{1}{\eta_c}} \left(C_t^{m,*}\right)^{\frac{\eta_c^*-1}{\eta_c^*}} \right]^{\frac{\eta_c^*}{\eta_c^*-1}}, \tag{11.4}$$

$$I_t^* + a\left(u_t^*\right) K_t^{p,*} = \Psi_t^* \left[(1-\omega_i^*)^{\frac{1}{\eta_i^*}} \left(I_t^{d,*}\right)^{\frac{\eta_i^*-1}{\eta_i^*}} + (\omega_i^*)^{\frac{1}{\eta_i}} \left(I_t^{m,*}\right)^{\frac{\eta_i^*-1}{\eta_i^*}} \right]^{\frac{\eta_i^*}{\eta_i^*-1}}. \tag{11.5}$$

As noted above, however, the small open-economy assumption implies that imports represent a negligible share in the consumption and investment bundles, such that

$$\begin{aligned} \omega_c^* &\rightarrow 0 \\ \omega_i^* &\rightarrow 0. \end{aligned}$$

Hence, (11.4) and (11.5) become:

$$C_t^{xe,*} = C_t^{d,*}, \tag{11.6}$$

$$I_t^* + a\left(u_t^*\right) K_t^{p,*} = \Psi_t^* I_t^{d,*}. \tag{11.7}$$

The representative final consumption good firm takes the price of output $P_t^{c,*}$ and the prices of inputs $P_t^{d,*}$, and $P_t^{ce,*}$ as given. It faces the following budget constraint:

$$P_t^{c,*} C_t^* = P_t^{cxe,*} C_t^{xe,*} + P_t^{ce,*} C_t^{e,*}, \tag{11.8}$$

where $P_t^{cxe,*} C_t^{xe,*}$ denotes the expenditures on non-energy goods and $P_t^{ce,*} C_t^{e,*}$ the expenditures on energy. Just as in the domestic economy, we have that

$$C_t^{xe,*} = (1-\omega_e^*) \left[\frac{P_t^{cxe,*}}{P_t^{c,*}} \right]^{-\eta_e^*} C_t^*, \tag{11.9}$$

$$C_t^{e,*} = \omega_e^* \left[\frac{P_t^{ce,*}}{P_t^{c,*}} \right]^{-\eta_e^*} C_t^*. \tag{11.10}$$

Moreover, we have that

$$P_t^{c,*} = \left[(1-\omega_e^*) \left(P_t^{cxe,*}\right)^{1-\eta_e^*} + \omega_e^* \left(P_t^{ce,*}\right)^{1-\eta_e^*} \right]^{1/(1-\eta_e^*)}, \tag{11.11}$$

$$P_t^{cxe,*} = \left[(1-\omega_c^*) \left(P_t^{d,*}\right)^{1-\eta_c^*} + \omega_c^* \left(P_t^{m,c,*}\right)^{1-\eta_c^*} \right]^{1/(1-\eta_c^*)}. \tag{11.12}$$

Letting $\omega_c^* \rightarrow 0$, we get that

$$P_t^{cxe,*} = P_t^{d,*}. \tag{11.13}$$

Using the definitions

$$p_t^{c,*} = \frac{P_t^{c,*}}{P_t^{d,*}}, \tag{11.14}$$

$$p_t^{cxe,*} = \frac{P_t^{cxe,*}}{P_t^{d,*}}, \tag{11.15}$$

$$p_t^{ce,*} = \frac{P_t^{ce,*}}{P_t^{d,*}}, \tag{11.16}$$

we can rewrite (11.11) as

$$p_t^{c,*} = \left[(1-\omega_e^*) + \omega_e^* \left(p_t^{ce,*}\right)^{1-\eta_e^*}\right]^{1/(1-\eta_e^*)}. \tag{11.17}$$

The rate of inflation of the foreign aggregate CPI is then given by

$$\pi_t^{c,*} = \frac{P_t^{c,*}}{P_{t-1}^{c,*}} = \left[\frac{(1-\omega_e^*)\left(P_t^{cxe,*}\right)^{1-\eta_e^*} + \omega_e^*\left(P_t^{ce,*}\right)^{1-\eta_e^*}}{(1-\omega_e^*)\left(P_{t-1}^{cxe,*}\right)^{1-\eta_e^*} + \omega_e^*\left(P_{t-1}^{ce,*}\right)^{1-\eta_e^*}}\right]^{\frac{1}{1-\eta_e^*}}, \tag{11.18}$$

or, in terms of relative prices,

$$\begin{aligned}\pi_t^{c,*} &= \left(\frac{P_t^{c,*}}{P_{t-1}^{c,*}}\frac{P_{t-1}^{d,*}}{P_t^{d,*}}\right)\frac{P_t^{d,*}}{P_{t-1}^{d,*}} = \frac{p_t^{c,*}}{p_{t-1}^{c,*}}\pi_t^{d,*} \\ &= \pi_t^{d,*}\left[\frac{(1-\omega_e^*) + \omega_e^*\left(p_t^{ce,*}\right)^{1-\eta_e^*}}{(1-\omega_e^*) + \omega_e^*\left(p_{t-1}^{ce,*}\right)^{1-\eta_e^*}}\right]^{\frac{1}{1-\eta_e^*}}. \end{aligned} \tag{11.19}$$

We note also that

$$\pi_t^{cxe,*} = \pi_t^{d,*}, \tag{11.20}$$

$$\pi_t^{c,*} = \frac{p_t^{c,*}}{p_{t-1}^{c,*}}\pi_t^{d,*}, \tag{11.21}$$

and that

$$\pi_t^{ce,*} = \frac{p_t^{ce,*}}{p_{t-1}^{ce,*}}\pi_t^{d,*}. \tag{11.22}$$

For investment we have that

$$P_t^{i,*} = \frac{1}{\Psi_t^*}\left[(1-\omega_i^*)\left(P_t^{d,*}\right)^{1-\eta_i^*} + \omega_i^*\left(P_t^{m,i,*}\right)^{1-\eta_i^*}\right]^{1/\left(1-\eta_i^*\right)}.$$

Letting $\omega_i^* \to 0$, we get that

$$P_t^{i,*} = \frac{1}{\Psi_t^*}P_t^{d,*}. \tag{11.23}$$

Using the definition

$$p_t^{i,*} \equiv \frac{\Psi_t^* P_t^{i,*}}{P_t^{d,*}},$$

(11.23) becomes

$$p_t^{i,*} = 1. \tag{11.24}$$

Moreover, the rate of inflation of the foreign investment good is given by

$$\pi_t^{i,*} = \frac{P_t^{i,*}}{P_{t-1}^{i,*}} = \frac{\pi_t^{d,*}}{\mu_{\Psi^*,t}}. \tag{11.25}$$

We note that, as the foreign economy is closed, the good Y_t^* is the single produced good in the economy (other than energy). It is sold at the price $P_t^{d,*}$, and used directly for consumption and investment by the households. The reason that investment price inflation may deviate from the consumption (and domestic) price inflation is the fact that we allow for a potentially different trend in the investment production technology, just as in Section 3.4 of the domestic problem earlier.

In scaled form, for consumption, we obtain

$$c_t^* = \left[(1-\omega_e^*)^{\frac{1}{\eta_e^*}} \left(c_t^{xe,*}\right)^{\frac{\eta_e^*-1}{\eta_e^*}} + (\omega_e^*)^{\frac{1}{\eta_c^*}} \left(c_t^{e,*}\right)^{\frac{\eta_e^*-1}{\eta_e^*}}\right]^{\frac{\eta_e^*}{\eta_e^*-1}}, \tag{11.26}$$

where

$$c_t^{xe,*} = (1-\omega_e^*)\left[\frac{P_t^{cxe,*}}{P_t^{c,*}}\right]^{-\eta_e^*} c_t^*, \tag{11.27}$$

$$c_t^{e,*} = \omega_e^*\left[\frac{P_t^{ce,*}}{P_t^{c,*}}\right]^{-\eta_e^*} c_t^*. \tag{11.28}$$

Using the definitions of the relative prices $p_t^{c,*}$, $p_t^{cxe,*}$ and $p_t^{ce,*}$ specified above, we have

$$c_t^{xe,*} = (1-\omega_e^*)\left(\frac{1}{p_t^{c,*}}\right)^{-\eta_e^*} c_t^*, \tag{11.29}$$

$$c_t^{e,*} = \omega_e^*\left(\frac{p_t^{ce,*}}{p_t^{c,*}}\right)^{-\eta_e^*} c_t^*. \tag{11.30}$$

For investment, we have

$$i_t^{d,*} = i_t^* + \frac{a\left(u_t^*\right) k_t^{p,*}}{\mu_{z^{+,*},t}\mu_{\Psi^*,t}}. \tag{11.31}$$

Before turning to the foreing household's optimization problem, we first log-linearize the few optimal conditions that we just derived for the foreign economy and that differ from the domestic ones. From (11.29) and (11.30) we directly obtain

$$\hat{c}_t^{xe,*} = \eta_e^*\hat{p}_t^{c,*} + \hat{c}_t^*, \tag{11.32}$$

$$\hat{c}_t^{e,*} = -\eta_e^*\left(\hat{p}_t^{ce,*} - \hat{p}_t^{c,*}\right) + \hat{c}_t^*. \tag{11.33}$$

We recall also that the following holds:

$$\hat{c}_t^{xe,*} = \hat{c}_t^{d,*}. \tag{11.34}$$

To obtain a log-linear expression for the foreign aggregate CPI, we can log-linearize equation (11.11) in levels to obtain

$$\hat{P}_t^{c,*} = (1-\omega_e^*)\left(\frac{1}{p^{c,*}}\right)^{1-\eta_e^*} \hat{P}_t^{cxe,*} + \omega_e^*\left(\frac{p^{ce,*}}{p^{c,*}}\right)^{1-\eta_e^*} \hat{P}_t^{ce,*}, \tag{11.35}$$

where we have used the definitions of relative prices $p_t^{c,*}$ and $p^{ce,*}$ specified above. Lagging one period and differencing, and using that the definitions of the inflation rates $\pi_t^{cxe,*}$, $\pi_t^{c,*}$ and $\pi_t^{ce,*}$ in Section 2.2 imply that

$$\hat{\pi}_t^{cxe,*} = \hat{P}_t^{cxe,*} - \hat{P}_{t-1}^{cxe,*}, \tag{11.36}$$

$$\hat{\pi}_t^{c,*} = \hat{P}_t^{c,*} - \hat{P}_{t-1}^{c,*}, \tag{11.37}$$

$$\hat{\pi}_t^{ce,*} = \hat{P}_t^{ce,*} - \hat{P}_{t-1}^{ce,*}, \tag{11.38}$$

we obtain the following log-linear expression for the foreign CPI inflation in terms of non-energy and energy price inflation,

$$\hat{\pi}_t^{c,*} = (1-\omega_e^*)\left(\frac{1}{p^{c,*}}\right)^{1-\eta_e^*}\hat{\pi}_t^{cxe,*} + \omega_e^*\left(\frac{p^{ce,*}}{p^{c,*}}\right)^{1-\eta_e^*}\hat{\pi}_t^{ce,*}. \tag{11.39}$$

We note also that taking logs of expression (11.20) gives

$$\hat{\pi}_t^{cxe,*} = \hat{\pi}_t^{d,*}. \tag{11.40}$$

From equation (11.22), we have

$$\hat{\pi}_t^{ce,*} = \hat{p}_t^{ce,*} - \hat{p}_{t-1}^{ce,*} + \pi_t^{d,*}. \tag{11.41}$$

Just as for the domestic economy, we assume that the relative price of energy evolves as an exogenous process, so that

$$\log p_t^{ce,*} = \left(1-\rho_{p^{ce,*}}\right)\log p^{ce,*} + \rho_{p^{ce,*}}\log p_{t-1}^{ce,*} + \sigma_{p^{ce,*}}\varepsilon_{p^{ce,*},t}. \tag{11.42}$$

We also log-linearize the relative price expression in equation (11.17), which yields the following expression:

$$\widehat{p}_t^{c,*} = \omega_e^*\left(\frac{p^{ce,*}}{p^{c,*}}\right)^{1-\eta_e^*}\widehat{p}_t^{ce,*}. \tag{11.43}$$

(11.31) yields

$$\hat{i}_t^{d,*} = \frac{1}{i^*}\left(i^*\hat{i}_t^* + \frac{\sigma_b^* k^{p,*}}{\mu_{z^+,*}\mu_{\Psi^*}}\hat{u}_t^*\right). \tag{11.44}$$

Finally, from equations (11.24) and (11.25), we have

$$\hat{p}_t^{i,*} = 0, \tag{11.45}$$

and

$$\hat{\pi}_t^{i,*} = \hat{\pi}_t^{d,*} - \hat{\mu}_{\Psi^*,t}. \tag{11.46}$$

11.1.2 Households

The household problem in the foreign economy is very similar to the domestic economy one, but for the following exception: the small open-economy assumption implies that foreign households do not have access to the domestic bond markets. As a result, the budget contraint for foreign households is slightly different from (4.10), and is given by:

$$\begin{aligned} & P_t^{c,*}C_t^* + P_t^{i,*}\left(I_t^* + a\left(u_t^*\right)K_t^{p,*}\right) + P_{k',t}^*\Delta_t^* + B_{t+1}^* \\ = & \int_0^1\int_0^{n_{j,t}^*} W_{j,k,t}^* dk dj + R_t^{k,*}u_t^* K_t^{p,*} + R_{t-1}^*\chi_t^* B_t^* + \Pi_t^* + TR_t^*, \end{aligned} \tag{11.47}$$

where all variables are defined analogously to those in the domestic economy. The household's utility maximization problem, analogous to (4.29), thus results in first-order conditions for the foreign economy corresponding to (4.30), (4.31), and (4.33)–(4.36). The preference shifters are specified in an analogous way to (4.5), (4.6) and (4.7). Moreover, the preference shocks pertaining to the foreign economy's household problem, $\zeta_t^{\beta,*}$, $\zeta_t^{c,*}$ and $\zeta_t^{n,*}$, are defined as in (4.2), (4.3) and (4.4), and the risk premium shock χ_t^* as in (4.12).

The law of motion for captital is given by the process specified in (4.21), and efficient capital is assumed to relate to physical capital as in (4.28).

Unemployment and labour supply are specified as in (4.38) and (4.39).

The wage setting problem is analogous to the domestic one in Section 4.6. Households set wages maximizing its future discounted utility, and then inelastically supply the firm's demand for labour at the going wage rate. Just as for prices, there is a time-varying wage markup shock, $\lambda_t^{w,*}$, defined as in (4.41). The wage setting problem yields expressions corresponding to the optimal wage equations (4.46) and (4.47), the relationship between aggregate homogeneous labour and aggregate household labour (4.46), and the wage dispersion expression (4.51).

After scaling and log-linearization, we end up with the starred version of the following eighteen equations from the household side: the expression determining the labour participation in equation (4.100), the unemployment rate in equation (4.99), the natural rate of unemployment in equation (4.110), the endogenous preference shifter in equation (4.80), the trend consumption in equation (4.81), the marginal utility of consumption in equation (4.88), the marginal rate of substitution in equation (4.83), the wage markup in equation (4.101), the consumption Euler equation (4.89), the first-order conditions with respect to capital and investment (4.92) and (4.93), the expression for the capital utilization rate (4.95), the law of motion for capital (4.96), the relationship between efficient and physical capital (4.97), the wage inflation expression (4.106), the wage Phillips curve (4.111), the expression for aggregate household labour (4.114), and the wage dispersion expression (4.115).

We restate and solve the foreign-economy wage problem below. Just as in the domestic economy, the foreign households are monopoly suppliers of differentiated labour services hired by the firm. As such, they can determine their wages. Households are subject to Calvo wage setting frictions, facing a probability $1-\xi_w^*$ in each period that it can reoptimize its nominal wage. If the union reporesenting the j^{th} labour type is not able to reoptimize in period t, the wage it will charge in period $t+1$ will be set according to the following indexation rule:

$$\begin{cases} W_{j,t+1}^* = \tilde{\pi}_{t+1}^{w,*} W_{j,t}^* \\ \tilde{\pi}_{t+1}^{w,*} \equiv \left(\pi_t^{c,*}\right)^{\kappa_w^*} \left(\bar{\pi}_{t+1}^{c,*}\right)^{1-\kappa_w^*-\varkappa_w^*} \left(\breve{\pi}^*\right)^{\varkappa_w^*} \left(\mu_{z^+,*}\right)^{\vartheta_w^*} . \end{cases} \tag{11.48}$$

The household's wage optimization problem is given by

$$\max_{\tilde{W}_{j,t}^*} \quad E_t \sum_{s=0}^{\infty} \left(\beta^* \xi_w^*\right)^s \zeta_{t+s}^{\beta,*} \left[\begin{array}{c} -\zeta_{t+s}^{n,*} \frac{\Theta_{t+s}^*}{1+\varphi^*} \left[\left(\frac{\tilde{\pi}_{t+s}^{w,*} \ldots \tilde{\pi}_{t+1}^{w,*}}{W_{t+s}^*} \right)^{\frac{\lambda_{t+s}^{w,*}}{1-\lambda_{t+s}^{w,*}}} N_{t+s}^* \right]^{1+\varphi^*} \left(\tilde{W}_{j,t}^* \right)^{\frac{\lambda_{t+s}^{w,*}(1+\varphi^*)}{1-\lambda_{t+s}^{w,*}}} \\ +\upsilon_{t+s}^* \left(W_{t+s}^*\right)^{-\frac{\lambda_{t+s}^{w,*}}{1-\lambda_{t+s}^{w,*}}} N_{t+s}^* \left(\tilde{\pi}_{t+s}^{w,*} \ldots \tilde{\pi}_{t+1}^{w,*}\right)^{\frac{1}{1-\lambda_{t+s}^{w,*}}} \left(\tilde{W}_{j,t}^*\right)^{\frac{1}{1-\lambda_{t+s}^{w,*}}} \end{array} \right].$$

Optimization w.r.t. $\tilde{W}_{j,t}^*$ yields

$$\left(\tilde{w}_t^*\right)^{\frac{1-\lambda_{t+s}^{w,*}(1+\varphi^*)}{1-\lambda_{t+s}^{w,*}}} = \frac{\lambda_{t+s}^{w,*} E_t \sum_{s=0}^{\infty} \left(\beta^* \xi_w^*\right)^s \zeta_{t+s}^{\beta,*} \zeta_{t+s}^{n,*} \Theta_{t+s}^* \left[\left(\frac{W_t^* \tilde{\pi}_{t+s}^{w,*} \ldots \tilde{\pi}_{t+1}^{w,*}}{W_{t+s}^*} \right)^{\frac{\lambda_{t+s}^{w,*}}{1-\lambda_{t+s}^{w,*}}} N_{t+s}^* \right]^{1+\varphi^*}}{E_t \sum_{s=0}^{\infty} \left(\beta^* \xi_w^*\right)^s \zeta_{t+s}^{\beta,*} \upsilon_{t+s}^* W_{t+s}^* N_{t+s}^* \left(\frac{W_t^* \tilde{\pi}_{t+s}^{w,*} \ldots \tilde{\pi}_{t+1}^{w,*}}{W_{t+s}^*} \right)^{\frac{1}{1-\lambda_{t+s}^{w,*}}}}. \tag{11.49}$$

where $\tilde{w}_t^* = \frac{\tilde{W}_t^*}{W_t^*}$. Note that, due to the Calvo assumption on wage setting frictions, we can then rewrite the wage index as follows:

$$\begin{aligned} \left(W_t^*\right)^{\frac{1}{1-\lambda_{t+s}^{w,*}}} &= \int_0^1 \left(W_{j,t}^*\right)^{\frac{1}{1-\lambda_{t+s}^{w,*}}} dj \\ &= \int_0^{\xi_w^*} \left(\tilde{\pi}_t^{w,*} W_{j,t-1}^*\right)^{\frac{1}{1-\lambda_{t+s}^{w,*}}} dj + \int_{\xi_w^*}^1 \left(\tilde{W}_t^*\right)^{\frac{1}{1-\lambda_{t+s}^{w,*}}} dj \\ &= \xi_w^* \left(\tilde{\pi}_t^{w,*} W_{t-1}^*\right)^{\frac{1}{1-\lambda_{t+s}^{w,*}}} + \left(1-\xi_w^*\right) \left(\tilde{W}_t^*\right)^{\frac{1}{1-\lambda_{t+s}^{w,*}}} . \end{aligned}$$

Dividing both sides by $(W_t^*)^{\frac{1}{1-\lambda_{t+s}^{w,*}}}$, we obtain

$$
\begin{aligned}
1 &= \xi_w^* \left(\frac{\tilde{\pi}_t^{w,*}}{\pi_t^{w,*}}\right)^{\frac{1}{1-\lambda_{t+s}^{w,*}}} + (1-\xi_w^*)(\tilde{w}_t^*)^{\frac{1}{1-\lambda_{t+s}^{w,*}}} \\
\tilde{w}_t^* &= \left[\frac{1-\xi_w^* \left(\frac{\tilde{\pi}_t^{w,*}}{\pi_t^{w,*}}\right)^{\frac{1}{1-\lambda_{t+s}^{w,*}}}}{(1-\xi_w^*)}\right]^{1-\lambda_t^{w,*}},
\end{aligned} \tag{11.50}
$$

where

$$
\pi_t^{w,*} = \frac{W_t^*}{W_{t-1}^*} = \frac{\bar{w}_t^* z_t^{+,*} P_t^{d,*}}{\bar{w}_{t-1}^* z_{t-1}^{+,*} P_{t-1}^{d,*}} = \frac{\bar{w}_t^* \mu_{z^{+,*},t} \pi_t^{d,*}}{\bar{w}_{t-1}^*}, \tag{11.51}
$$

and $\bar{w}_t^* = \frac{W_t^*}{z_t^{+,*} P_t^{d,*}}$ is the scaled real wage.

We next scale the wage setting equation. First, note that

$$
\frac{W_t^* \tilde{\pi}_{t+s}^{w,*} \dots \tilde{\pi}_{t+1}^{w,*}}{W_{t+s}^*} = \frac{W_t^* \tilde{\pi}_{t+s}^{w,*} \dots \tilde{\pi}_{t+1}^{w,*}}{\bar{w}_{t+s}^* z_{t+s}^{+,*} P_{t+s}^{d,*}} = \frac{\bar{w}_t^*}{\bar{w}_{t+s}^*} \frac{\tilde{\pi}_{t+s}^{w,*} \dots \tilde{\pi}_{t+1}^{w,*}}{\mu_{z^{+,*},t+1} \dots \mu_{z^{+,*},t+s} \pi_{t+1}^{d,*} \dots \pi_{t+s}^{d,*}}.
$$

We then have that

$$
(\tilde{w}_t^*)^{\frac{1-\lambda_{t+s}^{w,*}(1+\varphi^*)}{1-\lambda_{t+s}^{w,*}}} = \frac{\lambda_{t+s}^{w,*} E_t \sum_{s=0}^{\infty} (\beta^* \xi_w^*)^s \zeta_{t+s}^{\beta,*} \zeta_{t+s}^{n,*} \Theta_{t+s}^* \left[\left(\frac{\bar{w}_t^*}{\bar{w}_{t+s}^*} \frac{\tilde{\pi}_{t+s}^{w,*} \dots \tilde{\pi}_{t+1}^{w,*}}{\mu_{z^{+,*},t+1} \dots \mu_{z^{+,*},t+s} \pi_{t+1}^{d,*} \dots \pi_{t+s}^{d,*}}\right)^{\frac{\lambda_{t+s}^{w,*}}{1-\lambda_{t+s}^{w,*}}} N_{t+s}^*\right]^{1+\varphi^*}}{E_t \sum_{s=0}^{\infty} (\beta^* \xi_w^*)^s \zeta_{t+s}^{\beta,*} \psi_{z^{+,*},t+s} \bar{w}_{t+s}^* N_{t+s}^* \left(\frac{\bar{w}_t^*}{\bar{w}_{t+s}^*} \frac{\tilde{\pi}_{t+s}^{w,*} \dots \tilde{\pi}_{t+1}^{w,*}}{\mu_{z^{+,*},t+1} \dots \mu_{z^{+,*},t+s} \pi_{t+1}^{d,*} \dots \pi_{t+s}^{d,*}}\right)^{\frac{1}{1-\lambda_{t+s}^{w,*}}}}. \tag{11.52}
$$

For the log-linearization, we need to recall that $\tilde{\pi}_{t+1}^{w,*} = (\pi_t^{c,*})^{\kappa_w^*} (\bar{\pi}_{t+1}^{c,*})^{1-\kappa_w^*-\varkappa_w^*} (\breve{\pi}^*)^{\varkappa_w^*} (\mu_{z^{+,*}})^{\vartheta_w^*}$. From the steady-state computations, we have that $\pi^{c,*} = \bar{\pi}^{c,*} = \pi^{d,*}$, which implies that $\tilde{\pi}^{w,*} = (\pi^{d,*})^{1-\varkappa_w^*} (\breve{\pi}^*)^{\varkappa_w^*} (\mu_{z^{+,*}})^{\vartheta_w^*}$. Under the additional assumptions that $\varkappa_w^* = 0$ and $\vartheta_w^* = 1$, we have that

$$
\tilde{\pi}^{w,*} = \pi^{d,*} \mu_{z^{+,*}}.
$$

We moreover have that.

$$
\pi^{w,*} = \frac{\bar{w}^* \mu_{z^{+,*}} \pi^{d,*}}{\bar{w}^*} = \pi^{d,*} \mu_{z^{+,*}},
$$

and, thus, $\frac{\tilde{\pi}^{w,*}}{\pi_t^{w,*}} = 1$, which yields

$$
\tilde{w}^* = 1.
$$

Using these steady-state relationships, we can obtain the following log-linear expression of the optimal wage equation:

$$
\begin{aligned}
\frac{1-\lambda^{w,*}(1+\varphi^*)}{1-\lambda^{w,*}} \left(\hat{\tilde{w}}_t^* + \hat{\bar{w}}_t^*\right) &= (1-\beta^* \xi_w^*) \left(\hat{\zeta}_t^{n,*} + \hat{\Theta}_t^* + \varphi^* \hat{N}_t^* - \hat{\psi}_{z^+,t}^* + \hat{\lambda}_t^{w,*} - \frac{\lambda^{w,*} \varphi^*}{1-\lambda^{w,*}} \hat{\bar{w}}_t^*\right) \\
&\quad + \beta^* \xi_w^* \frac{1-\lambda^{w,*}(1+\varphi^*)}{1-\lambda^{w,*}} E_t \left(\hat{\tilde{w}}_{t+1}^* + \hat{\bar{w}}_{t+1}^*\right) \\
&\quad + (1-\beta^* \xi_w^*) \sum_{s=0}^{\infty} (\beta^* \xi_w^*)^s \frac{\lambda^{w,*}(1+\varphi^*) - 1}{1-\lambda^{w,*}} E_t \left(\hat{\tilde{\pi}}_{t+1}^{w,*} - \hat{\pi}_{t+1}^{d,*} - \hat{\mu}_{z^{+,*},t+1}\right).
\end{aligned}
$$

Log-linearizing the expression for $\tilde{w}_t$ that we derived from the aggregate wage index, we obtain

$$
\hat{\tilde{w}}_t^* = \frac{\xi_w^*}{1-\xi_w^*} \left(\pi_t^{w,*} - \hat{\tilde{\pi}}_t^{w,*}\right).
$$

Log-linearizing the expressions for $\pi_t^{w,*}$ and $\tilde{\pi}_t^{w,*}$, under the assumption that $\varkappa_w^* = 0$, we get

$$
\begin{aligned}
\widehat{\tilde{\pi}}_t^{w,*} &= \kappa_w^* \hat{\pi}_{t-1}^{c,*} + (1-\kappa_w^*)\,\widehat{\bar{\pi}}_t^{c,*} \\
\hat{\pi}_t^{w,*} &= \widehat{\bar{w}}_t^* - \widehat{\bar{w}}_{t-1}^* + \hat{\pi}_t^{d,*} + \hat{\mu}_{z^{+,*},t}.
\end{aligned}
$$

Combining and rearranging as in Section 4.8.5, we finally get the following wage setting equation

$$
\begin{aligned}
&\hat{\pi}_t^{w,*} = \beta^* E_t \hat{\pi}_{t+1}^{w,*} + \\
&+ (1-\beta^*\rho_{\bar{\pi}^*})\,\widehat{\bar{\pi}}_t^{c,*} + \kappa_w^* \left(\hat{\pi}_{t-1}^{c,*} - \widehat{\bar{\pi}}_t^{c,*}\right) - \beta^*\kappa_w^* E_t \left(\hat{\pi}_t^{c,*} - \widehat{\bar{\pi}}_{t+1}^{c,*}\right) \\
&- d_w^* \varphi^* \left(\hat{U}_t^* - \hat{U}_t^{n,*}\right),
\end{aligned}
\tag{11.53}
$$

where we have defined

$$
b_w^* = \frac{\lambda^{w,*}(1+\varphi^*) - 1}{(1-\beta^*\xi_w^*)(1-\xi_w^*)}
$$

and

$$
d_w^* = \frac{(1-\beta^*\xi_w^*)(1-\xi_w^*)}{\xi_w^*} \frac{\lambda^{w,*} - 1}{\lambda^{w,*}(1+\varphi^*) - 1} = \frac{\lambda^{w,*} - 1}{\xi_w^* b_w^*},
\tag{11.54}
$$

and where

$$
\hat{U}_t^{n,*} = \frac{1}{\varphi^*} \hat{\lambda}_t^{w,*}.
\tag{11.55}
$$

11.1.3 Monetary and fiscal authorities

We assume that foreign monetary policy is also conducted according to an instrument rule, which is analogous to the one described in Section 5.1:

$$
\begin{aligned}
\log\left(\frac{R_t^*}{R^*}\right) &= \rho_{R^*} \log\left(\frac{R_{t-1}^*}{R^*}\right) + (1-\rho_{R^*}) \left[\log\left(\frac{\bar{\pi}_t^{c,*}}{\bar{\pi}^{c,*}}\right) + r_{\pi^*} \log\left(\frac{\pi_{t-1}^{c,*}}{\bar{\pi}_t^{c,*}}\right)\right. \\
&\left.+ r_{RU^*}\left(U_{t-1}^* - U^*\right)\right] + r_{\Delta\pi^*} \Delta \log\left(\frac{\pi_t^{c,*}}{\pi^{c,*}}\right) + r_{\Delta RU^*} \Delta U_t^* + \log \varepsilon_{R^*,t}
\end{aligned}
\tag{11.56}
$$

where all variables are defined as in Section 5.1. Note that, due to the assumption that the foreign economy is approximately closed, we have excluded the exchange rate term from the rule.

We model foreign government consumption as follows

$$
G_t^* = g_t^* z_t^{+,*},
\tag{11.57}
$$

where g_t^* is an exogenous stochastic process defined as in (5.4). Here, $g^* = \eta_g^* Y^*$, where η_g^* denotes the steady-state foreign government consumption as a fraction of foreign GDP.

11.1.4 The aggregate resource constraint

Following Section 6, the foreign aggregate resource constraint writes:

$$
G_t^* + C_t^{d,*} + C_t^{e,*} + I_t^{d,*} = \left(\mathring{p}_t^{d,*}\right)^{\frac{\lambda_t^*}{\lambda_t^*-1}} \left[(z_t^*)^{1-\alpha^*} \epsilon_t^* (K_t^*)^{\alpha^*} (N_t^*)^{1-\alpha^*} - z_t^{+,*}\phi^*\right].
\tag{11.58}
$$

Scaling the resource constraint yields

$$
g_t^* + c_t^{d,*} + c_t^{e,*} + i_t^{d,*} = \left(\mathring{p}_t^{d,*}\right)^{\frac{\lambda_t^*}{\lambda_t^*-1}} \left[\epsilon_t^* \left(\frac{k_t^*}{\mu_{z^{+,*},t}\mu_{\Psi^*,t}}\right)^{\alpha^*} (N_t^*)^{1-\alpha^*} - \phi^*\right].
$$

Substituting N_t^* using the foreign version of (4.49), we have

$$\begin{aligned} & g_t^* + c_t^{d,*} + c_t^{e,*} + i_t^{d,*} \\ = & \left(\mathring{p}_t^{d,*}\right)^{\frac{\lambda_t^*}{\lambda_t^*-1}} \left[\epsilon_t^* \left(\frac{k_t^*}{\mu_{z+,*,t}\mu_{\Psi^*,t}}\right)^{\alpha^*} \left(n_t^* \left(\mathring{w}_t^*\right)^{-\frac{\lambda_t^{w,*}}{1-\lambda_t^{w,*}}}\right)^{1-\alpha^*} - \phi^*\right]. \end{aligned} \tag{11.59}$$

We now turn to the log-linearization. We first consider the left-hand side, i.e. the uses of the foreign good:

$$y_t^* = g_t^* + c_t^{d,*} + c_t^{e,*} + i_t^{d,*}.$$

We can substitute for $c_t^{d,*}$ to obtain

$$y_t^* = g_t^* + c_t^{xe,*} + c_t^{e,*} + i_t^{d,*}.$$

Log-linearization yields

$$\hat{y}_t^* = \frac{g^*}{y^*}\hat{g}_t^* + \frac{c^{xe,*}}{y^*}\hat{c}_t^{xe,*} + \frac{c^{e,*}}{y^*}\hat{c}_t^{e,*} + \frac{i^{d,*}}{y^*}\hat{i}_t^{d,*}. \tag{11.60}$$

Moving on to the right hand side, log-linearization yields

$$\begin{aligned} y^*\hat{y}_t^* = & \left[\left(\frac{k^*}{\mu_{z+,*}\mu_{\Psi^*}}\right)^{\alpha^*} (n^*)^{1-\alpha^*} - \phi^*\right] \times \left[\frac{\lambda^*}{1-\lambda^*}\widehat{\mathring{p}}_t^{d,*}\right] \\ & + \left(\frac{k^*}{\mu_{z+,*}\mu_{\Psi^*}}\right)^{\alpha^*} (n*)^{1-\alpha^*} \times \left[\hat{\epsilon}_t^* + \alpha^*\left(\hat{k}_t^* - \hat{\mu}_{z+,*,t} - \hat{\mu}_{\Psi^*,t}\right)\right. \\ & \left. + (1-\alpha^*)\left(\hat{n}_t^* - \frac{\lambda^{w,*}}{1-\lambda^{w,*}}\widehat{\mathring{w}}_t^*\right)\right]. \end{aligned}$$

Under full indexation (where $\mathring{p}^{d,*} = \mathring{w}^* = 1$), we also have that (foreign and steady-state version of (6.8))

$$y^* = \left(\frac{k^*}{\mu_{z+,*}\mu_{\Psi^*}}\right)^{\alpha^*} (n^*)^{1-\alpha^*} - \phi^*,$$

so the aggregate resource constraint from the production side writes

$$\begin{aligned} \hat{y}_t^* = & \frac{\lambda^*}{1-\lambda^*}\widehat{\mathring{p}}_t^{d,*} + \frac{1}{y^*}\left(\frac{k^*}{\mu_{z+,*}\mu_{\Psi^*}}\right)^{\alpha^*} (n^*)^{1-\alpha^*} \times \\ & \times \left[\hat{\epsilon}_t^* + \alpha^*\left(\hat{k}_t^* - \hat{\mu}_{z+,*,t} - \hat{\mu}_{\Psi^*,t}\right) + (1-\alpha^*)\left(\hat{n}_t^* - \frac{\lambda^{w,*}}{1-\lambda^{w,*}}\widehat{\mathring{w}}_t^*\right)\right]. \end{aligned} \tag{11.61}$$

11.1.5 Exogenous processes

The foreign model contains a total of 15 exogenous processes, all given by AR(1) processes:[44] the neutral stationary technology shock

$$\hat{\epsilon}_t^* = \rho_{\epsilon^*}\hat{\epsilon}_{t-1}^* + \sigma_{\epsilon^*}\varepsilon_{\epsilon^*,t}, \tag{11.62}$$

the investment-specific stationary technology shock

$$\hat{\Upsilon}_t^* = \rho_{\Upsilon^*}\hat{\Upsilon}_{t-1}^* + \sigma_{\Upsilon^*}\varepsilon_{\Upsilon^*,t}, \tag{11.63}$$

[44] Just as for the domestic-economy model in Section 10, we here restrict the discussion to the theoretical model which was used as the starting point for the estimations, and leave empirical considerations aside. In practice, it may be empirically motivated to model some of the shock processes differently, or turn them off in the estimation.

the shock to neutral technology growth

$$\hat{\mu}_{z^*,t} = \rho_{\mu_{z^*}} \hat{\mu}_{z^*,t-1} + \sigma_{\mu_{z^*}} \varepsilon_{\mu_{z^*},t}, \tag{11.64}$$

the shock to investment-specific technology growth

$$\hat{\mu}_{\Psi^*,t} = \rho_{\mu_{\Psi^*}} \hat{\mu}_{\Psi^*,t-1} + \sigma_{\mu_{\Psi^*}} \varepsilon_{\mu_{\Psi^*},t}, \tag{11.65}$$

the domestic price markup shock

$$\hat{\lambda}_t^* = \rho_{\lambda^*} \hat{\lambda}_{t-1}^* + \sigma_{\lambda^*} \varepsilon_{\lambda^*,t}, \tag{11.66}$$

the wage markup shock

$$\hat{\lambda}_t^{w,*} = \rho_{\lambda^{w,*}} \hat{\lambda}_{t-1}^{w,*} + \sigma_{\lambda^{w,*}} \varepsilon_{\lambda^{w,*},t}, \tag{11.67}$$

the fraction of the foreign firms' costs that has to be financed in advance

$$\hat{\nu}_t^{wc,*} = \rho_{\nu^{wc,*}} \hat{\nu}_{t-1}^{wc,*} + \sigma_{\nu^{wc,*}} \varepsilon_{\nu^{wc,*},t}, \tag{11.68}$$

the shock to the household's discount rate

$$\hat{\zeta}_t^{\beta,*} = \rho_{\zeta^{\beta,*}} \hat{\zeta}_{t-1}^{\beta,*} + \sigma_{\zeta^{\beta,*}} \varepsilon_{\zeta^{\beta,*},t}, \tag{11.69}$$

the shock to consumption preferences

$$\hat{\zeta}_t^{c,*} = \rho_{\zeta^{c,*}} \hat{\zeta}_{t-1}^{c,*} + \sigma_{\zeta^{c,*}} \varepsilon_{\zeta^{c,*},t}, \tag{11.70}$$

the labour supply shock

$$\hat{\zeta}_t^{n,*} = \rho_{\zeta^{n,*}} \hat{\zeta}_{t-1}^{n,*} + \sigma_{\zeta^{n,*}} \varepsilon_{\zeta^{n,*},t}, \tag{11.71}$$

the household risk premium shock

$$\hat{\chi}_t^* = \rho_{\chi^*}^* \hat{\chi}_{t-1}^* + \sigma_{\chi^*} \varepsilon_{\chi^*,t}, \tag{11.72}$$

the monetary policy shock

$$\hat{\varepsilon}_{R^*,t} = \rho_{\varepsilon_{R^*}} \hat{\varepsilon}_{R^*,t-1} + \sigma_{\varepsilon_{R^*}} \varepsilon_{\varepsilon_{R^*},t}, \tag{11.73}$$

the inflation target shock

$$\hat{\bar{\pi}}_t^{c,*} = \rho_{\bar{\pi}^{c,*}} \hat{\bar{\pi}}_{t-1}^{c,*} + \sigma_{\bar{\pi}^{c,*}} \varepsilon_{\bar{\pi}^{c,*},t}, \tag{11.74}$$

government consumption expenditures

$$\hat{g}_t^* = \rho_{g^*} \hat{g}_{t-1}^* + \sigma_{g^*} \varepsilon_{g^*,t}, \tag{11.75}$$

and the evolution of the relative price of energy

$$\hat{p}_t^{ce,*} = \rho_{p^{ce,*}} \hat{p}_{t-1}^{ce,*} + \sigma_{p^{ce,*}} \varepsilon_{p^{ce,*},t}. \tag{11.76}$$

11.2 Modelled as a VAR

In this section, we present on alternative model of the foreign economy, which is similar to the foreign-economy modelling in earlier Riksbank models. This version is not used at present, but only included for documentation purposes.

In Ramses II, the foreign economy block is specified by the following system of equations:

$$
\begin{aligned}
X_t^* &= AX_{t-1}^* + C\varepsilon_t \\
\begin{pmatrix} \log\left(\frac{y_t^*}{y^*}\right) \\ \pi_t^* - \pi^* \\ R_t^* - R^* \\ \log\left(\frac{\mu_{z,t}}{\mu_z}\right) \\ \log\left(\frac{\mu_{\Psi,t}}{\mu_\Psi}\right) \end{pmatrix} &= \begin{bmatrix} a_{11} & a_{12} & a_{13} & 0 & 0 \\ a_{21} & a_{22} & a_{23} & a_{24} & \frac{a_{24}\alpha}{1-\alpha} \\ a_{31} & a_{32} & a_{33} & a_{34} & \frac{a_{34}\alpha}{1-\alpha} \\ 0 & 0 & 0 & \rho_{\mu_z} & 0 \\ 0 & 0 & 0 & 0 & \rho_{\mu_\Psi} \end{bmatrix} \begin{pmatrix} \log\left(\frac{y_{t-1}^*}{y^*}\right) \\ \pi_{t-1}^* - \pi^* \\ R_{t-1}^* - R^* \\ \log\left(\frac{\mu_{z,t-1}}{\mu_z}\right) \\ \log\left(\frac{\mu_{\Psi,t-1}}{\mu_\Psi}\right) \end{pmatrix} \\
&+ \begin{bmatrix} \sigma_{y^*} & 0 & 0 & 0 & 0 \\ c_{21} & \sigma_{\pi^*} & 0 & c_{24} & \frac{c_{24}\alpha}{1-\alpha} \\ c_{31} & c_{32} & \sigma_{R^*} & c_{34} & \frac{c_{34}\alpha}{1-\alpha} \\ 0 & 0 & 0 & \sigma_{\mu_z} & 0 \\ 0 & 0 & 0 & 0 & \sigma_{\mu_\Psi} \end{bmatrix} \begin{pmatrix} \varepsilon_{y^*,t} \\ \varepsilon_{\pi^*,t} \\ \varepsilon_{R^*,t} \\ \varepsilon_{\mu_z,t} \\ \varepsilon_{\mu_\Psi,t} \end{pmatrix},
\end{aligned}
$$

where the ε_t's are mean-zero, unit variance i.i.d. processes uncorrelated with each other. It takes into account that foreign output, Y_t^*, is affected by disturbances to z_t^+, as

$$
\begin{aligned}
\log Y_t^* &= \log y_t^* + \log z_t^+ \\
&= \log y_t^* + \log z_t + \frac{\alpha}{1-\alpha}\log \Psi_t,
\end{aligned}
$$

where $\log(y_t^*)$ is asumed to be a stationary process. As the matrix C has 10 elements, the order condition for identification is satisfied. The documentation of Ramses II discusses the intuition behind the zero restrictions in A and C. Here, it suffices to note that the above system is estimated together with the rest of the model, implying that the estimation of the parameters in A and C is affected by domestic as well as foreign observed variables, and that the past few years' experience with estimation has taught us that the parameters pertaining to the foreign economy block tend to be rather unstable. Moreover, shocks to the investment technology process have been switched of since the original implementation of Ramses II. In the latest versions of Ramses II, even the neutral technology process has been detached from the rest of the foreign VAR, rendering the modelling of the foreign economy block similar to that in Ramses I.

In Ramses II, the two permanent technology processes are assumed to be global. As Sweden is a small-open economy, we usually assume exogeneity of foreign variables. In other words, the standard small-open-economy assumption implies that Swedish economic developments should be affected by, but not affect, the developments in the rest of the world. It is not obvious that global technology processes should be estimated in a model of the Swedish economy with restrictions on Swedish variables following global shocks, such as for example Ramses II, as the results will be driven to a considerable extent by data and restrictions pertaining to the domestic economy block.

For the above discussed reasons, in the model presented here, we will deviate from the assumptions made in Ramses II. We will instead assume that the foreign economy block looks like the one in Ramses I. To this end, we will shut off any shocks to the investment-specific technology, as in the implemented version of Ramses II, and assume that the global neutral technology growth is given by an AR(1) process.

The foreign economy in this model is specified as a VAR model of foreign inflation, output and interest rates, assumed to be exogenously given. Defining

$$X_t^* \equiv \left[\begin{array}{ccc} \pi_t^* & \hat{y}_t^* & R_t^* \end{array}\right]',$$

where π_t^* and R_t^* are quarterly foreign inflation and interest rates, and $\hat{y}_t^*$ the foreign output gap[45], we can write the model as

$$F_0 X_t^* = F(L)\, X_{t-1}^* + \varepsilon_{x^*,t}, \tag{11.77}$$

[45] In Ramses I, it is the HP-filtered output.

where $\varepsilon_{x^*,t} \sim N(0, \Sigma_{x^*})$. In Ramses I, it is assumed that F_0 has the following structure:

$$F_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\gamma^*_{\pi,0} & -\gamma^*_{y,0} & 1 \end{bmatrix}.$$

This structure is equivalent to assuming predetermined expectations in the Phillips curve and output equation, and could be not rejected in the estimation of Ramses I. Note that this specification of the foreign block is complemented with two exogenous processes for the evolution of the domestic and foreign technological processes $\hat{\mu}_{z^+,t}$ and $\widehat{\tilde{z}}_t^{+,*}$, specified in Section 10 below. The parameters in F_0 and $F(L)$ can be estimated outside of the model, and then calibrated prior to the estimation of the parameters pertaining to the domestic economy block. Note that the foreign variables are still needed as observed variables when the model is estimated, in order to enable idenfitication of the asymmetric technology shock $\tilde{z}_t^{+,*}$. In Ramses I, HP-filtered output is used in the estimation of the parameters in (11.77), while the vector of observed variables in the model estimation includes foreign output in growth rates.[46]

Note, finally, that in the case the foreign economy is modelled as in the above VAR, we need to assume that total demand by foreigners for domestic exports takes the following form:

$$X_t = \left(\frac{P_t^x}{P_t^*}\right)^{-\eta_f} Y_t^*, \tag{11.78}$$

as foreign GDP components are no longer explicitly modelled.

12 Steady state

In this section we solve for the non-stochastic steady state. We apply the convention that variables without a time subscript represent steady-state values. We first present the solution for the domestic economy, before proceding with the foreign one. Note that in practice, however, the steady state for the foreign economy model is solved first, as the domestic steady state partly relies on the foreign economy steady-state solution.

12.1 Steady state of the domestic-economy model

We begin by solving the steady state for the monetary policy rule. With the exception of consumer price inflation, all variables are in log-deviation from their steady-state values. We have, in steady state, that

$$\begin{aligned} \log\left(\frac{R_t}{R}\right) &= \rho_R \log\left(\frac{R_{t-1}}{R}\right) + (1-\rho_R)\left[\log\left(\frac{\bar{\pi}_t^c}{\bar{\pi}^c}\right) + r_\pi \log\left(\frac{\pi_{t-1}^c}{\bar{\pi}_t^c}\right)\right. \\ &\left. + r_{RU}\left(U_{t-1} - U\right) + r_q \log\left(\frac{q_{t-1}}{q}\right)\right] + r_{\Delta\pi}\Delta\log\left(\frac{\pi_t^c}{\pi^c}\right) + r_{\Delta RU}\Delta U_t + \log\varepsilon_{R,t}, \end{aligned} \tag{12.1}$$

$$\begin{aligned} \log\left(\frac{R}{R}\right) &= \rho_R \log\left(\frac{R}{R}\right) + (1-\rho_R)\left[\log\left(\frac{\bar{\pi}^c}{\bar{\pi}^c}\right) + r_\pi \log\left(\frac{\pi^c}{\bar{\pi}^c}\right)\right. \\ &\left. + r_{RU}\left(U - U\right)\right] + r_{\Delta\pi}\Delta\log\left(\frac{\pi^c}{\pi^c}\right) + r_{\Delta RU}\Delta U, \end{aligned}$$

yielding

$$\begin{aligned} \log\left(\frac{\pi^c}{\bar{\pi}^c}\right) &= 0 \\ \Rightarrow \pi^c &= \bar{\pi}^c. \end{aligned} \tag{12.2}$$

[46] Foreign inflation and interest rates are included in levels in both cases.

As will be clarified in the following section, the foreign policy maker follows a similar rule for the foreign economy and, hence,

$$\pi^{c,*} = \bar{\pi}^{c,*}.$$

The relationship between the aggregate and domestic inflation rates and the relative price of aggregate consumption then implies

$$\pi^{d,*} = \pi^{c,*}.$$

Solving for (8.1)–(8.10), we obtain:

$$\begin{aligned}
\pi^{d} &= \pi^{c} &(12.3)\\
\pi^{cxe} &= \pi^{d} &(12.4)\\
\pi^{ce} &= \pi^{d} &(12.5)\\
\pi^{d,ce} &= \pi^{d} &(12.6)\\
\pi^{i} &= \frac{\pi^{d}}{\mu_{\Psi}} &(12.7)\\
\pi^{x} &= \pi^{d,*} &(12.8)\\
\pi^{m,c} &= \pi^{d} &(12.9)\\
\pi^{m,i} &= \pi^{d} &(12.10)\\
\pi^{m,x} &= \pi^{d} &(12.11)\\
\pi^{m,ce} &= \pi^{d}. &(12.12)
\end{aligned}$$

Solving for (3.18), (3.81) and (3.190) yields

$$\begin{aligned}
\tilde{\pi}^{d} &= \left(\pi^{d}\right)^{\kappa_d} \left(\bar{\pi}^{c}\right)^{1-\kappa_d-\varkappa_d} \left(\breve{\pi}\right)^{\varkappa_d} = \left(\pi^{d}\right)^{1-\varkappa_d} \left(\breve{\pi}\right)^{\varkappa_d}\\
\tilde{\pi}^{m,j} &= \left(\pi^{m,j}\right)^{\kappa_{m,j}} \left(\bar{\pi}^{c}\right)^{1-\kappa_{m,j}-\varkappa_{m,j}} \left(\breve{\pi}\right)^{\varkappa_{m,j}} = \left(\pi^{d}\right)^{1-\varkappa_{m,j}} \left(\breve{\pi}\right)^{\varkappa_{m,j}}, \quad j = c, i, x, ce\\
\tilde{\pi}^{x} &= \left(\pi^{x}\right)^{\kappa_x} \left(\bar{\pi}^{*}\right)^{1-\kappa_x-\varkappa_x} \left(\breve{\pi}\right)^{\varkappa_x} = \left(\pi^{x}\right)^{1-\varkappa_x} \left(\breve{\pi}\right)^{\varkappa_x}.
\end{aligned}$$

Assuming full indexation, i.e. that $\varkappa_d = \varkappa_{m,j} = \varkappa_x = 0$, the above three expressions simplify to

$$\begin{aligned}
\tilde{\pi}^{d} &= \pi^{d} &(12.13)\\
\tilde{\pi}^{m,j} &= \pi^{d}, \quad j = c, i, x, ce &(12.14)\\
\tilde{\pi}^{x} &= \pi^{x}. &(12.15)
\end{aligned}$$

From equation (4.48), we can solve for wage inflation

$$\pi^{w} = \frac{w\mu_{z^+}\pi^{d}}{w} = \mu_{z^+}\pi^{d}, \tag{12.16}$$

and from (4.44) for wage indexation to obtain

$$\tilde{\pi}^{w} = \left(\pi^{c}\right)^{\kappa_w} \left(\bar{\pi}^{c}\right)^{1-\kappa_w-\varkappa_w} \left(\breve{\pi}\right)^{\varkappa_w} \left(\mu_{z^+}\right)^{\vartheta_w} = \left(\pi^{d}\right)^{1-\varkappa_w} \left(\breve{\pi}\right)^{\varkappa_w} \left(\mu_{z^+}\right)^{\vartheta_w}.$$

Assuming full indexation, i.e. that $\varkappa_w = 0$, and $\vartheta_w = 1$, wage indexation simplifies to

$$\tilde{\pi}^{w} = \mu_{z^+}\pi^{d} = \pi^{w}. \tag{12.17}$$

From (3.9), we have that

$$\mu_{z^+} = \mu_{\Psi}^{\frac{\alpha}{1-\alpha}}\mu_{z}. \tag{12.18}$$

From the Euler equation (4.58), we can then solve for the steady-state interest rate

$$R = \frac{\mu_{z+}}{\beta}\pi^d. \tag{12.19}$$

We then have that the real rate is given by

$$\bar{R} = \frac{R}{\pi^c} = \frac{\mu_{z+}}{\beta}, \tag{12.20}$$

where we have used that $\pi^d = \pi^c$ in steady state.[47] Correspondingly, R^* is determined by the foreign economy Euler equation, as given by

$$R^* = \frac{\mu_{z+,*}}{\beta^*}\pi^{d,*}. \tag{12.21}$$

With expressions for R and R^* at hand, we can now solve for the steady-state gross effective nominal interest rates faced by the different types of firms in equations (3.10), (3.62) and (3.185):

$$\begin{aligned} R^{wc,d} &= \nu^{wc,d}R + 1 - \nu^{wc,d} && (12.22)\\ R^{wc,m} &= \nu^{wc,m}R^* + 1 - \nu^{wc,m} && (12.23)\\ R^{wc,x} &= \nu^{wc,x}R + 1 - \nu^{wc,x}. && (12.24) \end{aligned}$$

From equation (9.1), we can solve for the the steady-state value of the nominal exchange rate growth rate:

$$s = \frac{\pi^c}{\pi^{c,*}} = \frac{\pi^d}{\pi^{d,*}}. \tag{12.25}$$

From the UIP condition (4.66), we have

$$R = R^*\Phi s.$$

Combining (12.19), (12.21) and (12.25), we have

$$\begin{aligned} \Phi &= \frac{\mu_{z+}}{\mu_{z+,*}}\frac{\beta^*}{\beta}\frac{\pi^d}{\pi^{d,*}} = \frac{1}{s}\frac{\mu_{z+}}{\mu_{z+,*}}\frac{\beta^*}{\beta}\frac{\pi^d}{\pi^{d,*}}\frac{\pi^{d,*}}{\pi^d} \\ \Phi &= \frac{\mu_{z+}}{\mu_{z+,*}}\frac{\beta^*}{\beta}. && (12.26) \end{aligned}$$

Under the assumption of equal steady-state growth rates and discount factors in the two economies, we have that

$$\Phi = 1.$$

It is clear from equation (4.19) that we denote the steady-state value of net foreign assets by $\bar{a}$. We assume that $\bar{a} = \eta_a y$.[48]

[47] Note also that we assume that the neutral interest rate, or the time-varying real interest rate trend, are equal in steady state, so that:

$$\bar{R}^t = \bar{R}.$$

See also Section 14.

[48] We will generally set η_a to 0. This is relevant for the linearlization of the model, as level deviations must be used for deviations of $\bar{a}_t$ from steady state instead of log deviations. The linearizations in earlier sections were done under this assumption.

Next, we solve for the steady-state marginal cost of domestic goods producers. We start with the scaled expression of the optimal price (3.40):

$$\begin{aligned}
\tilde{p}^d &= \frac{\lambda^d mc^d \psi_{z+}y \sum\limits_{s=0}^{\infty} (\beta\xi_d)^s \left[\left(\frac{\tilde{\pi}^d}{\pi^d}\right)^s\right]^{\frac{\lambda^d}{1-\lambda^d}}}{\psi_{z+}y \sum\limits_{s=0}^{\infty} (\beta\xi_d) \left[\left(\frac{\tilde{\pi}^d}{\pi^d}\right)^s\right]^{\frac{1}{1-\lambda^d}}} \\
\tilde{p}^d &= \lambda^d mc^d \frac{\sum\limits_{s=0}^{\infty} \left[\beta\xi_d \left(\frac{\tilde{\pi}^d}{\pi^d}\right)^{\frac{\lambda^d}{1-\lambda^d}}\right]^s}{\sum\limits_{s=0}^{\infty} \left[\beta\xi_d \left(\frac{\tilde{\pi}^d}{\pi^d}\right)^{\frac{1}{1-\lambda^d}}\right]^s} = \lambda^d mc^d \frac{1-\beta\xi_d \left(\frac{\tilde{\pi}^d}{\pi^d}\right)^{\frac{1}{1-\lambda^d}}}{1-\beta\xi_d \left(\frac{\tilde{\pi}^d}{\pi^d}\right)^{\frac{\lambda^d}{1-\lambda^d}}}.
\end{aligned}$$

Using the second expression of the optimal price (3.23), which we derived from the aggregate price, we obtain

$$mc^d = \frac{1}{\lambda^d} \frac{1-\beta\xi_d \left(\frac{\tilde{\pi}^d}{\pi^d}\right)^{\frac{\lambda^d}{1-\lambda^d}}}{1-\beta\xi_d \left(\frac{\tilde{\pi}^d}{\pi^d}\right)^{\frac{1}{1-\lambda^d}}} \left[\frac{1-\xi_d \left(\frac{\tilde{\pi}^d}{\pi^d}\right)^{\frac{1}{1-\lambda^d}}}{(1-\xi_d)}\right]^{1-\lambda^d}. \tag{12.27}$$

Assuming full indexation, implying that $\tilde{\pi}^d = \pi^d$, this simplifies to

$$mc^d = \frac{1}{\lambda^d}. \tag{12.28}$$

Note that λ^d is related to the elasticity of substitution between the different domestic goods, which we denote by η_d, in the following way:

$$\lambda^d = \frac{\eta_d}{1-\eta_d}.$$

We note that λ^d is calibrated. Similarly, for importing and exporting firms, we have

$$mc^x = \frac{1}{\lambda^x} \frac{1-\beta\xi_x \left(\frac{\tilde{\pi}^x}{\pi^x}\right)^{\frac{\lambda^x}{1-\lambda^x}}}{1-\beta\xi_x \left(\frac{\tilde{\pi}^x}{\pi^x}\right)^{\frac{1}{1-\lambda^x}}} \left[\frac{1-\xi_x \left(\frac{\tilde{\pi}^x}{\pi^x}\right)^{\frac{1}{1-\lambda^x}}}{(1-\xi_x)}\right]^{1-\lambda^x} \tag{12.29}$$

and

$$mc^{m,j} = \frac{1}{\lambda^{m,j}} \frac{1-\beta\xi_{m,j} \left(\frac{\tilde{\pi}^{m,j}}{\pi^{m,j}}\right)^{\frac{\lambda^{m,j}}{1-\lambda^{m,j}}}}{1-\beta\xi_{m,j} \left(\frac{\tilde{\pi}^{m,j}}{\pi^{m,j}}\right)^{\frac{1}{1-\lambda^{m,j}}}} \left[\frac{1-\xi_{m,j} \left(\frac{\tilde{\pi}^{m,j}}{\pi^{m,j}}\right)^{\frac{1}{1-\lambda^{m,j}}}}{(1-\xi_{m,j})}\right]^{1-\lambda^{m,j}}. \tag{12.30}$$

Assuming full indexation, so that $\tilde{\pi}^x = \pi^x$ and $\tilde{\pi}^{m,j} = \pi^{m,j}$, we get

$$mc^x = \frac{1}{\lambda^x} \tag{12.31}$$

and

$$mc^{m,j} = \frac{1}{\lambda^{m,j}}. \tag{12.32}$$

We again note that the markups λ^x and $\lambda^{m,j}$ are related to the elasticities of substitution between differentiated goods in the export and import aggregates, respectively, as follows:

$$\begin{aligned}
\lambda^x &= \frac{\eta_x}{1-\eta_x}, \\
\lambda^{m,j} &= \frac{\eta_{m,j}}{1-\eta_{m,j}}.
\end{aligned}$$

We note that λ^x and $\lambda^{m,j}$ for $j = c, i, x, ce$ are calibrated.

We can now also solve for price and wage dispersion, as well as expressions for the different firms' fixed costs. We start with the domestic firm. $\mathring{p}^d$ is given by the steady-state version of (6.4):

$$\mathring{p}^d = \left[\frac{(1-\xi_d)\left(\frac{1-\xi_d\left(\frac{\tilde{\pi}^d}{\pi^d}\right)^{\frac{1}{1-\lambda^d}}}{1-\xi_d}\right)^{\lambda^d}}{\left(1-\xi_d\left(\frac{\tilde{\pi}^d}{\pi^d}\right)^{\frac{\lambda^d}{1-\lambda^d}}\right)} \right]^{\frac{1-\lambda^d}{\lambda^d}}. \tag{12.33}$$

Assuming full indexation implies that $\tilde{\pi}^d = \pi^d$, so that $\mathring{p}^d = 1$. The fixed cost term, ϕ^d, is computed such that profits equal zero in steady state. Using equation (3.41) evaluated in steady state, and equating total profits to zero, we obtain

$$\phi^d = y\left(\frac{1}{mc^d} - \left(\mathring{p}^d\right)^{\frac{\lambda^d}{1-\lambda^d}}\right). \tag{12.34}$$

Assuming full indexation and using (12.28) we have

$$\phi^d = y\left(\lambda^d - 1\right). \tag{12.35}$$

For the importing firms, we can evaluate (3.89) at steady state, for $j = c, i, x, ce$, to get

$$\mathring{p}^{m,j} = \left[\frac{(1-\xi_{m,j})\left(\frac{1-\xi_{m,j}\left(\frac{\tilde{\pi}^{m,j}}{\pi^{m,j}}\right)^{\frac{1}{1-\lambda^{m,j}}}}{1-\xi_{m,j}}\right)^{\lambda^{m,j}}}{\left(1-\xi_{m,j}\left(\frac{\tilde{\pi}^{m,j}}{\pi^{m,j}}\right)^{\frac{\lambda^{m,j}}{1-\lambda^{m,j}}}\right)} \right]^{\frac{1-\lambda^{m,j}}{\lambda^{m,j}}}. \tag{12.36}$$

Assuming full indexation implies that

$$\mathring{p}^{m,j} = 1, \tag{12.37}$$

for $j = c, i, x, ce$. Evaluating (3.196) at steady state, for the exporting firm we have

$$\mathring{p}^x = \left[\frac{(1-\xi_x)\left(\frac{1-\xi_x\left(\frac{\tilde{\pi}^x}{\pi^x}\right)^{\frac{1}{1-\lambda^x}}}{1-\xi_x}\right)^{\lambda^x}}{1-\xi_x\left(\frac{\tilde{\pi}^x}{\pi^x}\right)^{\frac{\lambda^x}{1-\lambda^x}}} \right]^{\frac{1-\lambda^x}{\lambda^x}}. \tag{12.38}$$

Assuming full indexation implies that $\tilde{\pi}^x = \pi^x$, so that

$$\mathring{p}^x = 1. \tag{12.39}$$

We also need to pin down the expressions for the various fixed costs associated with imports and exports. These costs are computed such that profits equal zero in steady state. In the case of the importers of consumption goods, we can use (3.93) together with the zero profit condition to obtain:

$$\phi^{m,c} = c^m\left(\frac{1}{mc^{m,c}} - (\mathring{p}^{m,c})^{\frac{\lambda^{m,c}}{1-\lambda^{m,c}}}\right). \tag{12.40}$$

Similarly, we use (3.94), (3.95) and (3.96) to obtain:

$$\phi^{m,i} = i^m \left(\frac{1}{mc^{m,i}} - \left(\mathring{p}^{m,i}\right)^{\frac{\lambda^{m,i}}{1-\lambda^{m,i}}} \right), \tag{12.41}$$

$$\phi^{m,x} = x^m \left(\frac{1}{mc^{m,x}} - \left(\mathring{p}^{m,x}\right)^{\frac{\lambda^{m,x}}{1-\lambda^{m,x}}} \right), \tag{12.42}$$

$$\phi^{m,ce} = c^{e,m} \left(\frac{1}{mc^{m,ce}} - \left(\mathring{p}^{m,ce}\right)^{\frac{\lambda^{m,ce}}{1-\lambda^{m,ce}}} \right). \tag{12.43}$$

Assuming full indexation, we can combine with (12.32) to obtain

$$\phi^{m,c} = c^m \left(\lambda^{m,c} - 1\right), \tag{12.44}$$
$$\phi^{m,i} = i^m \left(\lambda^{m,i} - 1\right), \tag{12.45}$$
$$\phi^{m,x} = x^m \left(\lambda^{m,c} - 1\right), \tag{12.46}$$
$$\phi^{m,ce} = c^{e,m} \left(\lambda^{m,ce} - 1\right). \tag{12.47}$$

For the exporters' fixed costs, we evaluate (3.208) at steady state and impose zero profits to obtain:

$$\begin{aligned} 1 &= \frac{\phi^x}{x}\mathring{p}^{x,temp} + mc^x \left(\mathring{p}^x\right)^{\frac{\lambda^x}{1-\lambda^x}} \\ \phi^x &= \frac{x}{\mathring{p}^{x,temp}} \left(1 - mc^x \left(\mathring{p}^x\right)^{\frac{\lambda^x}{1-\lambda^x}}\right). \end{aligned}$$

As already mentioned in Section 3.5, under full indexation we have that $\mathring{p}^{x,temp} = 1$. To prove this, we need to evaluate (3.207) at steady state, which yields:

$$\mathring{p}^{x,temp} = \frac{\left(1 - \xi_x\right) \tilde{p}^x}{\left(1 - \xi_x \frac{\tilde{\pi}^x}{\pi^x}\right)}.$$

Recall from (12.15) that, under full indexation, $\tilde{\pi}^x = \pi^x$. We also know that $\tilde{p}^x = 1$ under full indexation. Hence, exporters' fixed costs are given by

$$\phi^x = x \left(1 - mc^x \left(\mathring{p}^x\right)^{\frac{\lambda^x}{1-\lambda^x}}\right). \tag{12.48}$$

Assuming full indexation, and using equation (12.31), we have

$$\phi^x = x \left(\frac{\lambda^x - 1}{\lambda^x} \right). \tag{12.49}$$

Finally, we find the steady-state solution for $\mathring{w}$ using (4.51) as follows:

$$\mathring{w} = \left[\frac{\left(1 - \xi_w\right) \left(\frac{1 - \xi_w \left(\frac{\tilde{\pi}^w}{\pi^w}\right)^{\frac{1}{1-\lambda^w}}}{\left(1 - \xi_w\right)} \right)^{\lambda^w}}{\left(1 - \xi_w \left(\frac{\tilde{\pi}^w}{\pi^w}\right)^{\frac{\lambda^w}{1-\lambda^w}}\right)} \right]^{\frac{1-\lambda^w}{\lambda^w}}. \tag{12.50}$$

Under full indexation, we have that $\tilde{\pi}^w = \pi^w$, which implies that

$$\mathring{w} = 1. \tag{12.51}$$

Up to here, we were able to solve analytically for the steady-state expressions of some variables. The rest of the steady state needs instead to be solved as a system of equations in the remaining variables of the model. We list the needed steady-state equations below, starting with the steady-state relative prices of consumption, investment and exports. We can use import producers' marginal costs in equations (3.61), (3.68) and (3.73), together with the definitions of relative prices in Section 2.2, to obtain the following expression for $p^{m,j}$, $j = c, i, x,$:

$$\begin{aligned} p^{m,j} &= \frac{P^{m,j}}{P^d}\frac{P^c}{P^c}\frac{P^{c,*}}{P^{c,*}} = \frac{1}{mc^{m,j}}\frac{SP^{c,*}}{P^c}\frac{P^{d,*}}{P^{c,*}}\frac{P^c}{P^d}R^{wc,m} \\ p^{m,j} &= \frac{qp^c R^{wc,m}}{p^{c,*}mc^{m,j}}. \end{aligned} \tag{12.52}$$

Similarly, using (3.78), we get the following expression for $p^{m,ce}$:

$$\begin{aligned} p^{m,ce} &= \frac{P^{m,ce}}{P^d}\frac{P^c}{P^c}\frac{P^{c,*}}{P^{c,*}}\frac{P^{d,*}}{P^{d,*}} = \frac{1}{mc^{m,ce}}\frac{S_t P^{c,*}}{P^c}\frac{P^c}{P^d}\frac{P^{ce,*}}{P^{d,*}}\frac{P^{d,*}}{P^{c,*}}R^{wc,m} \\ p^{m,ce} &= \frac{qp^c p^{ce,*} R^{wc,m}}{p^{c,*}mc^{m,ce}}. \end{aligned} \tag{12.53}$$

From (3.136), we have

$$p^{cxe} = \left[(1-\omega_c) + \omega_c \left(p^{m,c}\right)^{1-\eta_c}\right]^{1/(1-\eta_c)}, \tag{12.54}$$

while from (3.137), we have

$$p^{ce} = \left[(1-\omega_{em})\left(p^{d,ce}\right)^{1-\eta_{em}} + \omega_{em}\left(p^{m,ce}\right)^{1-\eta_{em}}\right]^{1/(1-\eta_{em})}. \tag{12.55}$$

From (3.138), next, we have

$$p^c = \left[(1-\omega_e)\left(p^{cxe}\right)^{1-\eta_e} + \omega_e\left(p^{ce}\right)^{1-\eta_e}\right]^{1/(1-\eta_e)}. \tag{12.56}$$

From the investment price index, (3.170), we have

$$p^i = \left[(1-\omega_i) + \omega_i\left(p^{m,i}\right)^{1-\eta_i}\right]^{\frac{1}{1-\eta_i}}. \tag{12.57}$$

We solve for p^x using the expression for the exporting firms' marginal costs, (3.202), evaluated in steady state, which yields

$$p^x = \frac{R^{wc,x}p^{c,*}}{qp^c mc^x}\left[\omega_x\left(p^{m,x}\right)^{1-\eta_x} + (1-\omega_x)\right]^{\frac{1}{1-\eta_x}}. \tag{12.58}$$

We note that we are still missing an expression for the steady-state real exchange rate q, which is why we cannot solve analytically for any of the relative prices.

We now derive expressions for the price of physical capital and the rental rate of capital. From the households' FOC w.r.t investment and bond holdings, (4.71) and (4.65), we have

$$p^i = \breve{p}_{k'}\Upsilon F_1(i,i) + \frac{\pi^d}{R}\breve{p}_{k'}\frac{1}{\mu_\Psi}\Upsilon F_2(i,i),$$

and we know from Section 4.3 that $\tilde{S}(x) = \tilde{S}'(x) = 0$ is assumed to hold in steady state, with $x = \mu_{z^+}\mu_\Psi$. Thus, (4.74) and (4.75) imply

$$\begin{aligned} F_1(i,i) &= 1 - \tilde{S}\left(\frac{\mu_{z^+}\mu_\Psi i}{i}\right) - \tilde{S}'\left(\frac{\mu_{z^+}\mu_\Psi i}{i}\right)\frac{\mu_{z^+}\mu_\Psi i}{i} = 1 \\ F_2(i,i) &= \tilde{S}'\left(\frac{\mu_{z^+}\mu_\Psi i}{i}\right)\left(\frac{\mu_{z^+}\mu_\Psi i}{i}\right)^2 = 0, \end{aligned}$$

which allows us to solve for the steady-state price of physical capital

$$\breve{p}_{k'} = \frac{p^i}{\Upsilon}. \tag{12.59}$$

Using (4.69) together with (4.65), and using that $a(u) = 0$, we can solve for $\bar{r}^k$ to obtain

$$\bar{r}^k = \breve{p}_{k'} \left(\frac{R}{\pi^d} \mu_\Psi - (1 - \delta) \right). \tag{12.60}$$

From the households' FOC w.r.t. capital utilization (4.73), we can then derive an expression for $\sigma_b = a'(u)$:

$$\sigma_b = \frac{\bar{r}^k}{p^i}. \tag{12.61}$$

Note that σ_b is a parameter, determined by the values of $\bar{r}^k$ and p^i.

We can find a solution for $\bar{w}$ using the expression for domestic intermediate goods producers' marginal costs, (3.30), which gives

$$\bar{w} = \left[\frac{1}{mc^d} \frac{\left(R^{wc,d}\right)^{1-\alpha}}{(1-\alpha)^{1-\alpha}} \frac{\left(\bar{r}^k\right)^{\alpha}}{\alpha^\alpha \epsilon} \right]^{-\frac{1}{1-\alpha}}. \tag{12.62}$$

The steady-state capital-to-labour ratio is computed using a second expression for the domestic intermerdiate goods producers' marginal costs, (3.31):

$$\frac{k}{N} = \mu_\Psi \mu_{z^+} \left(\frac{1}{mc^d} \frac{\bar{w} R^{wc,d}}{\epsilon (1-\alpha)} \right)^{\frac{1}{\alpha}}. \tag{12.63}$$

Note that this equation implicitly solves for k, as we will derive another expression to solve for N below. Before proceeding with our calculations, we need also to note the following. In steady state, we impose that the capital utilization rate, u, is 1, so using the relation between physical and efficient capital, (4.28), in scaled form we have

$$k = k^p. \tag{12.64}$$

We next consider the aggregate ressource constraint, (6.11), evaluated in steady state:

$$y = g + c^d + c^{e,d} + i^d + x^d, \tag{12.65}$$

where $g = \eta_g y$. From equation (3.145), we get the following steady-state expression for c^d:

$$c^d = (1 - \omega_c) \left(p^{cxe}\right)^{\eta_c} c^{xe}, \tag{12.66}$$

while from (3.147), we get the following steady-state expression for $c^{e,d}$:

$$c^{e,d} = (1 - \omega_{em}) \left[\frac{p^{d,ce}}{p^{ce}} \right]^{-\eta_{em}} c^e. \tag{12.67}$$

We here also note that equations (3.149) and (3.150) evaluated in steady state give

$$c^{xe} = (1 - \omega_e) \left[\frac{p^{cxe}}{p^c} \right]^{-\eta_e} c, \tag{12.68}$$

and

$$c^e = \omega_e \left[\frac{p^{ce}}{p^c} \right]^{-\eta_e} c, \tag{12.69}$$

respectively. From equation (3.174), we have that

$$i^d = (1-\omega_i)\left(p^i\right)^{\eta_i}\left(i + a(u)\frac{k^p}{\mu_{z^+}\mu_\Psi}\right).$$

Recall that $a(u) = 0$. The steady-state expression for i^d then becomes:

$$i^d = (1-\omega_i)\left(p^i\right)^{\eta_i} i. \tag{12.70}$$

From (3.203), we have the following steady-state expression for x^d:

$$x^d = \left(\omega_x\left(p^{m,x}\right)^{1-\eta_x} + (1-\omega_x)\right)^{\frac{\eta_x}{1-\eta_x}}(1-\omega_x)\left(\mathring{p}^x\right)^{\frac{\lambda^x}{1-\lambda^x}} x. \tag{12.71}$$

From equation (3.223), we have the following steady-state expression for exports

$$x = \left(p^x\right)^{-\eta_f}\left(c^{xe,*} + c^{e,*} + i^{d,*}\right), \tag{12.72}$$

We next derive expressions for y and i, which will allow for c to be implicitly determined by the resource constraint in equation (12.65), together with equations (12.66)–(12.72).

We begin with the steady-state expression for y. Evaluating in steady state the expression for total production in equation (6.8), we get

$$y = \left(\mathring{p}^d\right)^{\frac{\lambda^d}{\lambda^d-1}}\left[\epsilon\left(\frac{k}{\mu_\Psi\mu_{z^+}}\right)^\alpha (N)^{1-\alpha} - \phi^d\right]. \tag{12.73}$$

Combining (12.73) and (12.34), we have the following steady-state expression for aggregate production

$$y = \frac{\left(\mathring{p}^d\right)^{\frac{\lambda^d}{\lambda^d-1}}\epsilon\left(\frac{1}{\mu_\Psi\mu_{z^+}}\frac{k}{N}\right)^\alpha}{1 + \left(\mathring{p}^d\right)^{\frac{\lambda^d}{\lambda^d-1}}\left(\frac{1}{mc^d} - \left(\mathring{p}^d\right)^{\frac{\lambda^d}{1-\lambda^d}}\right)} N.$$

Assuming full indexation, we have that

$$y = \epsilon\left(\frac{1}{\mu_\Psi\mu_{z^+}}\frac{k}{N}\right)^\alpha mc^d N. \tag{12.74}$$

We proceed with deriving an expression for steady-state investment i. This can be derived from (4.76):

$$i = \frac{k}{\Upsilon}\left(1 - \frac{1-\delta}{\mu_{z^+}\mu_\Psi}\right), \tag{12.75}$$

where we have used that $k = k^p$.

We use the Euler equation (4.57) to derive an expression for ψ_{z^+}, yielding

$$\psi_{z^+} = \frac{1}{p^c}\frac{\zeta^c}{c\left(\mu_{z^+} - b\right)}\left(\mu_{z^+} - \beta b\right). \tag{12.76}$$

We can then derive an expression for steady-state employment, N, from the scaled optimal-wage expression in equation (4.79). The steady-state version of (4.79) is given by

$$\tilde{w}^{\frac{1-\lambda^w(1+\varphi)}{1-\lambda^w}} = \frac{\lambda^w\zeta^n\Theta N^\varphi}{\psi_{z^+}\bar{w}}\frac{1-\beta\xi_w\left(\frac{\tilde{\pi}^w}{\mu_{z^+}\pi^d}\right)^{\frac{1}{1-\lambda^w}}}{1-\beta\xi_w\left(\frac{\tilde{\pi}^w}{\mu_{z^+}\pi^d}\right)^{\frac{\lambda^w(1+\varphi)}{1-\lambda^w}}}.$$

Plugging in the steady-state version of the second expression of the optimal wage (4.47), which we derived from the aggregate wage, together with (4.48) which in steady state gives that $\pi^w = \mu_{z+}\pi^d$, we have

$$\left[\frac{1-\xi_w\left(\frac{\tilde{\pi}^w}{\pi^w}\right)^{\frac{1}{1-\lambda^w}}}{(1-\xi_w)}\right]^{1-\lambda^w(1+\varphi)} = \frac{\lambda^w\zeta^n\Theta N^\varphi}{\psi_{z+}\bar{w}}\frac{1-\beta\xi_w\left(\frac{\tilde{\pi}^w}{\pi^w}\right)^{\frac{1}{1-\lambda^w}}}{1-\beta\xi_w\left(\frac{\tilde{\pi}^w}{\pi^w}\right)^{\frac{\lambda^w(1+\varphi)}{1-\lambda^w}}}.$$

Assuming full indexation, this simplifies to

$$\bar{w} = \frac{\lambda^w\zeta^n\Theta N^\varphi}{\psi_{z+}}. \tag{12.77}$$

From this equation (in combination with the rest of the system of steady-state equations) we can solve for N.

Evaluating equation (4.52) in steady state, we get the following expression for the preference shifter Θ:

$$\Theta = z^C\bar{\upsilon}^N, \tag{12.78}$$

where from (4.53) and (4.54) we have

$$z^C = \frac{1}{\bar{\upsilon}^N}\left(\frac{1}{\mu_{z+}}\right)^{\frac{1-\nu}{\nu}} \tag{12.79}$$

and

$$\bar{\upsilon}^N = \frac{\zeta^\beta\zeta^c}{c\left(\mu_{z+}-b\right)}\left(\mu_{z+}-\beta b\right), \tag{12.80}$$

respectively. We note that we can combine equations (12.78) and (12.79) to obtain

$$\Theta = \frac{1}{\bar{\upsilon}^N}\left(\frac{1}{\mu_{z+}}\right)^{\frac{1-\nu}{\nu}}\bar{\upsilon}^N = \left(\frac{1}{\mu_{z+}}\right)^{\frac{1-\nu}{\nu}}, \tag{12.81}$$

which is a function only of variables which we have already solved for earlier.

We noted earlier, when deriving steady-state expressions for relative prices, that we were still missing an expression for the steady-state real exchange rate q. Adding the net foreign assets equation to the equation system above will allow us to solve for q together with the rest of the variables. Evaluating the expression for the evolution of net foreign assets (7.3) in steady state yields

$$\begin{aligned}
&\bar{a} + \frac{qp^c}{p^{c,*}}R^{wc,m}\left(c^m\left(\mathring{p}^{m,c}\right)^{\frac{\lambda^{m,c}}{1-\lambda^{m,c}}} + i^m\left(\mathring{p}^{m,i}\right)^{\frac{\lambda^{m,i}}{1-\lambda^{m,i}}} + x^m\left(\mathring{p}^{m,x}\right)^{\frac{\lambda^{m,x}}{1-\lambda^{m,x}}}\right.\\
&\left.+\phi^{m,c}+\phi^{m,i}+\phi^{m,x}\right) + \frac{qp^cp^{ce,*}}{p^{c,*}}R^{wc,m}\left(c^{e,m}\left(\mathring{p}^{m,ce}\right)^{\frac{\lambda_t^{m,ce}}{1-\lambda_t^{m,ce}}} + \phi^{m,ce}\right)\\
= \;& \frac{qp^cp^x}{p^{c,*}}\left(\left(\mathring{p}^x\right)^{\frac{\lambda^x}{1-\lambda^x}}x - \phi^x\right) + R^*\Phi\chi s\frac{\bar{a}}{\pi^d\mu_{z+}}.
\end{aligned}$$

Assuming full indexation implies that $\tilde{\pi}^{m,j} = \pi^{m,j}$, so that $\mathring{p}^{m,j} = 1$, for $j = c, i, x, ce$, and similarly $\mathring{p}^x = 1$. Simplifying the expression above and supposing that $\bar{a} = \eta_a y$, we obtain, under full indexation:

$$\begin{aligned}
&\eta_a y\frac{p^{c,*}}{qp^c}\left(1-\frac{R^*\Phi\chi s}{\pi^d\mu_{z+}}\right) + p^{ce,*}R^{wc,m}\left(c^{e,m}+\phi^{m,ce}\right)\\
&+R^{wc,m}\left(c^m+i^m+x^m+\phi^{m,c}+\phi^{m,i}+\phi^{m,x}\right)\\
= \;& p^x\left(x-\phi^x\right).
\end{aligned} \tag{12.82}$$

Under the assumption that $\eta_a = 0$, which we normally assume, we have that

$$R^{wc,m}\left(c^m+i^m+x^m+\phi^{m,c}+\phi^{m,i}+\phi^{m,x}\right) + p^{ce,*}R^{wc,m}\left(c^{e,m}+\phi^{m,ce}\right) = p^x\left(x-\phi^x\right).$$

Evaluating (3.146), (3.175), (3.204), and (3.148) in steady state to obtain expressions for c^m, i^m, x^m, and $c^{e,m}$, respectively, yields

$$\begin{aligned}
c^m &= \omega_c \left(\frac{p^{m,c}}{p^c}\right)^{-\eta_c} c, && (12.83)\\
i^m &= \omega_i \left(\frac{p^{m,i}}{p^i}\right)^{-\eta_i} \left(i + a(u)\frac{k^p}{\mu_{z+}\mu_\Psi}\right) = \omega_i \left(\frac{p^{m,i}}{p^i}\right)^{-\eta_i} i, && (12.84)\\
x^m &= \left(\omega_x + (1-\omega_x)(p^{m,x})^{\eta_x - 1}\right)^{\frac{\eta_x}{1-\eta_x}} \omega_x (\mathring{p}^x)^{\frac{\lambda^x}{1-\lambda^x}} x\\
&= \left(\omega_x + (1-\omega_x)(p^{m,x})^{\eta_x - 1}\right)^{\frac{\eta_x}{1-\eta_x}} \omega_x x, && (12.85)\\
c^{e,m} &= \omega_{em} \left(\frac{p^{m,ce}}{p^{ce}}\right)^{-\eta_{em}} c^e, && (12.86)
\end{aligned}$$

where the second expression uses the fact that $a(u) = 0$, and the third expression assumes full indexation implying $\mathring{p}^x = 1$.

We now have a complete system of equations, consisting of expressions (12.52) for $j = c, i, x, ce$, (12.54), (12.55), (12.56), (12.57), (12.58), (12.59), (12.60), (12.62), (12.63), (12.64), (12.65), (12.66), (12.67), (12.68), (12.69), (12.70), (12.71), (12.72), (12.74), (12.75), (12.76), (12.77), (12.82), (12.83), (12.84), (12.85), and (12.86) which, under the assumption of full indexation, can be used to solve for the steady-state values of the following variables:

$$\begin{aligned}
&p^{m,c}, p^{m,i}, p^{m,x}, p^{m,ce}, p^{cxe}, p^{ce}, p^c, p^i, p^x, \breve{p}_{k'}, \bar{r}^k, \bar{w}, k, k^p,\\
&c, c^d, c^{e,d}, c^{xe}, c^e, i^d, x^d,\\
&x, y, i, \psi_{z+}, N, q, c^m, i^m, x^m, c^{e,m},
\end{aligned}$$

given a steady-state solution of the structural foreign-economy model.

We can then solve for m using the total import demand equation (3.227) evaluated in steady state:

$$m = c^m + i^m + x^m + c^{e,m}. \qquad (12.87)$$

We can also solve for the steady-state households' aggregate labour, n, using (4.49):

$$n = N \mathring{w}^{\frac{\lambda^w}{1-\lambda^w}}.$$

Using (12.50) under the assumption of full indexation, we have that $\mathring{w} = 1$. Hence,

$$n = N. \qquad (12.88)$$

Using equation (4.56) and integrating over all labour types, we get the following expression for the steady-state marginal rate of substitution:

$$mrs = \frac{\zeta^\beta \zeta^n \Theta N^\varphi}{\bar{\upsilon}^N}. \qquad (12.89)$$

From (4.78), we have

$$\frac{\bar{w}}{p^c} \int_0^1 \frac{W_j}{W} dj = \zeta^\beta \zeta^n z^C \int_0^1 L_j^\varphi dj.$$

Assuming full indexation gives that $\int_0^1 \frac{W_j}{W} dj = 1$. We then have that $L = L_j$ for all j and

$$L = \left(\frac{\bar{w}}{p^c \zeta^\beta \zeta^n z^C}\right)^{\frac{1}{\varphi}}. \qquad (12.90)$$

Equation (4.38) implies that steady-state unemployment is given by

$$U = \frac{L - N}{L}. \tag{12.91}$$

Finally, we have that

$$U^n = U. \tag{12.92}$$

12.2 Steady state of the foreign-economy model

We begin to solve the steady state for the monetary policy rule. With the exception of consumer price inflation, all variables are in log-deviation from their steady-state values, or in deviation from its steady-state value in the case of unemployment. We have, in steady state, that

$$\begin{aligned} \log\left(\frac{R^*}{R^*}\right) &= \rho_{R^*}\log\left(\frac{R^*}{R^*}\right) + (1-\rho_{R^*})\left[\log\left(\frac{\bar{\pi}^{c,*}}{\bar{\pi}^{c,*}}\right) + r_{\pi^*}\log\left(\frac{\pi^{c,*}}{\bar{\pi}^{c,*}}\right)\right. \\ &\quad \left. + r_{RU^*}\left(U^* - U^*\right)\right] + r_{\Delta\pi^*}\Delta\log\left(\frac{\pi^{c,*}}{\pi^{c,*}}\right) + r_{\Delta RU^*}\Delta U^*, \end{aligned}$$

yielding

$$\begin{aligned} \log\left(\frac{\pi^{c,*}}{\bar{\pi}^{c,*}}\right) &= 0 \\ \Rightarrow \pi^{c,*} &= \bar{\pi}^{c,*}. \end{aligned} \tag{12.93}$$

From (11.20), (11.21) and (11.22), we have

$$\pi^{cxe,*} = \pi^{d,*}, \tag{12.94}$$

$$\pi^{c,*} = \pi^{d,*}, \tag{12.95}$$

$$\pi^{ce,*} = \pi^{d,*}. \tag{12.96}$$

From equation (11.25), we further have that

$$\pi^{i,*} = \frac{\pi^*}{\mu_{\Psi^*}}. \tag{12.97}$$

Moreover, (11.24) yields

$$p^{i,*} = 1.$$

Price indexation yields

$$\tilde{\pi}^{d,*} = \left(\pi^{d,*}\right)^{\kappa^*}\left(\bar{\pi}^{c,*}\right)^{1-\kappa^*-\varkappa^*}\left(\breve{\pi}^*\right)^{\varkappa^*} = \left(\pi^{d,*}\right)^{1-\varkappa^*}\left(\breve{\pi}^*\right)^{\varkappa^*}.$$

Assuming full indexation, i.e. that $\varkappa^* = 0$, this simplifies to

$$\tilde{\pi}^{d,*} = \pi^{d,*}. \tag{12.98}$$

This assumption also implies that the marginal cost in steady state is given by

$$mc^* = \frac{1}{\lambda^*}, \tag{12.99}$$

where λ^* is related to the elasticity of substitution between the different foreign goods, which we denote by η^*, in the following way:

$$\lambda^* = \frac{\eta^*}{1-\eta^*}.$$

From equation (11.51), we can solve for wage inflation

$$\pi^{w,*} = \frac{\bar{w}^* \mu_{z+,*} \pi^{d,*}}{\bar{w}^*} = \mu_{z+,*} \pi^{d,*}, \tag{12.100}$$

while wage indexation yields

$$\tilde{\pi}^{w,*} = \left(\pi^{c,*}\right)^{\kappa_w^*} \left(\bar{\pi}^{c,*}\right)^{1-\kappa_w^*-\varkappa_w^*} \left(\breve{\pi}^*\right)^{\varkappa_w^*} \left(\mu_{z+,*}\right)^{\vartheta_w^*}$$

$$\tilde{\pi}^{w,*} = \left(\pi^{d,*}\right)^{1-\varkappa_w^*} \left(\breve{\pi}^*\right)^{\varkappa_w^*} \left(\mu_{z+,*}\right)^{\vartheta_w^*}.$$

Under the additional assumptions that $\varkappa_w^* = 0$ and $\vartheta_w^* = 1$, we have that

$$\tilde{\pi}^{w,*} = \pi^{d,*} \mu_{z+,*}. \tag{12.101}$$

Given the expression for $\pi^{w,*}$ above, we moreover have that $\frac{\tilde{\pi}^{w,*}}{\pi_t^{w,*}} = 1$, which yields

$$\tilde{w}^* = 1. \tag{12.102}$$

From equation (11.17), we have that

$$p^{c,*} = \left[\left(1 - \omega_e^*\right) + \omega_e^* \left(p^{ce,*}\right)^{1-\eta_e^*}\right]^{1/\left(1-\eta_e^*\right)}, \tag{12.103}$$

where we recall that $p_t^{ce,*}$ is given by an exogenous process, and $p^{ce,*}$ is calibrated.

We have that

$$\mu_{z+,*} = \mu_{\Psi^*}^{\frac{\alpha^*}{1-\alpha^*}} \mu_{z^*}. \tag{12.104}$$

From the household's consumption Euler equation, we can then solve for the steady-state interest rate

$$R^* = \frac{\mu_{z+,*}}{\beta^*} \pi^{d,*}. \tag{12.105}$$

Having determined R^*, we can now solve for the steady-state gross effective nominal interest rate faced by the firms:

$$R^{wc,*} = \nu^{wc,*} R^* + 1 - \nu^{wc,*}. \tag{12.106}$$

From the household's FOC w.r.t investment and bond holdings, we have

$$p^{i,*} = \breve{p}_{k'}^* \Upsilon^* F_1\left(i^*, i^*\right) + \frac{\pi^{d,*}}{R^*} \breve{p}_{k'}^* \frac{1}{\mu_{\Psi^*}} \Upsilon^* F_2\left(i^*, i^*\right).$$

Assuming, just as for the domestic economy, that $\tilde{S}\left(x^*\right) = \tilde{S}'\left(x^*\right) = 0$ holds in steady state, with $x^* = \mu_{z+,*} \mu_{\Psi^*}$, we have that $F_1\left(i^*, i^*\right) = 1$ and $F_2\left(i^*, i^*\right) = 0$. The steady-state price of physical capital is then given by

$$\breve{p}_{k'}^* = \frac{p^{i,*}}{\Upsilon^*} = \frac{1}{\Upsilon^*}.$$

From the household's FOC, we can also solve for $\bar{r}^k$ as follows:

$$\bar{r}^{k,*} = \breve{p}_{k'}^* \left(\frac{R^*}{\pi^{d,*}} \mu_{\Psi^*} - \left(1 - \delta^*\right)\right), \tag{12.107}$$

where we have used that $a\left(u^*\right) = 0$ holds in steady state. We can also derive the following expression for $\sigma_b^* = a'\left(u^*\right)$:

$$\sigma_b^* = \bar{r}^{k,*}, \tag{12.108}$$

where we have used that $p^{i,*} = 1$.

From the foreign expressions for the firms' marginal costs, corresponding to (3.30) and (3.31), we can derive the following expressions for $\bar{w}^*$ and for the steady-state capital-to-labour ratio:

$$\bar{w}^* = \left[\frac{1}{mc^*} \frac{\left(R^{wc,*}\right)^{1-\alpha^*} \left(\bar{r}^{k,*}\right)^{\alpha^*}}{\left(1-\alpha^*\right)^{1-\alpha^*} \left(\alpha^*\right)^{\alpha^*} \epsilon^*} \right]^{\frac{1}{\alpha^*-1}}, \tag{12.109}$$

$$\frac{k^*}{N^*} = \mu_{\Psi^*} \mu_{z^+,*} \left(\frac{1}{mc^*} \frac{\bar{w}^* R^{wc,*}}{\epsilon^* \left(1-\alpha^*\right)} \right)^{\frac{1}{\alpha^*}}. \tag{12.110}$$

Just as for the domestic economy case, we impose that the capital utilization rate, u^*, is 1. Using the relation between physical and efficient capital, we then have

$$k^* = k^{p,*}. \tag{12.111}$$

As for the domestic economy, we will derive an expression for aggregate homogeneous labour, N^*, from the expressions for consumption and investment from the household optimization, combined with the aggregate resource constraint. We begin by deriving an expression for consumption as a function of hours. The optimal-wage expression corresponding to equation (4.79) for the domestic economy, evaluated in steady state under the assumption of full indexation, yields

$$\bar{w}^* = \frac{\lambda^{w,*} \zeta^{n,*} \Theta^* \left(N^*\right)^{\varphi^*}}{\psi_{z^+,*}}.$$

Using the consumption Euler equation, we can derive the following expression for $\psi_{z^+,*}$:

$$\psi_{z^+,*} = \frac{1}{p^{c,*}} \frac{\zeta^{c,*}}{c^* \left(\mu_{z^+,*} - b^*\right)} \left(\mu_{z^+,*} - \beta^* b^*\right). \tag{12.112}$$

Combining the two above equations, we obtain the following relationship determining the steady-state value of consumption:

$$c^* = \frac{\bar{w}^*}{p^{c,*} \lambda^{w,*} \zeta^{n,*} \Theta^* \left(N^*\right)^{\varphi^*}} \frac{\zeta^{c,*} \left(\mu_{z^+,*} - \beta^* b^*\right)}{\left(\mu_{z^+,*} - b^*\right)},$$

where

$$\Theta^* = \left(\frac{1}{\mu_{z^+,*}} \right)^{\frac{1-\nu^*}{\nu^*}},$$

or

$$c^* \left(N^*\right)^{\varphi^*} = \Xi_{c^* N^*}, \tag{12.113}$$

where

$$\Xi_{c^* H^*} = \frac{\bar{w}^* \left(\mu_{z^+,*}\right)^{\frac{1-\nu^*}{\nu^*}}}{p^{c,*} \lambda^{w,*} \zeta^{n,*}} \frac{\zeta^{c,*} \left(\mu_{z^+,*} - \beta^* b^*\right)}{\left(\mu_{z^+,*} - b^*\right)}.$$

From the law of motion for capital, we can derive the following expression for steady-state investment:

$$i^* = \frac{k^*}{\Upsilon^*} \left(1 - \frac{1-\delta^*}{\mu_{z^+,*} \mu_{\Psi^*}} \right). \tag{12.114}$$

Next, we derive a steady-state expression for the aggregate ressource constraint, which will allow us to determine N^* and, in continuation, y^* together with the above equations (12.113) and (12.114). For this, we first need to consider the expression for total production in equation (6.8). Evaluating in steady state, we get

$$y^* = \left(\mathring{p}^*\right)^{\frac{\lambda^*}{\lambda^*-1}} \left[\epsilon^* \left(\frac{k^*}{\mu_{\Psi^*} \mu_{z^+,*}} \right)^{\alpha^*} \left(N^*\right)^{1-\alpha^*} - \phi^* \right].$$

Assuming full indexation implies that $\mathring{p}^* = 1$. We can then obtain the following expression for the fixed cost term ϕ^*, setting profits to zero in steady state:

$$\phi^* = y^* \left(\frac{1}{mc^*} - 1 \right). \tag{12.115}$$

Combining the above two equations, using that $\epsilon^* = 1$, we get

$$y^* = \left(\frac{1}{\mu_{\Psi^*} \mu_{z^+,*}} \right)^{\alpha^*} \left(\frac{k^*}{N^*} \right)^{\alpha^*} N^* mc^*,$$

or

$$y^* = \Xi_{y^*} N^*, \tag{12.116}$$

where

$$\Xi_{y^*} = \left(\frac{1}{\mu_{\Psi^*} \mu_{z^+,*}} \right)^{\alpha^*} \left(\frac{k^*}{N^*} \right)^{\alpha^*} mc^*.$$

Note that we treat the capital-to-labour ratio as one variable, which we have already solved for above.

We have that the foreign good is allocated among the alternatives uses as follows:

$$y^* = g^* + c^{d,*} + c^{e,*} + i^{d,*},$$

where

$$c^{d,*} = c^{xe,*}$$

$$c^{xe,*} = (1 - \omega_e^*) \left(p^{c,*} \right)^{\eta_e^*} c^*, \tag{12.117}$$

$$c^{e,*} = \omega_e^* \left(\frac{p^{ce,*}}{p^{c,*}} \right)^{-\eta_e^*} c^*, \tag{12.118}$$

and

$$i_t^{d,*} = i_t^* + \frac{a(u_t^*) k_t^{p,*}}{\mu_{z^+,*,t} \mu_{\Psi^*,t}}.$$

Usign the assumption that $a(u^*) = 0$, we can write

$$y^* = g^* + \left((1 - \omega_e^*) \left(p^{c,*} \right)^{\eta_e^*} + \omega_e^* \left(\frac{p^{ce,*}}{p^{c,*}} \right)^{-\eta_e^*} \right) c^* + i^*. \tag{12.119}$$

Substituting in (12.113) and (12.114) into (12.119), and using that $g^* = \eta_g^* y^* = \eta_g^* \Xi_{y^*} N^*$, and that $\Upsilon^* = 1$, we can derive the following expression for steady-state aggregate homogeneous hours:

$$N^* = \left[\frac{\left((1 - \omega_e^*) \left(p^{c,*} \right)^{\eta_e^*} + \omega_e^* \left(\frac{p^{ce,*}}{p^{c,*}} \right)^{-\eta_e^*} \right) \Xi_{c^* N^*}}{\left(1 - \eta_g^* \right) \Xi_{y^*} - \left(1 - \frac{1 - \delta^*}{\mu_{z^+,*} \mu_{\Psi^*}} \right) \frac{k^*}{N^*}} \right]^{\frac{1}{1+\varphi^*}}.$$

Having found an expression for N^*, we can now easily solve for y^* from (12.116), c^* from (12.113), $c^{xe,*}$ from (12.117), $c^{e,*}$ from (12.118), $\psi_{z^+,*}$ from (12.112), k^* from (12.110). Having solved for k^*, we can use (12.114) to solve for i^*. We can also solve for aggregate household labour, using

$$n^* = N^* \left(\mathring{w}^* \right)^{\frac{\lambda^{w,*}}{1 - \lambda^{w,*}}}.$$

We note that full indexation implies that $\mathring{w}^* = 1$, and so

$$n^* = N^*.$$

13 Summary of the model with a structural foreign economy

We here list the set of log-linear equations needed to simulate the model with a structural foreign economy. Above each respective log-linear equations, we show the non-linear equation(s) from which it has been derived. This section summarizes the endogenous variables of the model. The exogenous variables are listed in Sections 10 for the domestic and 11.1.5 for the foreign economy.

13.1 Domestic equations

From the domestic intermediate good firms' problem, we have the gross effective nominal interest rate faced by domestic good firms in equation (3.42)

$$R_t^{wc,d} \equiv \nu_t^{wc,d} R_t + 1 - \nu_t^{wc,d}$$

$$\hat{R}_t^{wc,d} = \frac{\nu^{wc,d}(R-1)}{R^{wc,d}}\hat{\nu}_t^{wc,d} + \frac{\nu^{wc,d}R}{R^{wc,d}}\hat{R}_t, \tag{13.1}$$

marginal costs in equation (3.46)

$$mc_t^d = \frac{\bar{w}_t R_t^{wc,d}}{mpl_t}$$

$$\widehat{mc}_t^d = \hat{\bar{w}}_t + \hat{R}_t^{wc,d} - \widehat{mpl}_t, \tag{13.2}$$

the marginal product of labour in equation (3.48)

$$mpl_t = (1-\alpha)\,\epsilon_t\left(\frac{k_t}{\mu_{z^+,t}\mu_{\Psi,t}N_t}\right)^{\alpha}$$

$$\widehat{mpl}_t = \alpha\widehat{\left(\frac{k}{N}\right)_t} + \hat{\epsilon}_t, \tag{13.3}$$

the capital-to-labour ratio in equation (3.50)

$$\left(\frac{k}{N}\right)_t = \frac{k_t}{\mu_{z^+,t}\mu_{\Psi,t}N_t}$$

$$\widehat{\left(\frac{k}{N}\right)_t} = \hat{k}_t - \hat{N}_t - \left(\hat{\mu}_{z^+,t} + \hat{\mu}_{\Psi,t}\right), \tag{13.4}$$

marginal costs in equation (3.47)

$$mc_t^d = \frac{\bar{r}_t^k}{mpk_t}$$

$$\widehat{mc}_t^d = \hat{\bar{r}}_t^k - \widehat{mpk}_t, \tag{13.5}$$

the marginal product of capital in equation (3.49)

$$mpk_t = \alpha\epsilon_t\left(\frac{k_t}{\mu_{z^+,t}\mu_{\Psi,t}N_t}\right)^{-(1-\alpha)}$$

$$\widehat{mpk}_t = -(1-\alpha)\widehat{\left(\frac{k}{N}\right)_t} + \hat{\epsilon}_t, \tag{13.6}$$

and the domestic price Phillips curve in equation (3.56)[49]

$$\tilde{p}_t^d E_t \sum_{s=0}^{\infty} (\beta\xi_d)^s \zeta_{t+s}^{\beta} \psi_{z^+,t+s} y_{t+s} \left(\frac{\tilde{\pi}_{t+1}^d \cdots \tilde{\pi}_{t+s}^d}{\pi_{t+1}^d \cdots \pi_{t+s}^d} \right)^{\frac{1}{1-\lambda_{t+s}^d}}$$

$$= E_t \sum_{s=0}^{\infty} (\beta\xi_d)^s \zeta_{t+s}^{\beta} \psi_{z^+,t+s} y_{t+s} \lambda_{t+s}^d mc_{t+s}^d \left(\frac{\tilde{\pi}_{t+1}^d \cdots \tilde{\pi}_{t+s}^d}{\pi_{t+1}^d \cdots \pi_{t+s}^d} \right)^{\frac{\lambda_{t+s}^d}{1-\lambda_{t+s}^d}}$$

$$\tilde{\pi}_t^d = \left(\pi_{t-1}^d\right)^{\kappa_d} \left(\bar{\pi}_t^c\right)^{1-\kappa_d-\varkappa_d} \left(\breve{\pi}\right)^{\varkappa_d}$$

$$\tilde{p}_t^d = \left[\frac{1-\xi_d \left(\frac{\tilde{\pi}_t^d}{\pi_t^d} \right)^{\frac{1}{1-\lambda_t^d}}}{(1-\xi_d)} \right]^{1-\lambda_t^d}$$

$$\begin{aligned} \hat{\pi}_t^d - \widehat{\bar{\pi}}_t^c &= \frac{(1-\beta\xi_d)(1-\xi_d)}{\xi_d(1+\beta\kappa_d)} \left(\widehat{mc}_t^d + \hat{\lambda}_t^d \right) + \frac{\kappa_d}{1+\beta\kappa_d} \left(\hat{\pi}_{t-1}^d - \widehat{\bar{\pi}}_t^c \right) \\ &+ \frac{\beta}{1+\beta\kappa_d} E_t \left(\hat{\pi}_{t+1}^d - \widehat{\bar{\pi}}_{t+1}^c \right) - \frac{\beta\kappa_d}{1+\beta\kappa_d} E_t \left(\widehat{\bar{\pi}}_t^c - \widehat{\bar{\pi}}_{t+1}^c \right). \end{aligned} \tag{13.7}$$

We also include the evolution of the combination of investment-specific and neutral technology in equation (3.51)

$$\mu_{z^+,t} = \mu_{\Psi,t}^{\frac{\alpha}{1-\alpha}} \mu_{z,t}$$

$$\hat{\mu}_{z^+,t} = \frac{\alpha}{1-\alpha} \hat{\mu}_{\Psi,t} + \hat{\mu}_{z,t}, \tag{13.8}$$

and the expression for the domestic price dispersion in equation (3.57)

$$\mathring{p}_t^d = \left[\xi_d \left(\frac{\tilde{\pi}_t^d}{\pi_t^d} \mathring{p}_{t-1}^d \right)^{\frac{\lambda_t^d}{1-\lambda_t^d}} + (1-\xi_d) \left(\frac{1-\xi_d \left(\frac{\tilde{\pi}_t^d}{\pi_t^d} \right)^{\frac{1}{1-\lambda_t^d}}}{1-\xi_d} \right)^{\lambda_t^d} \right]^{\frac{1-\lambda_t^d}{\lambda_t^d}}$$

$$\widehat{\mathring{p}}_t^d = \xi_d \widehat{\mathring{p}}_{t-1}^d. \tag{13.9}$$

From the importing firms' problem, we have the gross effective nominal interest rate faced by importing firms in equation (3.97)

$$R_t^{wc,m} = \nu_t^{wc,m} R_t^* + 1 - \nu_t^{wc,m}$$

$$\hat{R}_t^{wc,m} = \frac{\nu^{wc,m}(R^*-1)}{R^{wc,m}} \hat{\nu}_t^{wc,m} + \frac{\nu^{wc,m} R^*}{R^{wc,m}} \hat{R}_t^*, \tag{13.10}$$

marginal costs for the import consumption, import investments, and import-to-export importers in equation (3.98)

$$mc_t^{m,j} = \frac{q_t p_t^c}{p_t^{c,*} p_t^{m,j}} R_t^{wc,m}, \quad j = c, i, x$$

[49] The log-linear Phillips curve is a combination of the log-linearized versions of the three included non-linear equations. Note that the log-linearized equation included here has been derived under the assumption that there is full indexation, that is assuming that $\varkappa_d = 0$.

$$\widehat{mc}_t^{m,c} = \hat{q}_t + \hat{p}_t^c - \hat{p}_t^{c,*} - \hat{p}_t^{m,c} + \hat{R}_t^{wc,m}, \tag{13.11}$$

$$\widehat{mc}_t^{m,i} = \hat{q}_t + \hat{p}_t^c - \hat{p}_t^{c,*} - \hat{p}_t^{m,i} + \hat{R}_t^{wc,m}, \tag{13.12}$$

$$\widehat{mc}_t^{m,x} = \hat{q}_t + \hat{p}_t^c - \hat{p}_t^{c,*} - \hat{p}_t^{m,x} + \hat{R}_t^{wc,m}, \tag{13.13}$$

marginal costs for the imported energy consumption in equation (3.99)

$$mc_t^{m,ce} = \frac{q_t p_t^c p_t^{ce,*}}{p_t^{c,*} p_t^{m,ce}} R_t^{wc,m}$$

$$\widehat{mc}_t^{m,ce} = \hat{q}_t + \hat{p}_t^c + \hat{p}_t^{ce,*} - \hat{p}_t^{c,*} - \hat{p}_t^{m,ce} + \hat{R}_t^{wc,m}, \tag{13.14}$$

the Phillips curves for the four different types of importers in equation (3.100)[50]

$$\begin{aligned}
& \tilde{p}_t^{m,j} E_t \sum_{s=0}^{\infty} (\beta\xi_{m,j})^s \zeta_{t+s}^{\beta} \psi_{z^+,t+s} p_t^{m,j} z_{t+s}^j \left(\frac{\tilde{\pi}_{t+1}^{m,j} \dots \tilde{\pi}_{t+s}^{m,j}}{\pi_{t+1}^{m,j} \dots \pi_{t+s}^{m,j}} \right)^{\frac{1}{1-\lambda_{t+s}^{m,j}}} \\
= \; & E_t \sum_{s=0}^{\infty} (\beta\xi_{m,j})^s \zeta_{t+s}^{\beta} \psi_{z^+,t+s} p_t^{m,j} z_{t+s}^j \lambda_{t+s}^{m,j} mc_{t+s}^{m,j} \left(\frac{\tilde{\pi}_{t+1}^{m,j} \dots \tilde{\pi}_{t+s}^{m,j}}{\pi_{t+1}^{m,j} \dots \pi_{t+s}^{m,j}} \right)^{\frac{\lambda_{t+s}^{m,j}}{1-\lambda_{t+s}^{m,j}}},
\end{aligned}$$

$$j = c, i, x, ce, \quad z_t^j = \begin{cases} c_t^m & if \;\; j = c \\ i_t^m & if \;\; j = i \\ x_t^m & if \;\; j = x \\ c_t^{e,m} & if \;\; j = ce \end{cases}$$

$$\tilde{\pi}_t^{m,j} \equiv \left(\pi_{t-1}^{m,j}\right)^{\kappa_{m,j}} \left(\bar{\pi}_t^c\right)^{1-\kappa_{m,j}-\varkappa_{m,j}} \left(\breve{\pi}\right)^{\varkappa_{m,j}}, \quad j = c, i, x, ce$$

$$\tilde{p}_t^{m,j} = \left[\frac{1 - \xi_{m,j} \left(\frac{\tilde{\pi}_t^{m,j}}{\pi_t^{m,j}} \right)^{\frac{1}{1-\lambda_t^{m,j}}}}{\left(1 - \xi_{m,j}\right)} \right]^{1-\lambda_t^{m,j}}, \quad j = c, i, x, ce$$

$$\begin{aligned}
\hat{\pi}_t^{m,c} - \widehat{\bar{\pi}}_t^c = \; & \frac{\left(1 - \beta\xi_{m,c}\right)\left(1 - \xi_{m,c}\right)}{\xi_{m,c}\left(1 + \beta\kappa_{m,c}\right)} \left(\widehat{mc}_t^{m,c} + \hat{\lambda}_t^{m,c}\right) + \frac{\kappa_{m,c}}{1 + \beta\kappa_{m,c}} \left(\hat{\pi}_{t-1}^{m,c} - \widehat{\bar{\pi}}_t^c\right) \\
& + \frac{\beta}{1 + \beta\kappa_{m,c}} E_t \left(\hat{\pi}_{t+1}^{m,c} - \widehat{\bar{\pi}}_{t+1}^c\right) - \frac{\beta\kappa_{m,c}}{1 + \beta\kappa_{m,c}} E_t \left(\widehat{\bar{\pi}}_t^c - \widehat{\bar{\pi}}_{t+1}^c\right),
\end{aligned} \tag{13.15}$$

$$\begin{aligned}
\hat{\pi}_t^{m,i} - \widehat{\bar{\pi}}_t^c = \; & \frac{\left(1 - \beta\xi_{m,i}\right)\left(1 - \xi_{m,i}\right)}{\xi_{m,i}\left(1 + \beta\kappa_{m,i}\right)} \left(\widehat{mc}_t^{m,i} + \hat{\lambda}_t^{m,i}\right) + \frac{\kappa_{m,i}}{1 + \beta\kappa_{m,i}} \left(\hat{\pi}_{t-1}^{m,i} - \widehat{\bar{\pi}}_t^c\right) \\
& + \frac{\beta}{1 + \beta\kappa_{m,i}} E_t \left(\hat{\pi}_{t+1}^{m,i} - \widehat{\bar{\pi}}_{t+1}^c\right) - \frac{\beta\kappa_{m,i}}{1 + \beta\kappa_{m,i}} E_t \left(\widehat{\bar{\pi}}_t^c - \widehat{\bar{\pi}}_{t+1}^c\right),
\end{aligned} \tag{13.16}$$

$$\begin{aligned}
\hat{\pi}_t^{m,x} - \widehat{\bar{\pi}}_t^c = \; & \frac{\left(1 - \beta\xi_{m,x}\right)\left(1 - \xi_{m,x}\right)}{\xi_{m,x}\left(1 + \beta\kappa_{m,x}\right)} \left(\widehat{mc}_t^{m,x} + \hat{\lambda}_t^{m,x}\right) + \frac{\kappa_{m,x}}{1 + \beta\kappa_{m,x}} \left(\hat{\pi}_{t-1}^{m,x} - \widehat{\bar{\pi}}_t^c\right) \\
& + \frac{\beta}{1 + \beta\kappa_{m,x}} E_t \left(\hat{\pi}_{t+1}^{m,x} - \widehat{\bar{\pi}}_{t+1}^c\right) - \frac{\beta\kappa_{m,x}}{1 + \beta\kappa_{m,x}} E_t \left(\widehat{\bar{\pi}}_t^c - \widehat{\bar{\pi}}_{t+1}^c\right),
\end{aligned} \tag{13.17}$$

[50] Just as for the domestic good producer, the log-linear Phillips curve for the importer is a combination of the log-linearized versions of the three included non-linear equations. Note that the log-linearized equation included here has been derived under the assumption that there is full indexation, that is assuming that $\varkappa_{m,j} = 0, \; j = c, i, x$.

$$\hat{\pi}_t^{m,ce} - \widehat{\bar{\pi}}_t^c = \frac{\left(1-\beta\xi_{m,ce}\right)\left(1-\xi_{m,ce}\right)}{\xi_{m,ce}\left(1+\beta\kappa_{m,ce}\right)}\left(\widehat{mc}_t^{m,ce} + \hat{\lambda}_t^{m,ce}\right) + \frac{\kappa_{m,ce}}{1+\beta\kappa_{m,ce}}\left(\hat{\pi}_{t-1}^{m,ce} - \widehat{\bar{\pi}}_t^c\right) \quad (13.18)$$
$$+\frac{\beta}{1+\beta\kappa_{m,ce}}E_t\left(\hat{\pi}_{t+1}^{m,ce} - \widehat{\bar{\pi}}_{t+1}^c\right) - \frac{\beta\kappa_{m,ce}}{1+\beta\kappa_{m,ce}}E_t\left(\widehat{\bar{\pi}}_t^c - \widehat{\bar{\pi}}_{t+1}^c\right),$$

and the expression for the import consumption, import investments, import-to-export, and energy import price dispersion in equation (3.102)

$$\mathring{p}_t^{m,j} = \left[\xi_{m,j}\left(\frac{\tilde{\pi}_t^{m,j}}{\pi_t^{m,j}}\mathring{p}_{t-1}^{m,j}\right)^{\frac{\lambda_t^{m,j}}{1-\lambda_t^{m,j}}} + \left(1-\xi_{m,j}\right)\left(\frac{1-\xi_{m,j}\left(\frac{\tilde{\pi}_t^{m,j}}{\pi_t^{m,j}}\right)^{\frac{1}{1-\lambda_t^{m,j}}}}{1-\xi_{m,j}}\right)^{\lambda_t^{m,j}}\right]^{\frac{1-\lambda_t^{m,j}}{\lambda_t^{m,j}}}, \quad j = c,i,x,ce$$

$$\widehat{\mathring{p}}_t^{m,c} = \xi_{m,c}\widehat{\mathring{p}}_{t-1}^{m,c}, \quad (13.19)$$
$$\widehat{\mathring{p}}_t^{m,i} = \xi_{m,i}\widehat{\mathring{p}}_{t-1}^{m,i}, \quad (13.20)$$
$$\widehat{\mathring{p}}_t^{m,x} = \xi_{m,x}\widehat{\mathring{p}}_{t-1}^{m,x}, \quad (13.21)$$
$$\widehat{\mathring{p}}_t^{m,ce} = \xi_{m,ce}\widehat{\mathring{p}}_{t-1}^{m,ce}. \quad (13.22)$$

From the exporting firms' problem, we have the gross effective nominal interest rate faced by exporting firms in equation (3.210)

$$R_t^{wc,x} = \nu_t^{wc,x}R_t + 1 - \nu_t^{wc,x}$$
$$\hat{R}_t^{wc,x} = \frac{\nu^{wc,x}\left(R-1\right)}{R^{wc,x}}\hat{\nu}_t^{wc,x} + \frac{\nu^{wc,x}R}{R^{wc,x}}\hat{R}_t, \quad (13.23)$$

marginal costs in equation (3.209)

$$mc_t^x = \frac{R_t^{wc,x}p_t^{c,*}}{q_t p_t^x p_t^c}\left(\omega_x\left(p_t^{m,x}\right)^{1-\eta_x} + \left(1-\omega_x\right)\right)^{\frac{1}{1-\eta_x}}$$
$$\widehat{mc}_t^x = \hat{R}_t^{wc,x} + \hat{p}_t^{c,*} - \hat{q}_t - \hat{p}_t^x - \hat{p}_t^c + \frac{\omega_x\left(p^{m,x}\right)^{1-\eta_x}}{\omega_x\left(p^{m,x}\right)^{1-\eta_x} + \left(1-\omega_x\right)}\hat{p}_t^{m,x}, \quad (13.24)$$

the export Phillips curve in equation (3.211)[51]

$$\tilde{p}_t^x E_t\sum_{s=0}^{\infty}(\beta\xi_x)^s\zeta_{t+s}^{\beta}\psi_{z^+,t+s}\frac{q_{t+s}p_{t+s}^c p_{t+s}^x}{p_{t+s}^{c,*}}x_{t+s}\left(\frac{\tilde{\pi}_{t+1}^x\cdots\tilde{\pi}_{t+s}^x}{\pi_{t+1}^x\cdots\pi_{t+s}^x}\right)^{\frac{1}{1-\lambda_{t+s}^x}}$$
$$= E_t\sum_{s=0}^{\infty}(\beta\xi_x)^s\zeta_{t+s}^{\beta}\psi_{z^+,t+s}\frac{q_{t+s}p_{t+s}^c p_{t+s}^x}{p_{t+s}^{c,*}}x_{t+s}\lambda_{t+s}^x mc_{t+s}^x\left(\frac{\tilde{\pi}_{t+1}^x\cdots\tilde{\pi}_{t+s}^x}{\pi_{t+1}^x\cdots\pi_{t+s}^x}\right)^{\frac{\lambda_{t+s}^x}{1-\lambda_{t+s}^x}}$$
$$\tilde{\pi}_t^x \equiv \left(\pi_{t-1}^x\right)^{\kappa_x}\left(\bar{\pi}_t^*\right)^{1-\kappa_x-\varkappa_x}\left(\breve{\pi}\right)^{\varkappa_x}$$
$$\tilde{p}_t^x = \left[\frac{1-\xi_x\left(\frac{\tilde{\pi}_t^x}{\pi_t^x}\right)^{\frac{1}{1-\lambda_t^x}}}{\left(1-\xi_x\right)}\right]^{1-\lambda_t^x}$$

[51] Just as for the domestic and import goods producers, the log-linear Phillips curve for the exporter is a combination of the log-linearized versions of the three included non-linear equations. Note that the log-linearized equation included here has been derived under the assumption that there is full indexation, that is assuming that $\varkappa_x = 0$.

$$
\begin{aligned}
\hat{\pi}^x_t - \widehat{\bar{\pi}}^*_t \quad = \quad & \frac{(1-\beta\xi_x)(1-\xi_x)}{\xi_x(1+\beta\kappa_x)}\left(\widehat{mc}^x_t + \hat{\lambda}^x_t\right) + \frac{\kappa_x}{1+\beta\kappa_x}\left(\hat{\pi}^x_{t-1} - \widehat{\bar{\pi}}^*_t\right) \\
& + \frac{\beta}{1+\beta\kappa_x} E_t\left(\hat{\pi}^x_{t+1} - \widehat{\bar{\pi}}^*_t\right) - \frac{\beta\kappa_x}{1+\beta\kappa_x} E_t\left(\widehat{\bar{\pi}}^*_t - \widehat{\bar{\pi}}^*_{t+1}\right),
\end{aligned} \tag{13.25}
$$

and the expression for the export price dispersion in equation (3.213)

$$
\mathring{p}^x_t = \left[\xi_x\left(\frac{\tilde{\pi}^x_t}{\pi^x_t}\mathring{p}^x_{t-1}\right)^{\frac{\lambda^x_t}{1-\lambda^x_t}} + (1-\xi_x)\left(\frac{1-\xi_x\left(\frac{\tilde{\pi}^x_t}{\pi^x_t}\right)^{\frac{1}{1-\lambda^x_t}}}{1-\xi_x}\right)^{\lambda^x_t}\right]^{\frac{1-\lambda^x_t}{\lambda^x_t}}
$$

$$
\widehat{\mathring{p}}^x_t = \xi_x \widehat{\mathring{p}}^x_{t-1}. \tag{13.26}
$$

We also include the demand equation for domestically produced goods used in export production in equation (3.214)

$$
x^d_t = \left(\omega_x \left(p^{m,x}_t\right)^{1-\eta_x} + (1-\omega_x)\right)^{\frac{\eta_x}{1-\eta_x}} (1-\omega_x)\left(\mathring{p}^x_t\right)^{\frac{\lambda^x_t}{1-\lambda^x_t}} x_t
$$

$$
\hat{x}^d_t = \frac{\eta_x\omega_x\left(p^{m,x}\right)^{1-\eta_x}}{\omega_x\left(p^{m,x}\right)^{1-\eta_x} + (1-\omega_x)}\hat{p}^{m,x}_t + \frac{\lambda^x}{1-\lambda^x}\widehat{\mathring{p}}^x_t + \hat{x}_t, \tag{13.27}
$$

and the one for imported goods used in export production in equation (3.215)

$$
x^m_t = \left(\omega_x + (1-\omega_x)\left(p^{m,x}_t\right)^{\eta_x-1}\right)^{\frac{\eta_x}{1-\eta_x}} \omega_x\left(\mathring{p}^x_t\right)^{\frac{\lambda^x_t}{1-\lambda^x_t}} x_t
$$

$$
\hat{x}^m_t = -\frac{\eta_x(1-\omega_x)\left(p^{m,x}\right)^{\eta_x-1}}{\omega_x + (1-\omega_x)\left(p^{m,x}\right)^{\eta_x-1}}\hat{p}^{m,x}_t + \frac{\lambda^x}{1-\lambda^x}\widehat{\mathring{p}}^x_t + \hat{x}_t. \tag{13.28}
$$

From the final consumption and investment good firms' problem, we include the expression for the aggregate non-energy consumption demand in equation (3.155)

$$
c^{xe}_t = (1-\omega_e)\left[\frac{p^{cxe}_t}{p^c_t}\right]^{-\eta_e} c_t
$$

$$
\hat{c}^{xe}_t = -\eta_e\left(\hat{p}^{cxe}_t - \hat{p}^c_t\right) + \hat{c}_t, \tag{13.29}
$$

and the expression for energy consumption demand in equation (3.156)

$$
c^e_t = \omega_e\left[\frac{p^{ce}_t}{p^c_t}\right]^{-\eta_e} c_t
$$

$$
\hat{c}^e_t = -\eta_e\left(\hat{p}^{ce}_t - \hat{p}^c_t\right) + \hat{c}_t, \tag{13.30}
$$

as well as demand for domestic consumption goods in equation (3.151)

$$
c^d_t = (1-\omega_c)\left(p^{cxe}_t\right)^{\eta_c} c^{xe}_t
$$

$$
\hat{c}^d_t = \eta_c\hat{p}^{cxe}_t + \hat{c}^{xe}_t, \tag{13.31}
$$

for imported consumption goods in equation (3.152).

$$
c^m_t = \omega_c\left[\frac{p^{m,c}_t}{p^{cxe}_t}\right]^{-\eta_c} c^{xe}_t
$$

$$
\hat{c}^m_t = -\eta_c\left(\hat{p}^{m,c}_t - \hat{p}^{cxe}_t\right) + \hat{c}^{xe}_t, \tag{13.32}
$$

for domestically produced energy in equation (3.153)

$$c_t^{e,d} = (1 - \omega_{em}) \left[\frac{p_t^{d,ce}}{p_t^{ce}} \right]^{-\eta_{em}} c_t^e$$

$$\hat{c}_t^{e,d} = -\eta_{em} \left(\hat{p}_t^{d,ce} - \hat{p}_t^{ce} \right) + \hat{c}_t^e, \tag{13.33}$$

and for imported energy in equation (3.154)

$$c_t^{e,m} = \omega_{em} \left[\frac{p_t^{m,ce}}{p_t^{ce}} \right]^{-\eta_{em}} c_t^e$$

$$\hat{c}_t^{e,m} = -\eta_{em} \left(\hat{p}_t^{m,ce} - \hat{p}_t^{ce} \right) + \hat{c}_t^e. \tag{13.34}$$

We further include demand for domestic investment goods in equation (3.176)

$$i_t^d = (1 - \omega_i) \left(p_t^i \right)^{\eta_i} \left(i_t + a(u_t) \frac{k_t^p}{\mu_{z^+,t} \mu_{\Psi,t}} \right)$$

$$\hat{i}_t^d = \eta_i \hat{p}_t^i + \hat{i}_t + \frac{1}{i} \frac{\sigma_b k^p}{\mu_{z^+} \mu_\Psi} \hat{u}_t, \tag{13.35}$$

and for imported investment goods in equation (3.177).

$$i_t^m = \omega_i \left(\frac{p_t^{m,i}}{p_t^i} \right)^{-\eta_i} \left(i_t + a(u_t) \frac{k_t^p}{\mu_{z^+,t} \mu_{\Psi,t}} \right)$$

$$\hat{i}_t^m = -\eta_i \left(\hat{p}_t^{m,i} - \hat{p}_t^i \right) + \hat{i}_t + \frac{1}{i} \frac{\sigma_b k^p}{\mu_{z^+} \mu_\Psi} \hat{u}_t. \tag{13.36}$$

We also include the expression for the CPI inflation in equation (3.159)

$$\pi_t^c = \left[\frac{(1 - \omega_e) \left(P_t^{cxe} \right)^{1-\eta_e} + \omega_e \left(P_t^{ce} \right)^{1-\eta_e}}{(1 - \omega_e) \left(P_{t-1}^{cxe} \right)^{1-\eta_e} + \omega_e \left(P_{t-1}^{ce} \right)^{1-\eta_e}} \right]^{1/(1-\eta_e)}$$

$$\hat{\pi}_t^c = (1 - \omega_e) \left(\frac{p^{cxe}}{p^c} \right)^{1-\eta_e} \hat{\pi}_t^{cxe} + \omega_e \left(\frac{p^{ce}}{p^c} \right)^{1-\eta_e} \hat{\pi}_t^{ce}, \tag{13.37}$$

as well as equation (3.157) for the non-energy consumer price inflation

$$\pi_t^{cxe} = \left[\frac{(1 - \omega_c) \left(P_t^d \right)^{1-\eta_c} + \omega_c \left(P_t^{m,c} \right)^{1-\eta_c}}{(1 - \omega_c) \left(P_{t-1}^d \right)^{1-\eta_c} + \omega_c \left(P_{t-1}^{m,c} \right)^{1-\eta_c}} \right]^{1/(1-\eta_c)}$$

$$\hat{\pi}_t^{cxe} = (1 - \omega_c) \left(\frac{1}{p^{cxe}} \right)^{1-\eta_c} \hat{\pi}_t^d + \omega_c \left(\frac{p^{m,c}}{p^{cxe}} \right)^{1-\eta_c} \hat{\pi}_t^{m,c}, \tag{13.38}$$

and equation (3.158) for the energy price inflation

$$\pi_t^{ce} = \frac{P_t^{ce}}{P_{t-1}^{ce}} = \left[\frac{(1 - \omega_{em}) \left(P_t^{d,ce} \right)^{1-\eta_{em}} + \omega_{em} \left(P_t^{m,ce} \right)^{1-\eta_{em}}}{(1 - \omega_{em}) \left(P_{t-1}^{d,ce} \right)^{1-\eta_{em}} + \omega_{em} \left(P_{t-1}^{m,ce} \right)^{1-\eta_{em}}} \right]^{1/(1-\eta_{em})}$$

$$\hat{\pi}_t^{ce} = (1 - \omega_{em}) \left(\frac{p^{d,ce}}{p^{ce}} \right)^{1-\eta_{em}} \hat{\pi}_t^{d,ce} + \omega_{em} \left(\frac{p^{m,ce}}{p^{ce}} \right)^{1-\eta_{em}} \hat{\pi}_t^{m,ce}. \tag{13.39}$$

Moreover, we need to include the expressions of the relative prices of the three differrent aggregate consumption goods in equations (3.162), (3.160) and (3.161),

$$p_t^c = \left[(1-\omega_e)\left(p_t^{cxe}\right)^{1-\eta_e} + \omega_e\left(p_t^{ce}\right)^{1-\eta_e}\right]^{1/(1-\eta_e)}$$

$$\widehat{p}_t^c = (1-\omega_e)\left(\frac{p^{cxe}}{p^c}\right)^{1-\eta_e}\widehat{p}_t^{cxe} + \omega_e\left(\frac{p^{ce}}{p^c}\right)^{1-\eta_e}\widehat{p}_t^{ce}, \tag{13.40}$$

$$p_t^{cxe} = \left[(1-\omega_c) + \omega_c\left(p_t^{m,c}\right)^{1-\eta_c}\right]^{1/(1-\eta_c)}$$

$$\hat{p}_t^{cxe} = \omega_c\left(\frac{p^{m,c}}{p^{cxe}}\right)^{1-\eta_c}\hat{p}_t^{m,c}, \tag{13.41}$$

$$p_t^{ce} = \left[(1-\omega_{em})\left(p_t^{d,ce}\right)^{1-\eta_{em}} + \omega_{em}\left(p_t^{m,ce}\right)^{1-\eta_{em}}\right]^{1/(1-\eta_{em})}$$

$$\hat{p}_t^{ce} = (1-\omega_{em})\left(\frac{p^{d,ce}}{p^{ce}}\right)^{1-\eta_{em}}\hat{p}_t^{d,ce} + \omega_{em}\left(\frac{p^{m,ce}}{p^{ce}}\right)^{1-\eta_{em}}\hat{p}_t^{m,ce}, \tag{13.42}$$

and the relative price of the aggregate investment good in equation (3.179)

$$p_t^i = \left[(1-\omega_i) + \omega_i\left(p_t^{m,i}\right)^{1-\eta_i}\right]^{1/(1-\eta_i)}$$

$$\hat{p}_t^i = \omega_i\left(\frac{p^{m,i}}{p^i}\right)^{1-\eta_i}\hat{p}_t^{m,i}. \tag{13.43}$$

From the household problem, we have the endogenous preference shifter in equation (4.80)

$$\Theta_t = z_t^C \bar{\upsilon}_t^N$$

$$\hat{\Theta}_t = \hat{z}_t^C + \widehat{\bar{\upsilon}}_t^N, \tag{13.44}$$

the trend consumption in equation (4.81)

$$z_t^C = \left(z_{t-1}^C \frac{1}{\mu_{z^+,t}}\right)^{1-\nu}\left(\frac{1}{\bar{\upsilon}_t^N}\right)^{\nu}$$

$$\hat{z}_t^C = (1-\nu)\left(\hat{z}_{t-1}^C - \hat{\mu}_{z^+,t}\right) - \nu\widehat{\bar{\upsilon}}_t^N, \tag{13.45}$$

the marginal utility of consumption in equation (4.88)

$$\bar{\upsilon}_t^N = \frac{\zeta_t^\beta \zeta_t^c}{c_t - bc_{t-1}\frac{1}{\mu_{z^+,t}}} - \beta b E_t \frac{\zeta_{t+1}^\beta \zeta_{t+1}^c}{c_{t+1}\mu_{z^+,t+1} - bc_t}$$

$$\begin{aligned}\widehat{\bar{\upsilon}}_t^N \;=\;& \mu_{z^+}\left(\mu_{z^+}-b\right)\hat{\zeta}_t^\beta + \mu_{z^+}\left(\mu_{z^+}-b\right)\hat{\zeta}_t^c - \mu_{z^+}^2\hat{c}_t + b\mu_{z^+}\hat{c}_{t-1} - b\mu_{z^+}\hat{\mu}_{z^+,t} \\ & -\beta b\left[\left(\mu_{z^+}-b\right)E_t\hat{\zeta}_{t+1}^\beta + \left(\mu_{z^+}-b\right)E_t\hat{\zeta}_{t+1}^c - \mu_{z^+}E_t\hat{c}_{t+1} + b\hat{c}_t - \mu_{z^+}E_t\hat{\mu}_{z^+,t+1}\right].\end{aligned} \tag{13.46}$$

the marginal rate of substitution in equation (4.83)

$$mrs_{j,t} = \frac{\zeta_t^\beta \zeta_t^n \Theta_t N_{j,t}^\varphi}{\bar{\upsilon}_t^N}$$

$$N_t = \left[\int_0^1 (N_{j,t})^{\frac{1}{\lambda_t^w}} dj\right]^{\lambda_t^w}$$

$$\widehat{mrs}_t = \hat{\zeta}_t^\beta + \hat{\zeta}_t^n + \hat{\Theta}_t + \varphi \hat{N}_t - \widehat{\bar{v}}_t^N, \tag{13.47}$$

We also have the UIP condition in equation (4.84)

$$R_t = R_t^* \Phi_t E_t s_{t+1}$$

$$\hat{R}_t = \hat{R}_t^* + \hat{\Phi}_t + E_t \hat{s}_{t+1} \tag{13.48}$$

and the expression for the endogenous risk premium term in equation (4.85)

$$\Phi_t = \exp\left(-\tilde{\phi}_a (\bar{a}_t - \bar{a}) - \tilde{\phi}_s \left(E_t s_{t+1} s_t - s^2\right) + \tilde{\phi}_t\right)$$

$$\hat{\Phi}_t = -\tilde{\phi}_a \breve{a}_t - \tilde{\phi}_s (E_t \hat{s}_{t+1} + \hat{s}_t) + \widehat{\tilde{\phi}}_t, \tag{13.49}$$

the log-linearized consumption Euler equation in (4.89)

$$\bar{v}_t^N = \beta E_t \frac{R_t \chi_t}{\pi_{t+1}^c \mu_{z^+,t+1}} \bar{v}_{t+1}^N$$

$$\widehat{\bar{v}}_t^N = \hat{R}_t + \hat{\chi}_t - E_t \hat{\pi}_{t+1}^c - E_t \hat{\mu}_{z^+,t+1} + E_t \widehat{\bar{v}}_{t+1}^N, \tag{13.50}$$

the first-order condition for capital in equation (4.70)

$$E_t\left[\bar{r}_{t+1}^k u_{t+1} - p_{t+1}^i a(u_{t+1}) + (1-\delta)\breve{p}_{k',t+1}\right] = E_t \frac{R_t \chi_t}{\pi_{t+1}^d} \mu_{\Psi,t+1} \breve{p}_{k',t}$$

$$\begin{aligned}\widehat{\breve{p}}_{k',t} &= \frac{\beta(1-\delta)}{\mu_{z^+}\mu_\Psi} \widehat{\breve{p}}_{k',t+1} + \frac{\beta \bar{r}^k}{\mu_{z^+}\mu_\Psi \breve{p}_{k'}} E_t\left(\widehat{\bar{r}}_{t+1}^k + \hat{u}_{t+1}\right) \\ &\quad - E_t \hat{\mu}_{\Psi,t+1} - E_t\left(\hat{R}_t - E_t \hat{\pi}_{t+1}^d + \hat{\chi}_t\right),\end{aligned} \tag{13.51}$$

the first-order condition for investment in equation (4.93)

$$\begin{aligned} p_t^i &= \breve{p}_{k',t} \Upsilon_t \left[1 - \tilde{S}\left(\frac{\mu_{z^+,t}\mu_{\Psi,t} i_t}{i_{t-1}}\right) - \tilde{S}'\left(\frac{\mu_{z^+,t}\mu_{\Psi,t} i_t}{i_{t-1}}\right) \frac{\mu_{z^+,t}\mu_{\Psi,t} i_t}{i_{t-1}}\right] \\ &\quad + \beta E_t \frac{\zeta_{t+1}^\beta}{\zeta_t^\beta} \frac{\psi_{z^+,t+1}}{\psi_{z^+,t}} \breve{p}_{k',t+1} \Upsilon_{t+1} \tilde{S}'\left(\frac{\mu_{z^+,t+1}\mu_{\Psi,t+1} i_{t+1}}{i_t}\right)\left(\frac{i_{t+1}}{i_t}\right)^2 \mu_{z^+,t+1}\mu_{\Psi,t+1}\end{aligned}$$

$$\widehat{i}_t - \widehat{i}_{t-1} + \hat{\mu}_{z^+,t} + \hat{\mu}_{\Psi,t} = \beta E_t\left(\widehat{i}_{t+1} - \widehat{i}_t + \hat{\mu}_{z^+,t+1} + \hat{\mu}_{\Psi,t+1}\right) + \frac{1}{(\mu_{z^+}\mu_\Psi)^2 \tilde{S}''}\left[\widehat{\breve{p}}_{k',t} + \hat{\Upsilon}_t - \frac{p^i}{\breve{p}_{k'}\Upsilon}\hat{p}_t^i\right], \tag{13.52}$$

and the expression determining the capital utilization in equation (4.95)

$$\bar{r}_t^k = p_t^i a'(u_t)$$

$$\widehat{\bar{r}}_t^k = \hat{p}_t^i + \sigma_a \hat{u}_t. \tag{13.53}$$

We moreover include the law of motion for capital in equation (4.96)

$$k_{t+1}^p = \frac{1-\delta}{\mu_{z^+,t}\mu_{\Psi,t}} k_t^p + \Upsilon_t\left(1 - \tilde{S}\left(\frac{\mu_{z^+,t}\mu_{\Psi,t} i_t}{i_{t-1}}\right)\right) i_t$$

$$\hat{k}^p_{t+1} = \frac{1-\delta}{\mu_{z^+}\mu_\Psi}\left(\hat{k}^p_t - \hat{\mu}_{z^+,t} - \hat{\mu}_{\Psi,t}\right) + \frac{\Upsilon i}{k^p}\left(\hat{\Upsilon}_t + \widehat{i}_t\right), \tag{13.54}$$

and the relationship between efficient and physical capital in equation (4.97)

$$k_t = u_t k^p_t$$

$$\hat{k}_t = \hat{u}_t + \hat{k}^p_t. \tag{13.55}$$

We also have the expression determining the labour participation rate in equation (4.100)

$$\frac{\bar{w}_t}{p^c_t}\int_0^1 \frac{W_{j,t}}{W_t}dj = \zeta^\beta_t \zeta^n_t z^C_t \int_0^1 L^\varphi_{j,t}dj$$

$$W_t^{\frac{1}{1-\lambda^w_t}} = \int_0^1 (W_{j,t})^{\frac{1}{1-\lambda^w_t}}\, dj$$

$$\widehat{\bar{w}}_t - \hat{p}^c_t = \hat{\zeta}^n_t + \hat{\zeta}^\beta_t + \hat{z}^C_t + \varphi\hat{L}_t, \tag{13.56}$$

the unemployment rate in equation (4.99)

$$U_t = \frac{L_t - N_t}{L_t}$$

$$\hat{U}_t = \hat{L}_t - \hat{N}_t, \tag{13.57}$$

the natural rate of unemployment in equation (4.110)

$$\hat{U}^n_t = \frac{1}{\varphi}\hat{\lambda}^w_t, \tag{13.58}$$

and the wage markup in equation (4.101)

$$\hat{\mu}_{w,t} = \varphi\hat{U}_t. \tag{13.59}$$

Finally, we include the expression for the wage inflation in equation (4.106)

$$\pi^w_t = \frac{\bar{w}_t \mu_{z^+,t}\pi^d_t}{\bar{w}_{t-1}}$$

$$\hat{\pi}^w_t = \widehat{\bar{w}}_t - \widehat{\bar{w}}_{t-1} + \hat{\pi}^d_t + \hat{\mu}_{z^+,t}, \tag{13.60}$$

the wage Phillips curve in equation (4.111)[52]

$$\tilde{w}_t^{\frac{1-\lambda^w_{t+s}(1+\varphi)}{1-\lambda^w_{t+s}}} = \frac{\lambda^w_{t+s} E_t \sum_{s=0}^{\infty} (\beta\xi_w)^s \zeta^\beta_{t+s}\zeta^n_{t+s}\Theta_{t+s}\left[\left(\frac{\bar{w}_t}{\bar{w}_{t+s}}\frac{\tilde{\pi}^w_{t+s}\cdots\tilde{\pi}^w_{t+1}}{\mu_{z^+,t+1}\cdots\mu_{z^+,t+s}\pi^d_{t+1}\cdots\pi^d_{t+s}}\right)^{\frac{\lambda^w_{t+s}}{1-\lambda^w_{t+s}}} N_{t+s}\right]^{1+\varphi}}{E_t \sum_{s=0}^{\infty} (\beta\xi_w)^s \zeta^\beta_{t+s}\psi_{z^+,t+s}\bar{w}_{t+s}N_{t+s}\left(\frac{\bar{w}_t}{\bar{w}_{t+s}}\frac{\tilde{\pi}^w_{t+s}\cdots\tilde{\pi}^w_{t+1}}{\mu_{z^+,t+1}\cdots\mu_{z^+,t+s}\pi^d_{t+1}\cdots\pi^d_{t+s}}\right)^{\frac{1}{1-\lambda^w_{t+s}}}}$$

$$\tilde{\pi}^w_{t+1} \equiv (\pi^c_t)^{\kappa_w}\left(\bar{\pi}^c_{t+1}\right)^{1-\kappa_w-\varkappa_w}(\breve{\pi})^{\varkappa_w}\left(\mu_{z^+}\right)^{\vartheta_w}$$

[52] Just as the log-linear price Phillips curves, the log-linear wage Phillips curve is a combination of the log-linearized versions of the three included non-linear equations. Note that the log-linearized wage equation included here has been derived under the assumption that there is full indexation, that is assuming that $\varkappa_w = 0$ and that $\vartheta_w = 1$. We have also used the log-linearized equations (4.100), (4.80), (4.81) and (4.90) to arrive at its final form.

$$\tilde{w}_t = \left[\frac{1 - \xi_w \left(\frac{\tilde{\pi}_t^w}{\pi_t^w} \right)^{\frac{1}{1-\lambda_t^w}}}{(1 - \xi_w)} \right]^{1-\lambda_t^w}$$

$$\begin{aligned} \hat{\pi}_t^w &= \beta E_t \hat{\pi}_{t+1}^w + (1 - \beta \rho_{\bar{\pi}}) \hat{\bar{\pi}}_t^c \\ &+ \kappa_w \left(\hat{\pi}_{t-1}^c - \hat{\bar{\pi}}_t^c \right) - \beta \kappa_w E_t \left(\hat{\pi}_t^c - \hat{\bar{\pi}}_{t+1}^c \right) - d_w \varphi \left(\hat{U}_t - \hat{U}_t^n \right), \\ d_w &= \frac{\lambda^w - 1}{\xi_w b_w},\ b_w = \frac{\lambda^w (1 + \varphi) - 1}{(1 - \beta \xi_w)(1 - \xi_w)}, \end{aligned} \tag{13.61}$$

the expression for aggregate household hours in terms of aggregate homogeneous hours worked in equation (4.114)

$$n_t = N_t (\mathring{w}_t)^{\frac{\lambda_t^w}{1-\lambda_t^w}}$$

$$\hat{n}_t = \hat{N}_t + \frac{\lambda^w}{1 - \lambda^w} \hat{\mathring{w}}_t, \tag{13.62}$$

and the expression for the wage dispersion in equation (4.115)

$$\mathring{w}_t = \left[\xi_w \left(\frac{\tilde{\pi}_t^w}{\pi_t^w} \mathring{w}_{t-1} \right)^{\frac{\lambda_t^w}{1-\lambda_t^w}} + (1 - \xi_w) \left(\frac{1 - \xi_w \left(\frac{\tilde{\pi}_t^w}{\pi_t^w} \right)^{\frac{1}{1-\lambda_t^w}}}{(1 - \xi_w)} \right)^{\lambda_t^w} \right]^{\frac{1-\lambda_t^w}{\lambda_t^w}}$$

$$\hat{\mathring{w}}_t = \xi_w \hat{\mathring{w}}_{t-1}. \tag{13.63}$$

We also need to include the central bank policy rule in equation (5.5)

$$\begin{aligned} \log \left(\frac{R_t}{R} \right) &= \rho_R \log \left(\frac{R_{t-1}}{R} \right) + (1 - \rho_R) \left[\log \left(\frac{\bar{\pi}_t^c}{\bar{\pi}^c} \right) + r_\pi \log \left(\frac{\pi_{t-1}^c}{\bar{\pi}_t^c} \right) \right. \\ &\left. + r_{RU} (U_{t-1} - U) + r_q \log \left(\frac{q_{t-1}}{q} \right) \right] + r_{\Delta\pi} \Delta \log \left(\frac{\pi_t^c}{\pi^c} \right) + r_{\Delta RU} \Delta U_t + \log \varepsilon_{R,t}, \end{aligned}$$

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1 - \rho_R) \left[\hat{\bar{\pi}}_t^c + r_\pi \left(\hat{\pi}_{t-1}^c - \hat{\bar{\pi}}_t^c \right) + r_{RU} \hat{U}_{t-1} + r_q \hat{q}_{t-1} \right] + r_{\Delta\pi} \Delta \hat{\pi}_t^c + r_{\Delta RU} \Delta \hat{U}_t + \hat{\varepsilon}_{R,t}, \tag{13.64}$$

the aggregate resource constraint in equations (6.12) and (6.13)

$$y_t = g_t + c_t^d + c_t^{e,d} + i_t^d + x_t^d$$

$$\hat{y}_t = \frac{g}{y} \hat{g}_t + \frac{c^d}{y} \hat{c}_t^d + \frac{c^{e,d}}{y} \hat{c}_t^{e,d} + \frac{i^d}{y} \hat{i}_t^d + \frac{x^d}{y} \hat{x}_t^d. \tag{13.65}$$

$$y_t = \left(\mathring{p}_t^d \right)^{\frac{\lambda_t^d}{\lambda_t^d - 1}} \left[\epsilon_t \left(\frac{k_t}{\mu_{\Psi,t} \mu_{z^+,t}} \right)^\alpha \left(\mathring{w}_t^{-\frac{\lambda_t^w}{1-\lambda_t^w}} n_t \right)^{1-\alpha} - \phi^d \right]$$

$$\begin{aligned} \hat{y}_t &= \frac{1}{y} \left(\frac{k}{\mu_\Psi \mu_{z^+}} \right)^\alpha n^{1-\alpha} \times \\ &\times \left[\hat{\epsilon}_t + \alpha \left(\hat{k}_t - \hat{\mu}_{\Psi,t} - \hat{\mu}_{z^+,t} \right) - \frac{\lambda^w (1 - \alpha)}{1 - \lambda^w} \hat{\mathring{w}}_t + (1 - \alpha) \hat{n}_t \right] \\ &- \frac{\lambda^d}{\lambda^d - 1} \hat{\mathring{p}}_t^d, \end{aligned} \tag{13.66}$$

the evolution of net foreign assets in equation (7.4)

$$\bar{a}_t + \frac{q_t p_t^c}{p_t^{c,*}} R_t^{wc,m} \left(c_t^m \left(\mathring{p}_t^{m,c}\right)^{\frac{\lambda_t^{m,c}}{1-\lambda_t^{m,c}}} + i_t^m \left(\mathring{p}_t^{m,i}\right)^{\frac{\lambda_t^{m,i}}{1-\lambda_t^{m,i}}} \right.$$

$$\left. + x_t^m \left(\mathring{p}_t^{m,x}\right)^{\frac{\lambda_t^{m,x}}{1-\lambda_t^{m,x}}} + \phi^{m,c} + \phi^{m,i} + \phi^{m,x} \right)$$

$$+ \frac{q_t p_t^c p_t^{ce,*}}{p_t^{c,*}} R_t^{wc,m} \left(c_t^{e,m} \left(\mathring{p}_t^{m,ce}\right)^{\frac{\lambda_t^{m,ce}}{1-\lambda_t^{m,ce}}} + \phi^{m,ce} \right)$$

$$= \frac{q_t p_t^x p_t^c}{p_t^{c,*}} \left(\left(\mathring{p}_t^x\right)^{\frac{\lambda_t^x}{1-\lambda_t^x}} x_t - \phi^x \right) + R_{t-1}^* \Phi_{t-1} \chi_{t-1} s_t \frac{\bar{a}_{t-1}}{\pi_t^d \mu_{z^+,t}},$$

$$\frac{p^{c,*}}{qp^c} \breve{a}_t = p^x \left(x - \phi^x\right) \hat{p}_t^x + p^x x \left(\hat{x}_t + \frac{\lambda^x}{1-\lambda^x} \widehat{\mathring{p}}_t^x \right) + \frac{p^{c,*}}{qp^c} \frac{R^* \Phi \chi s}{\pi^d \mu_{z^+}} \breve{a}_{t-1} \tag{13.67}$$
$$- p^x \left(x - \phi^x\right) \hat{R}_t^{wc,m} - p^{ce,*} R^{wc,m} \left(c^{e,m} + \phi^{m,ce}\right) \hat{p}_t^{ce,*}$$
$$- R^{wc,m} c^m \left(\hat{c}_t^m + \frac{\lambda^{m,c}}{1-\lambda^{m,c}} \widehat{\mathring{p}}_t^{m,c} \right) - R^{wc,m} i^m \left(\hat{i}_t^m + \frac{\lambda^{m,i}}{1-\lambda^{m,i}} \widehat{\mathring{p}}_t^{m,i} \right)$$
$$- R^{wc,m} x^m \left(\hat{x}_t^m + \frac{\lambda^{m,x}}{1-\lambda^{m,x}} \widehat{\mathring{p}}_t^{m,x} \right) - p^{ce,*} R^{wc,m} c^{e,m} \left(\hat{c}_t^{e,m} + \frac{\lambda^{m,ce}}{1-\lambda^{m,ce}} \widehat{\mathring{p}}_t^{m,ce} \right).$$

and total export demand in equation (3.224)

$$x_t = \left(p_t^x\right)^{-\eta_f} \left(c_t^*\right)^{\omega_c^x} \left(i_t^{d,*}\right)^{1-\omega_c^x}$$

$$\hat{x}_t = -\eta_f \hat{p}_t^x + \omega_c^x \hat{c}_t^* + \left(1 - \omega_c^x\right) \widehat{i}_t^{d,*}. \tag{13.68}$$

To complete the description of the domestic economy block, we also need to include the dynamics of relative prices in equations (8.11)[53] and (8.12)–(8.16)

$$\pi_t^{d,ce} = \frac{p_t^{d,ce}}{p_{t-1}^{d,ce}} \pi_t^d$$

$$\hat{\pi}_t^{d,ce} = \hat{p}_t^{d,ce} - \hat{p}_{t-1}^{d,ce} + \hat{\pi}_t^d, \tag{13.69}$$

$$p_t^x = \frac{\pi_t^x}{\pi_t^{d,*}} \left(\frac{\mu_{z^+,*,t}}{\mu_{z^+,t}} \right)^{-\frac{1}{\eta_f}} p_{t-1}^x$$

$$\hat{p}_t^x = \hat{\pi}_t^x - \hat{\pi}_t^{d,*} - \frac{1}{\eta_f} \left(\hat{\mu}_{z^+,*,t} - \hat{\mu}_{z^+,t} \right) + \hat{p}_{t-1}^x, \tag{13.70}$$

$$p_t^{m,c} = \frac{\pi_t^{m,c}}{\pi_t^d} p_{t-1}^{m,c}$$

$$\hat{p}_t^{m,c} = \hat{\pi}_t^{m,c} - \hat{\pi}_t^d + \hat{p}_{t-1}^{m,c}, \tag{13.71}$$

$$p_t^{m,i} = \frac{\pi_t^{m,i}}{\pi_t^d} p_{t-1}^{m,i}$$

$$\hat{p}_t^{m,i} = \hat{\pi}_t^{m,i} - \hat{\pi}_t^d + \hat{p}_{t-1}^{m,i}, \tag{13.72}$$

[53] Note that we will assume that the relative price of domestically produced energy evolves as an exogenous process, why we have rewritten equation (8.11) with inflation on the right hand side.

$$p_t^{m,x} = \frac{\pi_t^{m,x}}{\pi_t^d} p_{t-1}^{m,x}$$

$$\hat{p}_t^{m,x} = \hat{\pi}_t^{m,x} - \hat{\pi}_t^d + \hat{p}_{t-1}^{m,x}, \tag{13.73}$$

$$p_t^{m,ce} = \frac{\pi_t^{m,ce}}{\pi_t^d} p_{t-1}^{m,ce}$$

$$\hat{p}_t^{m,ce} = \hat{\pi}_t^{m,ce} - \hat{\pi}_t^d + \hat{p}_{t-1}^{m,ce}, \tag{13.74}$$

and the definition of the real exchange rate in equation (9.4)

$$q_t = \frac{s_t \pi_t^{c,*}}{\pi_t^c} q_{t-1}$$

$$\hat{q}_t = \hat{s}_t + \hat{\pi}_t^{c,*} - \hat{\pi}_t^c + \hat{q}_{t-1}. \tag{13.75}$$

We include some additional variables that are not needed to solve the above system, but that may be of interest. Specifically, for the domestic economy, we include the aggregate investment inflation in equation (3.178)

$$\pi_t^i = \frac{\Psi_{t-1}}{\Psi_t} \left[\frac{(1-\omega_i)\left(P_t^d\right)^{1-\eta_i} + \omega_i \left(P_t^{m,i}\right)^{1-\eta_i}}{(1-\omega_i)\left(P_{t-1}^d\right)^{1-\eta_i} + \omega_i \left(P_{t-1}^{m,i}\right)^{1-\eta_i}} \right]^{1/(1-\eta_i)}$$

$$\hat{\pi}_t^i = (1-\omega_i)\left(p^i\right)^{\eta_i - 1} \hat{\pi}_t^d + \omega_i \left(\frac{p^{m,i}}{p^i}\right)^{1-\eta_i} \hat{\pi}_t^{m,i} - \frac{1}{1-\eta_i} \hat{\mu}_{\Psi,t}, \tag{13.76}$$

and total import demand in equation (3.230)[54]

$$m_t = c_t^m \left(\mathring{p}_t^{m,c}\right)^{\frac{\lambda_t^{m,c}}{1-\lambda_t^{m,c}}} + i_t^m \left(\mathring{p}_t^{m,i}\right)^{\frac{\lambda_t^{m,i}}{1-\lambda_t^{m,i}}} + x_t^m \left(\mathring{p}_t^{m,x}\right)^{\frac{\lambda_t^{m,x}}{1-\lambda_t^{m,x}}} + c_t^{e,m} \left(\mathring{p}_t^{m,ce}\right)^{\frac{\lambda_t^{m,ce}}{1-\lambda_t^{m,ce}}}$$

$$\begin{aligned} \hat{m}_t \quad = \quad & \frac{c^m}{m}\left[\hat{c}_t^m + \frac{\lambda^{m,c}}{1-\lambda^{m,c}} \widehat{\mathring{p}}_t^{m,c}\right] + \frac{i^m}{m}\left[\hat{i}_t^m + \frac{\lambda^{m,i}}{1-\lambda^{m,i}} \widehat{\mathring{p}}_t^{m,i}\right] \\ & + \frac{x^m}{m}\left[\hat{x}_t^m + \frac{\lambda^{m,x}}{1-\lambda^{m,x}} \widehat{\mathring{p}}_t^{m,x}\right] + \frac{c^{e,m}}{m}\left[\hat{c}_t^{e,m} + \frac{\lambda^{m,ce}}{1-\lambda^{m,ce}} \widehat{\mathring{p}}_t^{m,ce}\right]. \end{aligned} \tag{13.77}$$

13.2 Foreign equations

We now turn to the foreign economy block. As the foreign economy is assumed closed, the below set of equations constitute a complete model that could be solved independently of the domestic economy block.

From the foreign intermediate good firm's problem, we have the gross effective nominal interest rate faced by the foreign firms

$$R_t^{wc,*} = \nu_t^{wc,*} R_t^* + 1 - \nu_t^{wc,*}$$

$$\hat{R}_t^{wc,*} = \frac{\nu^{wc,*}\left(R^* - 1\right)}{R^{wc,*}} \hat{\nu}_t^{wc,*} + \frac{\nu^{wc,*} R^*}{R^{wc,*}} \hat{R}_t^*, \tag{13.78}$$

[54] Note that this expression is the log-linearized version of total imports *used in* the domestic economy, as previously discussed in Section 3.2. Total import demand m_t and gross total imports $\tilde{m}_t$ differ by the inclusion of fixed costs, which affects their steady state values. The log-linear version of $\tilde{m}_t$ would look the same as the one for m_t, except for $\hat{m}_t$ and m being replaced by $\hat{\tilde{m}}_t$ and $\tilde{m}$, respectively.

marginal costs in terms of marginal product of labour

$$mc_t^* = \frac{\bar{w}_t^* R_t^{wc,*}}{mpl_t^*}$$

$$\widehat{mc}_t^* = \widehat{\bar{w}}_t^* + \hat{R}_t^{wc,*} - \widehat{mpl}_t^*, \tag{13.79}$$

the marginal product of labour

$$mpl_t^* = (1-\alpha^*)\,\epsilon_t^* \left(\frac{k_t^*}{\mu_{z^{+,*},t}\mu_{\Psi^*,t}N_t^*}\right)^{\alpha}$$

$$\widehat{mpl}_t^* = \alpha^* \widehat{\left(\frac{k}{N}\right)}_t^* + \hat{\epsilon}_t^*, \tag{13.80}$$

the capital-to-labour ratio

$$\left(\frac{k}{N}\right)_t^* = \frac{k_t^*}{\mu_{z^{+,*},t}\mu_{\Psi^*,t}N_t^*}$$

$$\widehat{\left(\frac{k}{N}\right)}_t^* = \hat{k}_t^* - \hat{N}_t^* - \left(\hat{\mu}_{z^{+,*},t} + \hat{\mu}_{\Psi^*,t}\right), \tag{13.81}$$

marginal costs in terms of marginal product of capital

$$mc_t^* = \frac{\bar{r}_t^{k,*}}{mpk_t^*}$$

$$\widehat{mc}_t^* = \widehat{\bar{r}}_t^{k,*} - \widehat{mpk}_t^*, \tag{13.82}$$

the marginal product of capital

$$mpk_t^* = \alpha^* \epsilon_t^* \left(\frac{k_t^*}{\mu_{z^{+,*},t}\mu_{\Psi^*,t}N_t^*}\right)^{-(1-\alpha^*)}$$

$$\widehat{mpk}_t^* = -(1-\alpha^*)\widehat{\left(\frac{k}{N}\right)}_t^* + \hat{\epsilon}_t^*. \tag{13.83}$$

and the price Phillips curve[55]

$$\begin{aligned} &\tilde{p}_t^{d,*} E_t \sum_{s=0}^{\infty} (\beta^*\xi^*)^s \zeta_{t+s}^{\beta,*} \psi_{z^{+,*},t+s} y_{t+s}^* \left(\frac{\tilde{\pi}_{t+1}^{d,*}\cdots\tilde{\pi}_{t+s}^{d,*}}{\pi_{t+1}^{d,*}\cdots\pi_{t+s}^{d,*}}\right)^{\frac{1}{1-\lambda_{t+s}^*}} \\ = \quad & E_t \sum_{s=0}^{\infty} (\beta^*\xi^*)^s \zeta_{t+s}^{\beta,*} \psi_{z^{+,*},t+s} y_{t+s}^* \lambda_{t+s}^* mc_{t+s}^* \left(\frac{\tilde{\pi}_{t+1}^{d,*}\cdots\tilde{\pi}_{t+s}^{d,*}}{\pi_{t+1}^{d,*}\cdots\pi_{t+s}^{d,*}}\right)^{\frac{\lambda_{t+s}^*}{1-\lambda_{t+s}^*}} \end{aligned}$$

$$\tilde{\pi}_t^{d,*} = \left(\pi_{t-1}^{d,*}\right)^{\kappa^*} \left(\bar{\pi}_t^{c,*}\right)^{1-\kappa^*-\varkappa^*} \left(\breve{\pi}^*\right)^{\varkappa^*},$$

$$\tilde{p}_t^{d,*} = \left[\frac{1-\xi^*\left(\frac{\tilde{\pi}_t^{d,*}}{\pi_t^{d,*}}\right)^{\frac{1}{1-\lambda_t^*}}}{(1-\xi^*)}\right]^{1-\lambda_t^*}$$

[55] The log-linear Phillips curve is a combination of the log-linearized versions of the three included non-linear equations. Note that the log-linearized equation included here has been derived under the assumption that there is full indexation, that is assuming that $\varkappa^* = 0$.

$$\hat{\pi}_t^{d,*} - \hat{\bar{\pi}}_t^{c,*} = \frac{(1-\beta^*\xi^*)(1-\xi^*)}{\xi^*(1+\beta^*\kappa^*)}\left(\widehat{mc}_t^* + \hat{\lambda}_t^*\right) + \frac{\kappa^*}{1+\beta^*\kappa^*}\left(\hat{\pi}_{t-1}^{d,*} - \hat{\bar{\pi}}_t^{c,*}\right) \tag{13.84}$$
$$+\frac{\beta^*}{1+\beta^*\kappa^*}E_t\left(\hat{\pi}_{t+1}^{d,*} - \hat{\bar{\pi}}_{t+1}^{c,*}\right) - \frac{\beta^*\kappa^*}{1+\beta^*\kappa^*}E_t\left(\hat{\bar{\pi}}_t^{c,*} - \hat{\bar{\pi}}_{t+1}^{c,*}\right).$$

We also need to include the evolution of the combination of investment-specific and neutral technology

$$\mu_{z+,*,t} = \left(\mu_{\Psi^*,t}\right)^{\frac{\alpha^*}{1-\alpha^*}}\mu_{z^*,t}$$
$$\hat{\mu}_{z+,*,t} = \frac{\alpha^*}{1-\alpha^*}\hat{\mu}_{\Psi^*,t} + \hat{\mu}_{z^*,t}, \tag{13.85}$$

and the price dispersion equation

$$\mathring{p}_t^{d,*} = \left[\xi^*\left(\frac{\tilde{\pi}_t^{d,*}}{\pi_t^{d,*}}\mathring{p}_{t-1}^{d,*}\right)^{\frac{\lambda_t^*}{1-\lambda_t^*}} + (1-\xi^*)\left(\frac{1-\xi^*\left(\frac{\tilde{\pi}_t^{d,*}}{\pi_t^{d,*}}\right)^{\frac{1}{1-\lambda_t^*}}}{1-\xi^*}\right)^{\lambda_t^*}\right]^{\frac{1-\lambda_t^*}{\lambda_t^*}}$$
$$\hat{\mathring{p}}_t^{d,*} = \xi^*\hat{\mathring{p}}_{t-1}^{d,*}. \tag{13.86}$$

From the final good firms' problem, we have the expression for non-energy consumption demand in equation (11.32)

$$c_t^{xe,*} = (1-\omega_e^*)\left(\frac{1}{p_t^{c,*}}\right)^{-\eta_e^*}c_t^*$$
$$\hat{c}_t^{xe,*} = \eta_e^*\hat{p}_t^{c,*} + \hat{c}_t^*, \tag{13.87}$$

and the expression for energy consumption demand in equation (11.33)

$$c_t^{e,*} = \omega_e^*\left(\frac{p_t^{ce,*}}{p_t^{c,*}}\right)^{-\eta_e^*}c_t^*$$
$$\hat{c}_t^{e,*} = -\eta_e^*\left(\hat{p}_t^{ce,*} - \hat{p}_t^{c,*}\right) + \hat{c}_t^*. \tag{13.88}$$

We further include demand for domestic investment goods in equation (11.44)[56]

$$i_t^{d,*} = i_t^* + \frac{a(u_t^*)k_t^{p,*}}{\mu_{z+,*,t}\mu_{\Psi^*,t}}$$
$$\hat{i}_t^{d,*} = \hat{i}_t^* + \frac{1}{i^*}\frac{\sigma_b^* k^{p,*}}{\mu_{z+,*}\mu_{\Psi^*}}\hat{u}_t^*. \tag{13.89}$$

We also include the expression for the CPI inflation in equation (11.39)

$$\pi_t^{c,*} = \frac{P_t^{c,*}}{P_{t-1}^{c,*}} = \left[\frac{(1-\omega_e^*)\left(P_t^{d,*}\right)^{1-\eta_e^*} + \omega_e^*\left(P_t^{ce,*}\right)^{1-\eta_e^*}}{(1-\omega_e^*)\left(P_{t-1}^{d,*}\right)^{1-\eta_e^*} + \omega_e^*\left(P_{t-1}^{ce,*}\right)^{1-\eta_e^*}}\right]^{\frac{1}{1-\eta_e^*}}$$
$$\hat{\pi}_t^{c,*} = (1-\omega_e^*)\left(\frac{1}{p^{c,*}}\right)^{1-\eta_e^*}\hat{\pi}_t^{d,*} + \omega_e^*\left(\frac{p^{ce,*}}{p^{c,*}}\right)^{1-\eta_e^*}\hat{\pi}_t^{ce,*}. \tag{13.90}$$

[56]Note that foreign firms do not use imported goods in their production. The demand for domestic investment goods still exceeds investment used in the accumulation of physical capital, i_t^*, however, as investment goods are assumed to be used also in capital maintenance, as illustrated by the second term on the righ hand side of the equation.

as well as the expressions for non-energy price inflation in equation (11.40)

$$\pi_t^{cxe,*} = \pi_t^{d,*}$$
$$\hat{\pi}_t^{cxe,*} = \hat{\pi}_t^{d,*}, \tag{13.91}$$

and for energy inflation in equation (11.41)

$$\pi_t^{ce,*} = \frac{p_t^{ce,*}}{p_{t-1}^{ce,*}}\pi_t^{d,*}$$
$$\hat{\pi}_t^{ce,*} = \hat{p}_t^{ce,*} - \hat{p}_{t-1}^{ce,*} + \pi_t^{d,*}. \tag{13.92}$$

Moreover, we need to include the expression of the relative prices of the aggregate consumption good in equation (11.43)

$$p_t^{c,*} = \left[(1-\omega_e^*) + \omega_e^*\left(p_t^{ce,*}\right)^{1-\eta_e^*}\right]^{1/(1-\eta_e^*)}$$
$$\widehat{p}_t^{c,*} = \omega_e^*\left(\frac{p^{ce,*}}{p^{c,*}}\right)^{1-\eta_e^*}\widehat{p}_t^{ce,*}. \tag{13.93}$$

and the relative price of the aggregate investment good in equation (11.45)

$$p_t^{i,*} = 1$$
$$\hat{p}_t^{i,*} = 0. \tag{13.94}$$

From the household problem, we have the endogenous preference shifter

$$\Theta_t^* = z_t^{C,*}\bar{\upsilon}_t^{N,*}$$
$$\hat{\Theta}_t^* = \hat{z}_t^{C,*} + \widehat{\bar{\upsilon}}_t^{N,*}, \tag{13.95}$$

the evolution of trend consumption

$$z_t^{C,*} = \left(z_{t-1}^{C,*}\frac{1}{\mu_{z^+,*,t}}\right)^{1-\nu^*}\left(\frac{1}{\bar{\upsilon}_t^{N,*}}\right)^{\nu,*}$$
$$\hat{z}_t^{C,*} = (1-\nu^*)\left(\hat{z}_{t-1}^{C,*} - \hat{\mu}_{z^+,*,t}\right) - \nu^*\widehat{\bar{\upsilon}}_t^{N,*}, \tag{13.96}$$

the marginal utility of consumption

$$\bar{\upsilon}_t^{N,*} = \frac{\zeta_t^{\beta,*}\zeta_t^{c,*}}{c_t^* - b^* c_{t-1}^*\frac{1}{\mu_{z^+,*,t}}} - \beta^* b^* E_t\frac{\zeta_{t+1}^{\beta,*}\zeta_{t+1}^{c,*}}{c_{t+1}^*\mu_{z^+,*,t+1} - b^* c_t^*}$$

$$\begin{aligned}\widehat{\bar{\upsilon}}_t^{N,*} \;=\;& \mu_{z^+,*}\left(\mu_{z^+,*} - b^*\right)\hat{\zeta}_t^{\beta,*} + \mu_{z^+,*}\left(\mu_{z^+,*} - b^*\right)\hat{\zeta}_t^{c,*} - \mu_{z^+,*}^2\hat{c}_t^* + b^*\mu_{z^+,*}\hat{c}_{t-1}^* - b^*\mu_{z^+,*}\hat{\mu}_{z^+,*,t} \\ & -\beta^* b^*\left[\left(\mu_{z^+,*} - b^*\right)E_t\hat{\zeta}_{t+1}^{\beta,*} + \left(\mu_{z^+,*} - b^*\right)E_t\hat{\zeta}_{t+1}^{c,*} - \mu_{z^+,*}E_t\hat{c}_{t+1}^* + b^*\hat{c}_t^* - \mu_{z^+,*}E_t\hat{\mu}_{z^+,*,t+1}\right].\end{aligned} \tag{13.97}$$

the marginal rate of substitution

$$mrs_{j,t}^* = \frac{\zeta_t^{\beta,*}\zeta_t^{n,*}\Theta_t^*\left(N_{j,t}^*\right)^{\varphi^*}}{\bar{\upsilon}_t^{N,*}}$$

$$N_t^* = \left[\int_0^1 \left(N_{j,t}^*\right)^{\frac{1}{\lambda_t^{w,*}}} dj\right]^{\lambda_t^{w,*}}$$

$$\widehat{mrs}_t^* = \hat{\zeta}_t^{\beta,*} + \hat{\zeta}_t^{n,*} + \hat{\Theta}_t^* + \varphi^* \hat{N}_t^* - \widehat{\bar{\upsilon}}_t^{N,*}, \tag{13.98}$$

the consumption Euler equation

$$\bar{\upsilon}_t^{N,*} = \beta^* E_t \frac{R_t^* \chi_t^*}{\pi_{t+1}^{c,*} \mu_{z^+,*,t+1}} \bar{\upsilon}_{t+1}^{N,*}$$

$$\widehat{\bar{\upsilon}}_t^{N,*} = E_t \widehat{\bar{\upsilon}}_{t+1}^{N,*} + \widehat{\bar{R}}_t^* - E_t \hat{\mu}_{z^+,*,t+1} + \chi_t^*, \tag{13.99}$$

the first-order condition for capital

$$E_t \left[\bar{r}_{t+1}^{k,*} u_{t+1}^* - p_{t+1}^{i,*} a\left(u_{t+1}^*\right) + (1-\delta^*) \breve{p}_{k',t+1}^* \right] = \frac{1}{\beta^*} E_t \frac{\zeta_t^{\beta,*}}{\zeta_{t+1}^{\beta,*}} \frac{\psi_{z^+,*,t}}{\psi_{z^+,*,t+1}} \mu_{z^+,*,t+1} \mu_{\Psi^*,t+1} \breve{p}_{k',t}^*$$

$$-\zeta_t^{\beta,*} \psi_{z^+,t}^* + \beta^* E_t \frac{\zeta_{t+1}^{\beta,*} \psi_{z^+,t+1}^*}{\mu_{z^+,*,t+1}} \frac{R_t^* \chi_t^*}{\pi_{t+1}^{d,*}} = 0$$

$$\begin{aligned} \widehat{\breve{p}}_{k',t}^* &= \frac{\beta^*(1-\delta^*)}{\mu_{z^+,*} \mu_{\Psi^*}} E_t \widehat{\breve{p}}_{k',t+1}^* + \frac{\beta^* \bar{r}^{k,*}}{\mu_{z^+,*} \mu_{\Psi^*} \breve{p}_{k'}^*} E_t \left(\widehat{\bar{r}}_{t+1}^{k,*} + \hat{u}_{t+1}^* \right) \\ &\quad - E_t \hat{\mu}_{\Psi^*,t+1} - E_t \left(\hat{R}_t^* - E_t \hat{\pi}_{t+1}^{d,*} + \chi_t^* \right), \end{aligned} \tag{13.100}$$

the first-order condition for investment[57]

$$\begin{aligned} p_t^{i,*} &= \breve{p}_{k',t}^* \Upsilon_t^* \left[1 - \tilde{S}\left(\frac{\mu_{z^+,*,t} \mu_{\Psi^*,t} i_t^*}{i_{t-1}^*} \right) - \tilde{S}'\left(\frac{\mu_{z^+,*,t} \mu_{\Psi^*,t} i_t^*}{i_{t-1}^*} \right) \frac{\mu_{z^+,*,t} \mu_{\Psi^*,t} i_t^*}{i_{t-1}^*} \right] \\ &\quad + \beta^* E_t \frac{\zeta_{t+1}^{\beta,*}}{\zeta_t^{\beta,*}} \frac{\psi_{z^+,*,t+1}}{\psi_{z^+,*,t}} \breve{p}_{k',t+1}^* \Upsilon_{t+1}^* \tilde{S}'\left(\frac{\mu_{z^+,*,t+1} \mu_{\Psi^*,t+1} i_{t+1}^*}{i_t^*} \right) \left(\frac{i_{t+1}^*}{i_t^*} \right)^2 \mu_{z^+,*,t+1} \mu_{\Psi^*,t+1} \end{aligned}$$

$$\widehat{i}_t^* - \widehat{i}_{t-1}^* + \hat{\mu}_{z^+,*,t} + \hat{\mu}_{\Psi,*,t} = \beta^* E_t \left(\widehat{i}_{t+1}^* - \widehat{i}_t^* + \hat{\mu}_{z^+,*,t+1} + \hat{\mu}_{\Psi,*,t+1} \right) + \frac{1}{\left(\mu_{z^+,*} \mu_{\Psi^*}\right)^2 \tilde{S}''^{,*}} \left[\widehat{\breve{p}}_{k',t}^* + \hat{\Upsilon}_t^* - \frac{p^{i,*}}{\breve{p}_{k'}^* \Upsilon^*} \hat{p}_t^{i,*} \right] \tag{13.101}$$

and the expression for the capital utilization

$$\bar{r}_t^{k,*} = p_t^{i,*} a'\left(u_t^*\right)$$

$$\widehat{\bar{r}}_t^{k,*} = \hat{p}_t^{i,*} + \sigma_a^* \hat{u}_t^*. \tag{13.102}$$

We further include the law of motion for capital

$$k_{t+1}^{p,*} = \frac{1-\delta^*}{\mu_{z^+,*,t} \mu_{\Psi^*,t}} k_t^{p,*} + \Upsilon_t^* \left(1 - \tilde{S}\left(\frac{\mu_{z^+,*,t} \mu_{\Psi^*,t} i_t^*}{i_{t-1}^*} \right) \right) i_t^*$$

$$\hat{k}_{t+1}^{p,*} = \frac{1-\delta^*}{\mu_{z^+,*} \mu_{\Psi^*}} \left(\hat{k}_t^{p,*} - \hat{\mu}_{z^+,*,t} - \hat{\mu}_{\Psi^*,t} \right) + \frac{\Upsilon^* i^*}{k^{p,*}} \left(\hat{\Upsilon}_t^* + \widehat{i}_t^* \right), \tag{13.103}$$

and the relationship between efficient and physical capital

$$k_t^* = u_t^* k_t^{p,*}$$

$$\hat{k}_t^* = \hat{u}_t^* + \hat{k}_t^{p,*}. \tag{13.104}$$

[57]Note that the second derivative of the function S is a function of steady state variables only and therefore treated as a parameter. This explains why is has a superscript $*$, even though the function S itself doesn't.

We also have the expression determining the labour participation rate

$$\frac{\bar{w}_t^*}{p_t^{c,*}}\int_0^1 \frac{W_{j,t}^*}{W_t^*}dj = \zeta_t^{\beta,*}\zeta_t^{n,*}z_t^{C,*}\int_0^1 \left(L_{j,t}^*\right)^{\varphi^*} dj$$

$$\left(W_t^*\right)^{\frac{1}{1-\lambda_t^{w,*}}} = \int_0^1 \left(W_{j,t}^*\right)^{\frac{1}{1-\lambda_t^{w,*}}} dj$$

$$\widehat{\bar{w}}_t^* - \hat{p}_t^{c,*} = \hat{\zeta}_t^{n,*} + \hat{\zeta}_t^{\beta,*} + \hat{z}_t^{C,*} + \varphi^* \hat{L}_t^*, \tag{13.105}$$

the unemployment rate

$$U_t^* = \frac{L_t^* - N_t^*}{L_t^*}$$

$$\hat{U}_t^* = \hat{L}_t^* - \hat{N}_t^*, \tag{13.106}$$

the natural rate of unemployment

$$\hat{U}_t^{n,*} = \frac{1}{\varphi^*}\hat{\lambda}_t^{w,*}, \tag{13.107}$$

and the wage markup

$$\hat{\mu}_{w,t}^* = \varphi^* \hat{U}_t^*. \tag{13.108}$$

Moreover, from the wage setting problem, we have the expression for the wage inflation

$$\pi_t^{w,*} = \frac{\bar{w}_t^* \mu_{z+,*,t}\pi_t^{d,*}}{\bar{w}_{t-1}^*}$$

$$\hat{\pi}_t^{w,*} = \widehat{\bar{w}}_t^* - \widehat{\bar{w}}_{t-1}^* + \hat{\pi}_t^{d,*} + \hat{\mu}_{z+,*,t}. \tag{13.109}$$

the wage Phillips curve in equation (11.53)[58]

$$\left(\tilde{w}_t^*\right)^{\frac{1-\lambda_{t+s}^{w,*}(1+\varphi^*)}{1-\lambda_{t+s}^{w,*}}} = \frac{\lambda_{t+s}^{w,*} E_t \sum_{s=0}^{\infty} \left(\beta^* \xi_w^*\right)^s \zeta_{t+s}^{\beta,*}\zeta_{t+s}^{n,*}\Theta_{t+s}^* \left[\left(\frac{\bar{w}_t^*}{\bar{w}_{t+s}^*} \frac{\tilde{\pi}_{t+s}^{w,*}\cdots\tilde{\pi}_{t+1}^{w,*}}{\mu_{z+,*,t+1}\cdots\mu_{z+,*,t+s}\pi_{t+1}^{d,*}\cdots\pi_{t+s}^{d,*}}\right)^{\frac{\lambda_{t+s}^{w,*}}{1-\lambda_{t+s}^{w,*}}} N_{t+s}^*\right]^{1+\varphi^*}}{E_t \sum_{s=0}^{\infty} \left(\beta^* \xi_w^*\right)^s \zeta_{t+s}^{\beta,*}\psi_{z+,*,t+s}\bar{w}_{t+s}^* N_{t+s}^* \left(\frac{\bar{w}_t^*}{\bar{w}_{t+s}^*} \frac{\tilde{\pi}_{t+s}^{w,*}\cdots\tilde{\pi}_{t+1}^{w,*}}{\mu_{z+,*,t+1}\cdots\mu_{z+,*,t+s}\pi_{t+1}^{d,*}\cdots\pi_{t+s}^{d,*}}\right)^{\frac{1}{1-\lambda_{t+s}^{w,*}}}}$$

$$\tilde{\pi}_{t+1}^{w,*} \equiv \left(\pi_t^{c,*}\right)^{\kappa_w^*}\left(\bar{\pi}_{t+1}^{c,*}\right)^{1-\kappa_w^*-\varkappa_w^*}\left(\breve{\pi}^*\right)^{\varkappa_w^*}\left(\mu_{z+,*}\right)^{\vartheta_w^*}$$

$$\tilde{w}_t^* = \left[\frac{1-\xi_w^*\left(\frac{\tilde{\pi}_t^{w,*}}{\pi_t^{w,*}}\right)^{\frac{1}{1-\lambda_{t+s}^{w,*}}}}{\left(1-\xi_w^*\right)}\right]^{1-\lambda_t^{w,*}}$$

$$\begin{aligned} \hat{\pi}_t^{w,*} &= \beta^* E_t \hat{\pi}_{t+1}^{w,*} + \left(1-\beta^*\rho_{\bar{\pi}^*}\right)\widehat{\bar{\pi}}_t^{c,*} \\ &+\kappa_w^*\left(\hat{\pi}_{t-1}^{c,*} - \widehat{\bar{\pi}}_t^{c,*}\right) - \beta^*\kappa_w^* E_t\left(\hat{\pi}_t^{c,*} - \widehat{\bar{\pi}}_{t+1}^{c,*}\right) - d_w^*\varphi^*\left(\hat{U}_t^* - \hat{U}_t^{n,*}\right), \\ d_w^* &= \frac{\lambda^{w,*}-1}{\xi_w^* b_w^*},\ b_w^* = \frac{\lambda^{w,*}\left(1+\varphi^*\right)-1}{\left(1-\beta^*\xi_w^*\right)\left(1-\xi_w^*\right)}, \end{aligned} \tag{13.110}$$

[58]Just as the log-linear price Phillips curves, the log-linear wage Phillips curve is a combination of the log-linearized versions of the three included non-linear equations, as well as a number of log-linearized equations from the household problem (as the domestic wage Phillips curve). Note that the log-linearized wage equation included here has been derived under the assumption that there is full indexation, that is assuming that $\varkappa_w = 0$ and that $\vartheta_w = 1$.

the expression for aggregate household hours in terms of aggregate homogeneous hours worked

$$n_t^* = N_t^* \left(\mathring{w}_t^*\right)^{\frac{\lambda_t^{w,*}}{1-\lambda_t^{w,*}}},$$

$$\hat{n}_t^* = \hat{N}_t^* + \frac{\lambda^{w,*}}{1-\lambda^{w,*}} \widehat{\mathring{w}}_t^*, \tag{13.111}$$

and the wage dispersion equation

$$\mathring{w}_t^* = \left[\xi_w^* \left(\frac{\tilde{\pi}_t^{w,*}}{\pi_t^{w,*}} \mathring{w}_{t-1}^* \right)^{\frac{\lambda_t^{w,*}}{1-\lambda_t^{w,*}}} + (1-\xi_w^*) \left(\frac{1 - \xi_w^* \left(\frac{\tilde{\pi}_t^{w,*}}{\pi_t^{w,*}} \right)^{\frac{1}{1-\lambda_t^{w,*}}}}{(1-\xi_w^*)} \right)^{\lambda_t^{w,*}} \right]^{\frac{1-\lambda_t^{w,*}}{\lambda_t^{w,*}}}$$

$$\widehat{\mathring{w}}_t^* = \xi_w^* \widehat{\mathring{w}}_{t-1}^*. \tag{13.112}$$

We next need to include the central bank policy rule

$$\begin{aligned} \log \left(\frac{R_t^*}{R^*} \right) &= \rho_{R^*} \log \left(\frac{R_{t-1}^*}{R^*} \right) + (1-\rho_{R^*}) \left[\log \left(\frac{\bar{\pi}_t^{c,*}}{\bar{\pi}^{c,*}} \right) + r_{\pi^*} \log \left(\frac{\pi_{t-1}^{c,*}}{\bar{\pi}_t^{c,*}} \right) \right. \\ & \left. + r_{RU^*} \left(U_{t-1}^* - U^* \right) \right] + r_{\Delta\pi^*} \Delta \log \left(\frac{\pi_t^{c,*}}{\pi^{c,*}} \right) + r_{\Delta RU^*} \Delta U_t^* + \log \varepsilon_{R^*,t} \end{aligned}$$

$$\hat{R}_t^* = \rho_{R^*} \hat{R}_{t-1}^* + (1-\rho_{R^*}) \left[\widehat{\bar{\pi}}_t^{c,*} + r_{\pi^*} \left(\hat{\pi}_{t-1}^{c,*} - \widehat{\bar{\pi}}_t^{c,*} \right) + r_{RU^*} \hat{U}_{t-1}^* \right] + r_{\Delta\pi^*} \Delta \hat{\pi}_t^{c,*} + r_{\Delta RU^*} \Delta \hat{U}_t^* + \hat{\varepsilon}_{R^*,t}, \tag{13.113}$$

and the following two equations from the derivations aggregate resource constraint:

$$y_t^* = g_t^* + c_t^{d,*} + c_t^{e,*} + i_t^{d,*}$$

$$\hat{y}_t^* = \frac{g^*}{y^*} \hat{g}_t^* + \frac{c^{xe,*}}{y^*} \hat{c}_t^{xe,*} + \frac{c^{e,*}}{y^*} \hat{c}_t^{e,*} + \frac{i^{d,*}}{y^*} \hat{i}_t^{d,*}, \tag{13.114}$$

$$y_t^* = \left(\mathring{p}_t^*\right)^{\frac{\lambda_t^*}{\lambda_t^*-1}} \left[\epsilon_t^* \left(\frac{k_t^*}{\mu_{z^+,*,t} \mu_{\Psi^*,t}} \right)^{\alpha^*} \left(n_t^* \left(\mathring{w}_t^*\right)^{-\frac{\lambda_t^{w,*}}{1-\lambda_t^{w,*}}} \right)^{1-\alpha^*} - \phi^* \right]$$

$$\begin{aligned} \hat{y}_t^* &= \frac{\lambda^*}{1-\lambda^*} \widehat{\mathring{p}}_t + \frac{1}{y^*} \left(\frac{k^*}{\mu_{z^+,*} \mu_{\Psi^*}} \right)^{\alpha^*} (n^*)^{1-\alpha^*} \times \\ & \times \left[\hat{\epsilon}_t^* + \alpha^* \left(\hat{k}_t^* - \hat{\mu}_{z^+,*,t} - \hat{\mu}_{\Psi^*,t} \right) + (1-\alpha^*) \left(\hat{n}_t^* - \frac{\lambda^{w,*}}{1-\lambda^{w,*}} \widehat{\mathring{w}}_t^* \right) \right]. \end{aligned} \tag{13.115}$$

We again include an additional variables of interest, that is not needed to solve the above system, namely the foreign investment inflation rate as given by the following expression:

$$\pi_t^{i,*} = \frac{\pi_t^{d,*}}{\mu_{\Psi^*,t}}$$

$$\hat{\pi}_t^{i,*} = \hat{\pi}_t^{d,*} - \hat{\mu}_{\Psi^*,t}. \tag{13.116}$$

13.3 The complete system

The equations listed in Sections 13.1 and 13.2 above constitute a system of equations for the following lite of variables:

$$
\begin{aligned}
&\hat{\pi}_t^d, \hat{\pi}_t^{m,c}, \hat{\pi}_t^{m,i}, \hat{\pi}_t^{m,x}, \hat{\pi}_t^{m,ce}, \hat{\pi}_t^x, \hat{\pi}_t^c, \hat{\pi}_t^{xe}, \hat{\pi}_t^{ce}, \hat{\pi}_t^{d,ce}, \hat{\pi}_t^w, \hat{\pi}_t^i,\\
&\widehat{mc}_t^d, \widehat{mc}_t^{m,c}, \widehat{mc}_t^{m,i}, \widehat{mc}_t^{m,x}, \widehat{mc}_t^{m,ce}, \widehat{mc}_t^x, \hat{R}_t^{wc,d}, \hat{R}_t^{wc,m}, \hat{R}_t^{wc,x},\\
&\widehat{mpl}, \widehat{mkp}_t, \widehat{\left(\frac{k}{N}\right)}_t, \hat{\bar{r}}_t^k, \hat{\bar{w}}_t, \hat{\Theta}_t, \hat{z}_t^C, \hat{\bar{\upsilon}}_t^N, \widehat{mrs}_t, \hat{L}_t, \hat{U}_t, \hat{U}_t^n, \hat{\mu}_{w,t}, \hat{k}_t, \hat{N}_t, \hat{R}_t, \hat{y}_t, \hat{\mu}_{z^+,t},\\
&\hat{c}_t, \hat{c}_t^{xe}, \hat{c}_t^e, \hat{c}_t^{e,d}, \hat{c}_t^{e,m}, \hat{c}_t^d, \hat{c}_t^m, \breve{a}_t, \hat{i}_t, \hat{i}_t^d, \hat{i}_t^m, \hat{x}_t, \hat{x}_t^d, \hat{x}_t^m, \hat{m}_t, \hat{u}_t, \hat{\breve{p}}_{k',t}, \hat{k}_{t+1}^p, \hat{n}_t,\\
&\hat{p}_t^c, \hat{p}_t^{xe}, \hat{p}_t^{ce}, \hat{p}_t^{m,c}, \hat{p}_t^{m,i}, \hat{p}_t^{m,x}, \hat{p}_t^{m,ce}, \hat{p}_t^x, \hat{p}_t^i, \hat{q}_t, \hat{s}_t, \hat{\Phi}_t,\\
&\hat{\mathring{p}}_t^d, \hat{\mathring{p}}_t^{m,c}, \hat{\mathring{p}}_t^{m,i}, \hat{\mathring{p}}_t^{m,x}, \hat{\mathring{p}}_t^{m,ce}, \hat{\mathring{p}}_t^x, \hat{\mathring{w}}_t,\\
&\hat{\pi}_t^{d,*}, \hat{\pi}_t^{cxe,*}, \hat{\pi}_t^{ce,*}, \hat{\pi}_t^{c,*}, \hat{\pi}_t^{i,*}, \hat{\pi}_t^{w,*}, \widehat{mc}_t^*, \hat{R}_t^{wc,*}, \widehat{mpl}^*, \widehat{mkp}_t^*, \widehat{\left(\frac{k}{N}\right)}_t^*,\\
&\hat{\bar{r}}_t^{k,*}, \hat{\bar{w}}_t^*, \hat{\Theta}_t^*, \hat{z}_t^{C,*}, \hat{\bar{\upsilon}}_t^{N,*}, \widehat{mrs}_t^*, \hat{L}_t^*, \hat{U}_t^*, \hat{U}_t^{n,*}, \hat{\mu}_{w,t}^*, \hat{k}_t^*, \hat{N}_t^*, \hat{R}_t^*, \hat{y}_t^*, \hat{\mu}_{z^+,*,t},\\
&\hat{c}_t^*, \hat{c}_t^{xe,*}, \hat{c}_t^{e,*}, \hat{i}_t^*, \hat{i}_t^{d,*}, \hat{u}_t^*, \hat{\breve{p}}_{k',t}^*, \hat{k}_{t+1}^{p,*}, \hat{n}_t^*,\\
&\hat{p}_t^{c,*}, \hat{p}_t^{i,*}, \hat{\mathring{p}}_t^*, \hat{\mathring{w}}_t^*.
\end{aligned}
$$

14 Time-varying neutral rate

In the model above, we assume that the monetary policy maker sets the interest rate, R_t, in relation to its steady-state value, R. In other words, the central bank reacts to the deviation of inflation, unemployment, etc. from their steady-state values by making the interest rate deviate from its steady-state value. Also, households take into account the deviations of the interest rate from steady state in their allocation decisions. An alternative assumption would be to assume that the agents in the economy instead consider deviations of the interest rate from some time-varying medium term value, R_t^t, which we shall refer to as the neutral rate and which, in turn, itself varies around the steady-state, R. The idea is that there are slow movements in the real interest rate that are driven by global factors, such as global savings developments or demographics, that do not affect allocations, as this seem to be in line with what we have observed in data in recent years. We think of the neutral rate as the rate that is neither expansionary nor contractionary if the economy operates near its potential. We specify a simple auxiliary model for the trend component of real and nominal Swedish and foreign interest rates. Importantly, we assume that shocks which affect the trend components have no effects on the cyclical components, and vice versa. The trend-cycle decomposition of interest rates will be jointly determined by the structural model and the auxiliary model.

Denoting by bar the real version of the corresponding nominal rate, we assume that

$$R_t^t = \bar{R}_t^t \bar{\pi}_t^c, \tag{14.1}$$

where R_t^t is the nominal neutral rate, $\bar{R}_t^t$ is the real neutral rate, and $\bar{\pi}_t^c$ is the central bank's inflation target. Similarly, for the foreign economy, we have that

$$R_t^{t,*} = \bar{R}_t^{t,*} \bar{\pi}_t^{c,*}, \tag{14.2}$$

where we have now used the foreign central bank's inflation target to derive the expression for the nominal neutral rate. The real interest rate trend is allowed to depend on technology growth, $\mu_{z^+,t}$. It is also allowed to depend on the risk premium shock, χ_t, in order to capture the effects of shifts in the demand for safe assets on policy rates, which could be viewed as an attempt to endogenize part of the variation in the real neutral rate, which would otherwise be attributed to exogenous factors. The resulting specification of the real interest rate trend is then given by

$$\log\left(\frac{\bar{R}_t^t}{R}\right) = \log R + r_{\mu_{z+}} \log\left(\frac{\mu_{z+,t}}{\mu_{z+}}\right) - r_\chi \log\left(\frac{\chi_t}{\chi}\right) + \log\left(\frac{z_t^R}{z^R}\right), \tag{14.3}$$

where z_t^R is a shock to the real interest rate trend, intended to capture non-modelled factors, such as the effects of demographic changes on the real interest rate. For the foreign economy, we similarly have

$$\log\left(\frac{\bar{R}_t^{t,*}}{R^*}\right) = \log R^* + r_{\mu_{z+,*}} \log\left(\frac{\mu_{z+,*,t}}{\mu_{z+,*}}\right) - r_{\chi^*} \log\left(\frac{\chi_t^*}{\chi^*}\right) + \log\left(\frac{z_t^{R,*}}{z^{R,*}}\right), \tag{14.4}$$

Note that, in steady state, the neutral rates are equal to the actual rates, so that $R^t = R$ and $R^{t,*} = R^*$.

The shocks to the real interest rate trends are assumed to evolve according to the following processes:

$$\log z_t^R = (1-\rho_{z^R}) \log z^R + \rho_{z^R} \log z_{t-1}^R + \sigma_{z^R}\varepsilon_{z^R,t}, \tag{14.5}$$

$$\log z_t^{R,*} = (1-\rho_{z^{R,*}}) \log z^{R,*} + \rho_{z^{R,*}} \log z_{t-1}^{R,*} + \sigma_{z^{R,*}}\varepsilon_{z^{R,*},t}. \tag{14.6}$$

An innovation to the real interest rate trend affects the policy rate but no other variables in the model, since it does not affect the policy rate gap. This specification of the real neutral rate is similar to its modelling in semi-structural models aimed to provide estimates of the neutral interest rate, as in e.g. Laubach and Williams (2015), and Holston, Laubach, and Williams (2016).

In case we want to include a time-varying neutral rate in the model, we need to log-linearize equations (14.1)–(14.6) above. This gives the following expressions: for the domestic nominal neutral rate

$$\hat{R}_t^t = \hat{\bar{R}}_t^{t,*} + \hat{\bar{\pi}}_t^c, \tag{14.7}$$

for the foreign nominal neutral rate

$$\hat{R}_t^{t,*} = \hat{\bar{R}}_t^{t,*} + \hat{\bar{\pi}}_t^{c,*}, \tag{14.8}$$

for the domestic real neutral rate trend

$$\hat{\bar{R}}_t^t = r_{\mu_{z+}} \hat{\mu}_{z+,t} - r_\chi \hat{\chi}_t + \hat{z}_t^R, \tag{14.9}$$

for the foreign real neutral rate trend

$$\hat{\bar{R}}_t^{t,*} = r_{\mu_{z+,*}} \hat{\mu}_{z+,*,t} - r_{\chi^*} \hat{\chi}_t^* + \hat{z}_t^{R,*}, \tag{14.10}$$

and for the exogenous processes

$$\hat{z}_t^R = \rho_{z^R} \hat{z}_{t-1}^R + \sigma_{z^R}\varepsilon_{z^R,t}, \tag{14.11}$$

$$\hat{z}_t^{R,*} = \rho_{z^{R,*}} \hat{z}_{t-1}^{R,*} + \sigma_{z^{R,*}}\varepsilon_{z^{R,*},t}. \tag{14.12}$$

We finally note that, in estimation, the measurement equations of the interest rates need to be changed accordingly. We think of the observed interest rate level as composed by two gaps: the deviation of the time-varying neutral rate from steady state, and the deviation from the actual interest rate from the time-varying neutral one. In the model, however, it is only the latter gap that is relevant, so the observed interest rate deviations need to map the difference between the actual interest rate and the time-varying neutral one.

15 The EHL model of the labour market

In this section, we describe how the model changes when the labour market is modelled as in Erceg, Henderson, and Levin (2000) (EHL), instead of as in Galí, Smets, and Wouters (2012) (GSW) as assumed above. The changes primarily concern the household optimization and wage setting, leaving the modelling of the firms largely unaffected.

15.1 Households

The EHL model assumes continuum of households in the economy, indexed by $j \in (0,1)$. They attain utility from consumption and leisure, as in Christiano, Trabandt, and Walentin (2011) and Adolfson et al. (2013). Compared to the specification in Section 4, the preferences with respect to consumption are unchanged. Household j has the following preferences:

$$E_0^j \sum_{t=0}^{\infty} \beta^t \zeta_t^\beta \left[\zeta_t^c \log \left(C_{j,t} - bC_{j,t-1} \right) - \zeta_t^n A_L \frac{h_{j,t}^{1+\sigma_L}}{1+\sigma_L} \right], \tag{15.1}$$

where $C_{j,t}$ and $h_{j,t}$ denote level of aggregate consumption and work effort, respectively, of household j at time t. The parameter σ_L is the inverse of the Frisch elasticity of labour supply, measuring the substitution effect of a change in the wage rate on labour supply, holding constant the marginal utility of wealth.[59] Finally, A_L is a labour disutility constant.

The budget constraint (4.10) is slightly modified due to the assumption of a continuum of households, rathert han a large representative one as in GSW. Here, household j optimizes its utility subject to the following budget constraint:

$$\begin{aligned} & P_t^c C_{j,t} + P_t^i \left(I_{j,t} + a\left(u_{j,t}\right) K_{j,t}^p \right) + P_{k',t} \Delta_{j,t} + B_{j,t+1} + S_t B_{j,t+1}^F \\ = \; & W_{j,t} h_{j,t} + R_t^k u_{j,t} K_{j,t}^p + R_{t-1} \chi_{t-1} B_{j,t} + R_{t-1}^* \Phi_{t-1} \chi_{t-1} S_t B_{j,t}^F + \Pi_t + TR_t, \end{aligned} \tag{15.2}$$

Just as in Section 4, the left-hand side contains the expenditure terms and the right-hand side the income terms. Note that $W_{j,t}$ denotes the wage set by household j. Note further that the wage is now defined as the wage per hour rather than the wage per worker, as reflected by the product of wages and hours worked replacing the integral over the household members' wages.

We finally note that hours enter the firm problem identically to household employment in the GSW model. Hence, the firm side of the economy need not be modified, with the exception that $N_{j,t}$ has to be replaced by $H_{j,t}$. As in (3.5), firm i is assumed to have the technology

$$Y_{i,t} = \left(z_t H_{i,t} \right)^{1-\alpha} \epsilon_t K_{i,t}^\alpha - z_t^+ \phi^d, \tag{15.3}$$

where $H_{i,t}$ is demand for hours worked of firm i. Firms buy $H_{i,t}$ from labour contractors as in the GSW version of the model.[60]

15.2 Wage setting

Just as in the GSW version of the model, we assume that household members are monopoly suppliers of differentiated labour services hired by the firm. Thus, households can determine their wages. We assume that the differentiated labour $h_{j,t}$ is sold by households to labour contractors who combine it into a homogeneous input good H_t using the following technology:

$$H_t = \left[\int_0^1 \left(h_{j,t} \right)^{\frac{1}{\lambda_t^w}} dj \right]^{\lambda_t^w}, \quad 1 \le \lambda_t^w < \infty, \tag{15.4}$$

where λ_t^w is a time-varying wage markup given by the following process (just as in equation 4.41):

$$\log \lambda_t^w = \left(1 - \rho_{\lambda^w}\right) \log \lambda^w + \rho_{\lambda^w} \log \lambda_{t-1}^w + \sigma_{\lambda^w} \varepsilon_{\lambda^w,t}. \tag{15.5}$$

[59] In general, wage changes also have wealth effects on labor supply. The Frisch elasticity, hence, does not capture the total effect on hours from wage shocks but only the component that is due to intertemporal substitution effects.

[60] Note that, for estimation purposes, the observed variables and the observation equations of the model need to be changed accordingly. The two observation equations for employment and unemployment are now replaced by one observation equation for hours. Moreover, even though the observation equation for wages itself doesn't change, the appropriate wage measure does.

These labour contractors take the price of the j^{th} differentiated labour input $W_{j,t}$, and the price of the homogeneous labour service W_t as given. Profit maximization writes

$$\max_{h_{j,t}} \quad W_t H_t - \int_0^1 W_{j,t} h_{j,t} dj$$

and leads to the following first-order condition:

$$h_{j,t} = \left(\frac{W_{j,t}}{W_t} \right)^{\frac{\lambda_t^w}{1-\lambda_t^w}} H_t, \tag{15.6}$$

which is a demand curve for the individual households' labour services. Integrating (4.42) and using the definition of H_t, we obtain the expression for the aggregate wage rate

$$W_t = \left[\int_0^1 (W_{j,t})^{\frac{1}{1-\lambda_t^w}} dj \right]^{1-\lambda_t^w} . \tag{15.7}$$

As in the GSW version of our model, we assume that households are subject to Calvo wage setting frictions as in Erceg, Henderson, and Levin (2000). If the j^{th} household is not able to reoptimize in period t, the wage it will charge in period $t+1$ will be set according to the following indexation rule:

$$\begin{cases} W_{j,t+1} = \tilde{\pi}_{t+1}^w W_{j,t} \\ \tilde{\pi}_{t+1}^w \equiv (\pi_t^c)^{\kappa_w} \left(\bar{\pi}_{t+1}^c\right)^{1-\kappa_w-\varkappa_w} (\breve{\pi})^{\varkappa_w} \left(\mu_{z^+}\right)^{\vartheta_w} . \end{cases} \tag{15.8}$$

Let us denote by $\tilde{W}_{j,t}$ the reoptimized nominal wage of household j set in period t, and consider that this household has not been able to reoptimize during s periods ahead. The wage in $t+s$ will be given by

$$W_{j,t+s} = \tilde{\pi}_{t+s}^w \ldots \tilde{\pi}_{t+1}^w \tilde{W}_{j,t}.$$

In period t, when setting its wage $\tilde{W}_{j,t}$, the j^{th} households will maximize its future discounted utility (i.e. individual utility, as opposed to the utility of all household members in the large representative household) subject to the budget constraint as in Section 4.4, taking into account that there is a probability ξ_w in each period that it cannot reoptimize. Using (15.1) and ignoring the irrelevant terms (of the utility function) for the wage setting problem, the problem becomes

$$\begin{cases} \max\limits_{\tilde{W}_{j,t}} \quad E_t \sum\limits_{s=0}^{\infty} (\beta\xi_w)^s \zeta_{t+s}^{\beta} \left[-\zeta_{t+s}^n A_L \frac{(h_{j,t+s})^{1+\sigma_L}}{1+\sigma_L} + \upsilon_{t+s} W_{j,t+s} h_{j,t+s} \right] \\ \qquad s.t. \quad h_{j,t} = \left(\frac{W_{j,t}}{W_t} \right)^{\frac{\lambda_t^w}{1-\lambda_t^w}} H_t \end{cases} . \tag{15.9}$$

Replacing both $h_{j,t}$ and the expression for the wage, we get

$$\max_{\tilde{W}_{j,t}} \quad E_t \sum_{s=0}^{\infty} (\beta\xi_w)^s \zeta_{t+s}^{\beta} \left[\begin{array}{c} -\zeta_{t+s}^n \frac{A_L}{1+\sigma_L} \left[\left(\frac{\tilde{\pi}_{t+s}^w \ldots \tilde{\pi}_{t+1}^w}{W_{t+s}} \right)^{\frac{\lambda_{t+s}^w}{1-\lambda_{t+s}^w}} H_{t+s} \right]^{1+\sigma_L} \tilde{W}_{j,t}^{\frac{\lambda_{t+s}^w (1+\sigma_L)}{1-\lambda_{t+s}^w}} \\ +\upsilon_{t+s} (W_{t+s})^{-\frac{\lambda_{t+s}^w}{1-\lambda_{t+s}^w}} H_{t+s} \left(\tilde{\pi}_{t+s}^w \ldots \tilde{\pi}_{t+1}^w \tilde{W}_{j,t} \right)^{\frac{1}{1-\lambda_{t+s}^w}} \end{array} \right].$$

Note that the objective is almost the same as in the GSW model. Instead of $\zeta_{t+s}^n \frac{\Theta_{t+s}}{1+\varphi}$ multiplying the first term in the summation, we instead have $\zeta_{t+s}^n \frac{A_L}{1+\sigma_L}$ and the term within the square brackets is raised to $1+\sigma_L$ instead of $1+\varphi$. The first-order condition are then also almost identical. Taking derivatives w.r.t. $\tilde{W}_{j,t}$ gives

$$\begin{aligned} & \left(\tilde{W}_{j,t}\right)^{1-\frac{\sigma_L \lambda_{t+s}^w}{1-\lambda_{t+s}^w}} E_t \sum_{s=0}^{\infty} (\beta\xi_w)^s \zeta_{t+s}^{\beta} \upsilon_{t+s} (W_{t+s})^{-\frac{\lambda_{t+s}^w}{1-\lambda_{t+s}^w}} H_{t+s} \left(\tilde{\pi}_{t+s}^w \ldots \tilde{\pi}_{t+1}^w\right)^{\frac{1}{1-\lambda_{t+s}^w}} \\ = \quad & E_t \sum_{s=0}^{\infty} (\beta\xi_w)^s \zeta_{t+s}^{\beta} \zeta_{t+s}^n A_L \lambda_{t+s}^w \left[\left(\frac{\tilde{\pi}_{t+s}^w \ldots \tilde{\pi}_{t+1}^w}{W_{t+s}} \right)^{\frac{\lambda_{t+s}^w}{1-\lambda_{t+s}^w}} H_{t+s} \right]^{1+\sigma_L} . \end{aligned}$$

As each household faces the same optimization problem, we can drop the index j. Proceeding as in Section 4.6, we get

$$\tilde{w}_t^{\frac{1-\lambda_{t+s}^w(1+\sigma_L)}{1-\lambda_{t+s}^w}} = \frac{\lambda_{t+s}^w A_L E_t \sum_{s=0}^{\infty} (\beta\xi_w)^s \zeta_{t+s}^\beta \zeta_{t+s}^n \left[\left(\frac{W_t \tilde{\pi}_{t+s}^w \ldots \tilde{\pi}_{t+1}^w}{W_{t+s}} \right)^{\frac{\lambda_{t+s}^w}{1-\lambda_{t+s}^w}} H_{t+s} \right]^{1+\sigma_L}}{E_t \sum_{s=0}^{\infty} (\beta\xi_w)^s \zeta_{t+s}^\beta \upsilon_{t+s} W_{t+s} H_{t+s} \left(\frac{W_t \tilde{\pi}_{t+s}^w \ldots \tilde{\pi}_{t+1}^w}{W_{t+s}} \right)^{\frac{1}{1-\lambda_{t+s}^w}}}. \tag{15.10}$$

Note that the expression is very similar to (4.46). Only A_L and σ_L are different.

15.3 Scaling

Proceeding as in Section 4.7.5, expression (15.10) becomes

$$\tilde{w}_t^{\frac{1-\lambda_{t+s}^w(1+\sigma_L)}{1-\lambda_{t+s}^w}} = \frac{\lambda_{t+s}^w A_L E_t \sum_{s=0}^{\infty} (\beta\xi_w)^s \zeta_{t+s}^\beta \zeta_{t+s}^n \left[\left(\frac{\bar{w}_t}{\bar{w}_{t+s}} \frac{\tilde{\pi}_{t+s}^w \ldots \tilde{\pi}_{t+1}^w}{\mu_{z^+,t+1} \ldots \mu_{z^+,t+s} \pi_{t+1}^d \ldots \pi_{t+s}^d} \right)^{\frac{\lambda_{t+s}^w}{1-\lambda_{t+s}^w}} H_{t+s} \right]^{1+\sigma_L}}{E_t \sum_{s=0}^{\infty} (\beta\xi_w)^s \zeta_{t+s}^\beta \psi_{z^+,t+s} \bar{w}_{t+s} H_{t+s} \left(\frac{\bar{w}_t}{\bar{w}_{t+s}} \frac{\tilde{\pi}_{t+s}^w \ldots \tilde{\pi}_{t+1}^w}{\mu_{z^+,t+1} \ldots \mu_{z^+,t+s} \pi_{t+1}^d \ldots \pi_{t+s}^d} \right)^{\frac{1}{1-\lambda_{t+s}^w}}}. \tag{15.11}$$

15.4 Log-linearization

To log-linearize equation (15.11), we start by re-expressing it as follows:

$$\begin{aligned} &\exp \left(\frac{1-\lambda_{t+s}^w (1+\sigma_L)}{1-\lambda_{t+s}^w} \log \tilde{w}_t \right) \times \\ &E_t \sum_{s=0}^{\infty} (\beta\xi_w)^s \zeta_{t+s}^\beta \psi_{z^+,t+s} \bar{w}_{t+s} H_{t+s} \exp \left(\frac{1}{1-\lambda_{t+s}^w} \log \left(\frac{\bar{w}_t}{\bar{w}_{t+s}} \frac{\tilde{\pi}_{t+s}^w \ldots \tilde{\pi}_{t+1}^w}{\mu_{z^+,t+1} \ldots \mu_{z^+,t+s} \pi_{t+1}^d \ldots \pi_{t+s}^d} \right) \right) \\ = \; &\lambda_{t+s}^w A_L E_t \sum_{s=0}^{\infty} (\beta\xi_w)^s \zeta_{t+s}^\beta \zeta_{t+s}^n H_{t+s}^{1+\sigma_L} \exp \left(\frac{\lambda_{t+s}^w (1+\sigma_L)}{1-\lambda_{t+s}^w} \log \left(\frac{\bar{w}_t}{\bar{w}_{t+s}} \frac{\tilde{\pi}_{t+s}^w \ldots \tilde{\pi}_{t+1}^w}{\mu_{z^+,t+1} \ldots \mu_{z^+,t+s} \pi_{t+1}^d \ldots \pi_{t+s}^d} \right) \right). \end{aligned}$$

Note that the right hand side is similar to the GSW model, with the exception that A_L replaces Θ_{t+s} and σ_L replaces φ. A first-order Taylor expansion of the left-hand side yields:

$$
\begin{aligned}
&\tilde{w}^{\frac{1-\lambda^w(1+\sigma_L)}{1-\lambda^w}} \sum_{s=0}^{\infty} (\beta\xi_w)^s \zeta^\beta \psi_{z^+} \bar{w} H \left(\frac{\tilde{\pi}^w}{\pi^d \mu_{z^+}} \right)^{\frac{s}{1-\lambda^w}} \\
&+\tilde{w}^{\frac{1-\lambda^w(1+\sigma_L)}{1-\lambda^w}} E_t \sum_{s=0}^{\infty} (\beta\xi_w)^s \psi_{z^+} \bar{w} H \left(\frac{\tilde{\pi}^w}{\pi^d \mu_{z^+}} \right)^{\frac{s}{1-\lambda^w}} \left(\zeta_{t+s}^\beta - \zeta^\beta \right) \\
&+\tilde{w}^{\frac{1-\lambda^w(1+\sigma_L)}{1-\lambda^w}} E_t \sum_{s=0}^{\infty} (\beta\xi_w)^s \zeta^\beta \bar{w} H \left(\frac{\tilde{\pi}^w}{\pi^d \mu_{z^+}} \right)^{\frac{s}{1-\lambda^w}} \left(\psi_{z^+,t+s} - \psi_{z^+} \right) \\
&+\tilde{w}^{\frac{1-\lambda^w(1+\sigma_L)}{1-\lambda^w}} E_t \sum_{s=0}^{\infty} (\beta\xi_w)^s \zeta^\beta \psi_{z^+} \bar{w} \left(\frac{\tilde{\pi}^w}{\pi^d \mu_{z^+}} \right)^{\frac{s}{1-\lambda^w}} \left(H_{t+s} - H \right) \\
&+\frac{1-\lambda^w(1+\sigma_L)}{1-\lambda^w} \tilde{w}^{\frac{-\lambda^w \sigma_L}{1-\lambda^w}} E_t \sum_{s=0}^{\infty} (\beta\xi_w)^s \zeta^\beta \psi_{z^+} \bar{w} H \left(\frac{\tilde{\pi}^w}{\pi^d \mu_{z^+}} \right)^{\frac{s}{1-\lambda^w}} \left(\tilde{w}_t - \tilde{w} \right) \\
&-\tilde{w}^{\frac{1-\lambda^w(1+\sigma_L)}{1-\lambda^w}} E_t \sum_{s=0}^{\infty} (\beta\xi_w)^s \zeta^\beta \psi_{z^+} \bar{w} H \left(\frac{\tilde{\pi}^w}{\pi^d \mu_{z^+}} \right)^{\frac{s}{1-\lambda^w}} \left[\frac{1}{\bar{w}} \frac{\lambda^w}{1-\lambda^w} \right] \left(\bar{w}_{t+s} - \bar{w} \right) \\
&+\tilde{w}^{\frac{1-\lambda^w(1+\sigma_L)}{1-\lambda^w}} E_t \sum_{s=0}^{\infty} (\beta\xi_w)^s \zeta^\beta \psi_{z^+} \bar{w} H \left(\frac{\tilde{\pi}^w}{\pi^d \mu_{z^+}} \right)^{\frac{s}{1-\lambda^w}} \left[\frac{1}{\bar{w}} \frac{1}{1-\lambda^w} \right] \left(\bar{w}_t - \bar{w} \right) \\
&+\tilde{w}^{\frac{1-\lambda^w(1+\sigma_L)}{1-\lambda^w}} E_t \sum_{s=0}^{\infty} (\beta\xi_w)^s \zeta^\beta \psi_{z^+} \bar{w} H \left(\frac{\tilde{\pi}^w}{\pi^d \mu_{z^+}} \right)^{\frac{s}{1-\lambda^w}} \times \\
&\times \left[\begin{array}{c} -\frac{1}{(1-\lambda^w)^2} \times \\ \left(\begin{array}{c} \log \left(\frac{\tilde{\pi}^w}{\pi^d \mu_{z^+}} \right)^s \\ +\left(1-\lambda^w(1+\sigma_L)\right) \log \tilde{w} \end{array} \right) \\ -(1+\sigma_L) \log \tilde{w} \frac{1}{1-\lambda^w} \end{array} \right] \left(\lambda_{t+s}^w - \lambda^w \right) \\
&+\tilde{w}^{\frac{1-\lambda^w(1+\sigma_L)}{1-\lambda^w}} E_t \sum_{s=0}^{\infty} (\beta\xi_w)^s \zeta^\beta \psi_{z^+} \bar{w} H \left(\frac{\tilde{\pi}^w}{\pi^d \mu_{z^+}} \right)^{\frac{s}{1-\lambda^w}} \frac{1}{1-\lambda^w} \times \\
&\times \left[\begin{array}{c} \frac{1}{\tilde{\pi}^w} \left(\tilde{\pi}_{t+1}^w + \ldots + \tilde{\pi}_{t+s}^w - s\tilde{\pi}^w \right) \\ -\frac{1}{\pi^d} \left(\pi_{t+1}^d + \ldots + \pi_{t+s}^d - s\pi^d \right) \\ -\frac{1}{\mu_{z^+}} \left(\mu_{z^+,t+1} + \ldots + \mu_{z^+,t+s} - s\mu_{z^+} \right) \end{array} \right].
\end{aligned}
$$

A first-order Taylor expansion of the right-hand side yields:

$$\sum_{s=0}^{\infty}(\beta\xi_w)^s\zeta^{\beta}\zeta^{n}\lambda^{w}A_LH^{1+\sigma_L}\left(\frac{\tilde{\pi}^w}{\pi^d\mu_{z^+}}\right)^{s\frac{\lambda^w(1+\sigma_L)}{1-\lambda^w}}$$

$$+E_t\sum_{s=0}^{\infty}(\beta\xi_w)^s\zeta^{n}\lambda^{w}A_LH^{1+\sigma_L}\left(\frac{\tilde{\pi}^w}{\pi^d\mu_{z^+}}\right)^{s\frac{\lambda^w(1+\sigma_L)}{1-\lambda^w}}\left(\zeta^{\beta}_{t+s}-\zeta^{\beta}\right)$$

$$+E_t\sum_{s=0}^{\infty}(\beta\xi_w)^s\zeta^{\beta}\lambda^{w}A_LH^{1+\sigma_L}\left(\frac{\tilde{\pi}^w}{\pi^d\mu_{z^+}}\right)^{s\frac{\lambda^w(1+\sigma_L)}{1-\lambda^w}}\left(\zeta^{n}_{t+s}-\zeta^{n}\right)$$

$$+E_t\sum_{s=0}^{\infty}(\beta\xi_w)^s\zeta^{\beta}\zeta^{n}\lambda^{w}A_L\left(1+\sigma_L\right)H^{\sigma_L}\left(\frac{\tilde{\pi}^w}{\pi^d\mu_{z^+}}\right)^{s\frac{\lambda^w(1+\sigma_L)}{1-\lambda^w}}\left(H_{t+s}-H\right)$$

$$+E_t\sum_{s=0}^{\infty}(\beta\xi_w)^s\zeta^{\beta}\zeta^{n}A_LH^{1+\sigma_L}\left(\frac{\tilde{\pi}^w}{\pi^d\mu_{z^+}}\right)^{s\frac{\lambda^w(1+\sigma_L)}{1-\lambda^w}}\times$$

$$\times\left[\begin{array}{c}1+\frac{1+\sigma_L}{(1-\lambda^w)^2}\times\\ \log\left(\frac{\tilde{\pi}^w}{\pi^d\mu_{z^+}}\right)^s\left(\frac{\tilde{\pi}^w}{\pi^d\mu_{z^+}}\right)^{s\frac{\lambda^w(1+\sigma_L)}{1-\lambda^w}}\end{array}\right]\left(\lambda^{w}_{t+s}-\lambda^{w}\right)$$

$$+E_t\sum_{s=0}^{\infty}(\beta\xi_w)^s\zeta^{\beta}\zeta^{n}\lambda^{w}A_LH^{1+\sigma_L}\left(\frac{\tilde{\pi}^w}{\pi^d\mu_{z^+}}\right)^{s\frac{\lambda^w(1+\sigma_L)}{1-\lambda^w}}\frac{\lambda^w\left(1+\sigma_L\right)}{1-\lambda^w}\times$$

$$\times\left[\begin{array}{c}\frac{1}{\tilde{\pi}^w}\left(\tilde{\pi}^w_{t+1}+\ldots+\tilde{\pi}^w_{t+s}\right)\\ -\frac{1}{\pi^d}\left(\pi^d_{t+1}+\ldots+\pi^d_{t+s}-s\pi^d\right)\\ -\frac{1}{\mu_{z^+}}\left(\mu_{z^+,t+1}+\ldots+\mu_{z^+,t+s}-s\mu_{z^+}\right)\end{array}\right].$$

Using steady-state relationships, we equate both sides of the optimal-wage equation derived above and, after simpliflying, we obtain

$$\frac{1-\lambda^w\left(1+\sigma_L\right)}{1-\lambda^w}\widehat{\bar{w}}_t\zeta^{\beta}\psi_{z^+}\bar{w}H\sum_{s=0}^{\infty}(\beta\xi_w)^s$$
$$+E_t\sum_{s=0}^{\infty}(\beta\xi_w)^s\zeta^{\beta}\psi_{z^+}\bar{w}H\times$$
$$\times\left[\hat{\zeta}^{\beta}_{t+s}+\hat{\psi}_{z^+,t+s}+\hat{H}_{t+s}+\frac{1}{1-\lambda^w}\left(\widehat{\bar{w}}_t-\lambda^w\widehat{\bar{w}}_{t+s}+\widehat{\tilde{\pi}}^w_{t+1}+\ldots+\widehat{\tilde{\pi}}^w_{t+s}\right.\right.$$
$$\left.\left.-\hat{\pi}^d_{t+1}-\ldots-\hat{\pi}^d_{t+s}-\hat{\mu}_{z^+,t+1}-\ldots-\hat{\mu}_{z^+,t+s}\right)\right]$$
$$=E_t\sum_{s=0}^{\infty}(\beta\xi_w)^s\zeta^{\beta}\zeta^{n}\lambda^{w}A_LH^{1+\sigma_L}\left[\hat{\zeta}^{\beta}_{t+s}+\hat{\zeta}^{n}_{t+s}+\left(1+\sigma_L\right)\hat{H}_{t+s}+\hat{\lambda}^w_{t+s}\right.$$
$$\left..+\frac{\lambda^w\left(1+\sigma_L\right)}{1-\lambda^w}\left(\widehat{\bar{w}}_t-\widehat{\bar{w}}_{t+s}+\widehat{\tilde{\pi}}^w_{t+1}+\ldots+\widehat{\tilde{\pi}}^w_{t+s}-\hat{\pi}^d_{t+1}-\ldots-\hat{\pi}^d_{t+s}-\hat{\mu}_{z^+,t+1}-\ldots-\hat{\mu}_{z^+,t+s}\right)\right].$$

Solving for steady state in equation (15.11) implies $\psi_{z^+}\bar{w}H=\lambda^wA_L\zeta^nH^{1+\sigma_L}$. Using this and rearranging the expressions above (proceeding as in Section 4.8.5) gives the loglinearized version of the wage Phillips curve (15.11):

$$
\begin{aligned}
& \xi_w b_w \widehat{\bar{w}}_{t-1} + \left(\lambda^w \sigma_L - b_w \left(1 + \beta \xi_w^2\right)\right) \widehat{\bar{w}}_t + \beta \xi_w b_w E_t \widehat{\bar{w}}_{t+1} - \xi_w b_w \left(\hat{\pi}_t^d - \widehat{\bar{\pi}}_t^c\right) \\
& + \beta \xi_w b_w E_t \left(\hat{\pi}_{t+1}^d - \widehat{\bar{\pi}}_{t+1}^c\right) + \kappa_w \xi_w b_w \left(\hat{\pi}_{t-1}^c - \widehat{\bar{\pi}}_t^c\right) - \beta \xi_w \kappa_w b_w E_t \left(\hat{\pi}_t^c - \widehat{\bar{\pi}}_{t+1}^c\right) \\
& + \left(1 - \lambda^w\right) \left(\hat{\psi}_{z^+,t} - \hat{\zeta}_t^n - \sigma_L \hat{H}_t - \hat{\lambda}_t^w\right) - \xi_w b_w \hat{\mu}_{z^+,t} + \beta \xi_w b_w E_t \hat{\mu}_{z^+,t+1} \\
= \;& 0, \qquad (15.12)
\end{aligned}
$$

where $b_w = \frac{\lambda^w(1+\sigma_L)-1}{(1-\beta\xi_w)(1-\xi_w)}$. Note that the last row of this equation contains the shock to labour supply and the wage markup shock.[61] Importantly, this is the only equation in the EHL where those two shocks enter, meaning that they are observationally equivalent and cannot both be identified.[62] We leave in both shocks in the final equation, nevertheless, for consistency with the GSW model, noting that one of them needs to be shut off for estimations of the EHL version of the model. The estimated shock can then be interpreted as either a labour supply shock or a wage markup shock – for estimation purposes this does not matter. However, it does matter for normative purposes, as it will affect the model-implied measures of the efficient output gap.[63] An alternative approach, explored in Sala, Söderström, and Trigari (2010), could be to assume that the processes of the two shocks are different, allowing us to potentially identify both shocks. Sala, Söderström, and Trigari (2010) find, however, that the one-shock and two-shock models are virtually identical in their estimations.

15.5 Foreign economy

For the foreign economy, we modify the household's problem and the wage setting in an analogous way to the domestic economy. Household preferences in the foreign economy are now given by

$$
E_0^j \sum_{t=0}^{\infty} \left(\beta^*\right)^t \zeta_t^{\beta,*} \left[\zeta_t^{c,*} \log\left(C_{j,t}^* - b^* C_{j,t-1}^*\right) - \zeta_t^{n,*} A_L^* \frac{\left(h_{j,t}^*\right)^{1+\sigma_L^*}}{1+\sigma_L^*}\right].
$$

The foreign wage setting problem results in the following equation, corresponding to equation (15.10) for the domestic economy:

$$
\left(\tilde{w}_t^*\right)^{\frac{1-\lambda_{t+s}^{w,*}\left(1+\sigma_L^*\right)}{1-\lambda_{t+s}^{w,*}}} = \frac{\lambda_{t+s}^{w,*} A_L^* E_t \sum_{s=0}^{\infty} \left(\beta^* \xi_w^*\right)^s \zeta_{t+s}^{\beta,*} \zeta_{t+s}^{n,*} \left[\left(\frac{\bar{w}_t^*}{\bar{w}_{t+s}^*} \frac{\tilde{\pi}_{t+s}^{w,*} \cdots \tilde{\pi}_{t+1}^{w,*}}{\mu_{z^+,*,t+1} \cdots \mu_{z^+,*,t+s} \pi_{t+1}^{d,*} \cdots \pi_{t+s}^{d,*}}\right)^{\frac{\lambda_{t+s}^{w,*}}{1-\lambda_{t+s}^{w,*}}} H_{t+s}^*\right]^{1+\sigma_L^*}}{E_t \sum_{s=0}^{\infty} \left(\beta^* \xi_w^*\right)^s \zeta_{t+s}^{\beta,*} \psi_{z^+,*,t+s} \bar{w}_{t+s}^* H_{t+s}^* \left(\frac{\bar{w}_t^*}{\bar{w}_{t+s}^*} \frac{\tilde{\pi}_{t+s}^{w,*} \cdots \tilde{\pi}_{t+1}^{w,*}}{\mu_{z^+,*,t+1} \cdots \mu_{z^+,*,t+s} \pi_{t+1}^{d,*} \cdots \pi_{t+s}^{d,*}}\right)^{\frac{1}{1-\lambda_{t+s}^{w,*}}}}. \qquad (15.13)
$$

15.6 Exogenous processes

In the EHL version of the model, the labour supply shock is assumed to evolve according to the following AR process:

$$
\hat{\zeta}_t^h = \rho_{\zeta^h} \hat{\zeta}_{t-1}^h + \sigma_{\zeta^h} \varepsilon_{\zeta^h,t}, \qquad (15.14)
$$

with ρ_{ζ^h} no longer being calibrated to a very high value as in the GSW model, but estimated.

[61] It contains also the discount rate shock, but this shock enters also in the consumption Euler equation from the household's problem.

[62] That the wage markup shock is isomorphic to the preference shock to disutility from labor in the log-linearized version is a standard problem in this class of models.

[63] As discussed in Sala, Söderström, and Trigari (2010), the the labor supply shock is efficient while the wage markup shock is not, as markups are zero in the effcient (flex-price) allocation. The latter shock thus affects the efficient output gap, while the former does not, with different implications for monetary policy.

15.7 Summary of the model with the EHL labour market

Comparing to the GSW model above, the inclusion of the EHL labour market removes the following variables from the model: the labour force, $\hat{L}_t$, employment, $\hat{N}_t$ and $\hat{n}_t$, the unemployment rate, $\hat{U}_t$, the natural rate of unemployment, $\hat{U}_t^n$, the endogenous preference shifter, $\hat{\Theta}_t$, the smooth trend consumption, $\hat{z}_t^C$, the marginal utility of consumption, $\widehat{\overline{\upsilon}}_t^N$, the marginal rate of substitution, $\widehat{mrs}_t$, and the average wage markup, $\hat{\mu}_{w,t}$. We thus remove the following equations: the expressions determining the labour participation rate in equation (4.100), the unemployment rate in equation (4.99), the natural rate of unemployment in equation (4.110), the endogenous preference shifter in equation (4.80), the trend consumption in equation (4.81), the marginal utility of consumption in equation (4.90), the marginal rate of substitution in equation (4.83), and the wage markup in equation (4.101). We instead add hours, $\hat{H}_t$ and $\hat{h}_t$.

We replace the wage setting in equation (13.61) by the wage setting expression in equation (15.12)

$$\tilde{w}_t^{\frac{1-\lambda_{t+s}^w(1+\sigma_L)}{1-\lambda_{t+s}^w}} = \frac{\lambda_{t+s}^w A_L E_t \sum_{s=0}^{\infty} (\beta\xi_w)^s \zeta_{t+s}^{\beta} \zeta_{t+s}^h \left[\left(\frac{\bar{w}_t}{\bar{w}_{t+s}} \frac{\tilde{\pi}_{t+s}^w \cdots \tilde{\pi}_{t+1}^w}{\mu_{z^+,t+1} \cdots \mu_{z^+,t+s} \pi_{t+1}^d \cdots \pi_{t+s}^d} \right)^{\frac{\lambda_{t+s}^w}{1-\lambda_{t+s}^w}} H_{t+s} \right]^{1+\sigma_L}}{E_t \sum_{s=0}^{\infty} (\beta\xi_w)^s \zeta_{t+s}^{\beta} \psi_{z^+,t+s} \bar{w}_{t+s} H_{t+s} \left(\frac{\bar{w}_t}{\bar{w}_{t+s}} \frac{\tilde{\pi}_{t+s}^w \cdots \tilde{\pi}_{t+1}^w}{\mu_{z^+,t+1} \cdots \mu_{z^+,t+s} \pi_{t+1}^d \cdots \pi_{t+s}^d} \right)^{\frac{1}{1-\lambda_{t+s}^w}}}$$

$$\tilde{\pi}_{t+1}^w \equiv (\pi_t^c)^{\kappa_w} \left(\bar{\pi}_{t+1}^c\right)^{1-\kappa_w-\varkappa_w} (\breve{\pi})^{\varkappa_w} (\mu_{z^+})^{\vartheta_w}$$

$$\tilde{w}_t = \left[\frac{1 - \xi_w \left(\frac{\tilde{\pi}_t^w}{\pi_t^w} \right)^{\frac{1}{1-\lambda_t^w}}}{(1-\xi_w)} \right]^{1-\lambda_t^w}$$

$$\begin{aligned} & \xi_w b_w \widehat{\bar{w}}_{t-1} + \left(\lambda^w \sigma_L - b_w \left(1 + \beta\xi_w^2\right)\right) \widehat{\bar{w}}_t + \beta\xi_w b_w E_t \widehat{\bar{w}}_{t+1} - \xi_w b_w \left(\hat{\pi}_t^d - \widehat{\bar{\pi}}_t^c\right) \\ & + \beta\xi_w b_w E_t \left(\hat{\pi}_{t+1}^d - \widehat{\bar{\pi}}_{t+1}^c\right) + \kappa_w \xi_w b_w \left(\hat{\pi}_{t-1}^c - \widehat{\bar{\pi}}_t^c\right) - \beta\xi_w \kappa_w b_w E_t \left(\hat{\pi}_t^c - \widehat{\bar{\pi}}_{t+1}^c\right) \\ & + (1-\lambda^w) \left(\hat{\psi}_{z^+,t} - \hat{\zeta}_t^h - \sigma_L \hat{H}_t - \hat{\lambda}_t^w\right) - \xi_w b_w \hat{\mu}_{z^+,t} + \beta\xi_w b_w E_t \hat{\mu}_{z^+,t+1} \\ = \; & 0, \qquad b_w = \frac{\lambda^w (1+\sigma_L) - 1}{(1-\beta\xi_w)(1-\xi_w)}. \end{aligned} \tag{15.15}$$

Note that hours in EHL replace the employment rate in the GSW model. Hence, the equations including employment in the GSW version are now replaced with corresponding expressions including hours. Going from the GSW to the EHL model, we replace the marginal and products of labour and capital, in equations (13.3) and (13.6), respectively, by

$$mpl_t = (1-\alpha)\, \epsilon_t \left(\frac{k_t}{\mu_{z^+,t} \mu_{\Psi,t} H_t} \right)^{\alpha}$$

$$\widehat{mpl}_t = \alpha \widehat{\left(\frac{k}{H}\right)}_t + \hat{\epsilon}_t \tag{15.16}$$

and

$$mpk_t = \alpha \epsilon_t \left(\frac{k_t}{\mu_{z^+,t} \mu_{\Psi,t} H_t} \right)^{-(1-\alpha)}$$

$$\widehat{mpk}_t = -(1-\alpha) \widehat{\left(\frac{k}{H}\right)}_t + \hat{\epsilon}_t, \tag{15.17}$$

and the capital-to-labour ratio in equation (13.4) by

$$\left(\frac{k}{H}\right)_t = \frac{k_t}{\mu_{z^+,t}\mu_{\Psi,t}H_t}$$

$$\widehat{\left(\frac{k}{H}\right)}_t = \hat{k}_t - \hat{H}_t - \left(\hat{\mu}_{z^+,t} + \hat{\mu}_{\Psi,t}\right). \tag{15.18}$$

We also replace the resource constraint in equation (13.66) by

$$y_t = \left(\mathring{p}_t^d\right)^{\frac{\lambda_t^d}{\lambda_t^d - 1}} \left[\epsilon_t \left(\frac{k_t}{\mu_{\Psi,t}\mu_{z^+,t}}\right)^{\alpha} \left(\mathring{w}_t^{-\frac{\lambda_t^w}{1-\lambda_t^w}} h_t\right)^{1-\alpha} - \phi^d\right]$$

$$\begin{aligned}
\hat{y}_t \quad = \quad & \frac{1}{y}\left(\frac{k}{\mu_\Psi \mu_{z^+}}\right)^{\alpha} h^{1-\alpha} \times \\
& \times \left[\hat{\epsilon}_t + \alpha\left(\hat{k}_t - \hat{\mu}_{\Psi,t} - \hat{\mu}_{z^+,t}\right) - \frac{\lambda^w (1-\alpha)}{1-\lambda^w}\widehat{\mathring{w}}_t + (1-\alpha)\hat{h}_t\right] \\
& - \frac{\lambda^d}{\lambda^d - 1}\widehat{\mathring{p}}_t^d,
\end{aligned} \tag{15.19}$$

and aggregate household labour in terms of aggregate homogeneous labour in equation (13.62) by the expression for aggregate household hours in terms of aggregate homogeneous hours worked

$$h_t = H_t \left(\mathring{w}_t\right)^{\frac{\lambda_t^w}{1-\lambda_t^w}}$$

$$\hat{h}_t = \hat{H}_t + \frac{\lambda^w}{1-\lambda^w}\widehat{\mathring{w}}_t. \tag{15.20}$$

Finally, as unemployment is dropped from the model, the Taylor rule is respecified accordingly. We now assume that monetary policy is conducted according to the following rule:

$$\begin{aligned}
\log\left(\frac{R_t}{R}\right) \quad = \quad & \rho_R \log\left(\frac{R_{t-1}}{R}\right) + (1-\rho_R)\left[\log\left(\frac{\bar{\pi}_t^c}{\bar{\pi}^c}\right) + r_\pi \log\left(\frac{\pi_{t-1}^c}{\bar{\pi}_t^c}\right)\right. \\
& \left. + r_y \log\left(\frac{h_{t-1}}{h}\right) + r_q \log\left(\frac{q_{t-1}}{q}\right)\right] + r_{\Delta\pi}\Delta\log\left(\frac{\pi_t^c}{\pi^c}\right) + r_{\Delta y}\Delta\log\left(\frac{h_t}{h}\right) + \varepsilon_{R,t}
\end{aligned}$$

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1-\rho_R)\left[\widehat{\bar{\pi}}_t^c + r_\pi\left(\hat{\pi}_{t-1}^c - \widehat{\bar{\pi}}_t^c\right) + r_y \hat{h}_{t-1} + r_q \hat{q}_{t-1}\right] + r_{\Delta\pi}\Delta\hat{\pi}_t^c + r_{\Delta y}\Delta\hat{h}_t + \varepsilon_{R,t}, \tag{15.21}$$

instead of the one specified in equation (13.64).

For the foreign economy, we add and replace the same equations as for the domestic. We thus have the wage setting equation

$$\left(\tilde{w}_t^*\right)^{\frac{1-\lambda_{t+s}^{w,*}(1+\sigma_L^*)}{1-\lambda_{t+s}^{w,*}}} = \frac{\lambda_{t+s}^{w,*} A_L^* E_t \sum_{s=0}^{\infty} \left(\beta^* \xi_w^*\right)^s \zeta_{t+s}^{\beta,*}\zeta_{t+s}^{h,*} \left[\left(\frac{\bar{w}_t^*}{\bar{w}_{t+s}^*}\frac{\tilde{\pi}_{t+s}^{w,*}\ldots\tilde{\pi}_{t+1}^{w,*}}{\mu_{z^+,*,t+1}\ldots\mu_{z^+,*,t+s}\pi_{t+1}^*\ldots\pi_{t+s}^*}\right)^{\frac{\lambda_{t+s}^{w,*}}{1-\lambda_{t+s}^{w,*}}} H_{t+s}^*\right]^{1+\sigma_L^*}}{E_t \sum_{s=0}^{\infty} \left(\beta^* \xi_w^*\right)^s \zeta_{t+s}^{\beta,*}\psi_{z^+,*,t+s}\bar{w}_{t+s}^* H_{t+s}^* \left(\frac{\bar{w}_t^*}{\bar{w}_{t+s}^*}\frac{\tilde{\pi}_{t+s}^{w,*}\ldots\tilde{\pi}_{t+1}^{w,*}}{\mu_{z^+,*,t+1}\ldots\mu_{z^+,*,t+s}\pi_{t+1}^*\ldots\pi_{t+s}^*}\right)^{\frac{1}{1-\lambda_{t+s}^{w,*}}}}$$

$$\tilde{\pi}_{t+1}^{w,*} = \left(\pi_t^{c,*}\right)^{\kappa_w^*}\left(\bar{\pi}_{t+1}^*\right)^{1-\kappa_w^*-\varkappa_w^*}\left(\breve{\pi}^*\right)^{\varkappa_w^*}\left(\mu_{z^+,*}\right)^{\vartheta_w^*}$$

$$\tilde{w}_t^* = \left[\frac{1-\xi_w^*\left(\frac{\tilde{\pi}_t^{w,*}}{\pi_t^{w,*}}\right)^{\frac{1}{1-\lambda_{t+s}^{w,*}}}}{\left(1-\xi_w^*\right)}\right]^{1-\lambda_t^{w,*}}$$

$$
\begin{aligned}
&\xi_w^* b_w^* \widehat{\bar{w}}_{t-1}^* + \left(\lambda^{w,*}\sigma_L^* - b_w^*\left(1+\beta^*\xi_w^{*^2}\right)\right)\widehat{\bar{w}}_t^* + \beta^*\xi_w^* b_w^* E_t \widehat{\bar{w}}_{t+1}^* - \xi_w^* b_w^*\left(\hat{\pi}_t^* - \widehat{\bar{\pi}}_t^*\right) \\
&+\beta^*\xi_w^* b_w^* E_t\left(\hat{\pi}_{t+1}^* - \widehat{\bar{\pi}}_{t+1}^*\right) + \kappa_w^*\xi_w^* b_w^*\left(\hat{\pi}_{t-1}^{c,*} - \widehat{\bar{\pi}}_t^*\right) - \beta^*\xi_w^*\kappa_w^* b_w^* E_t\left(\hat{\pi}_t^{c,*} - \widehat{\bar{\pi}}_{t+1}^*\right) \\
&+\left(1-\lambda^{w,*}\right)\left(\hat{\psi}_{z^+,*,t} - \hat{\zeta}_t^{h,*} - \sigma_L^*\hat{H}_t^* - \hat{\lambda}_t^{w,*}\right) - \xi_w^* b_w^*\hat{\mu}_{z^+,*,t} + \beta^*\xi_w^* b_w^* E_t\hat{\mu}_{z^+,*,t+1} \\
= \quad & 0, \qquad b_w^* = \frac{\lambda^{w,*}\left(1+\sigma_L^*\right)-1}{\left(1-\beta^*\xi_w^*\right)\left(1-\xi_w^*\right)},
\end{aligned}
\tag{15.22}
$$

the marginal and products of labour and capital,

$$
\begin{aligned}
mpl_t^* &= \left(1-\alpha^*\right)\epsilon_t^*\left(\frac{k_t^*}{\mu_{z^+,*,t}\mu_{\Psi^*,t}H_t^*}\right)^{\alpha} \\
\widehat{mpl}_t^* &= \alpha^*\widehat{\left(\frac{k}{H}\right)}_t^* + \hat{\epsilon}_t^*,
\end{aligned}
\tag{15.23}
$$

and

$$
\begin{aligned}
mpk_t^* &= \alpha^*\epsilon_t^*\left(\frac{k_t^*}{\mu_{z^+,*,t}\mu_{\Psi^*,t}H_t^*}\right)^{-(1-\alpha^*)} \\
\widehat{mpk}_t^* &= -\left(1-\alpha^*\right)\widehat{\left(\frac{k}{H}\right)}_t^* + \hat{\epsilon}_t^*.
\end{aligned}
\tag{15.24}
$$

the capital-to-labour ratio

$$
\begin{aligned}
\left(\frac{k}{H}\right)_t^* &= \frac{k_t^*}{\mu_{z^+,*,t}\mu_{\Psi^*,t}H_t^*} \\
\widehat{\left(\frac{k}{H}\right)}_t^* &= \hat{k}_t^* - \hat{H}_t^* - \left(\hat{\mu}_{z^+,*,t} + \hat{\mu}_{\Psi^*,t}\right),
\end{aligned}
\tag{15.25}
$$

the resource constraint from the supply side

$$
y_t^* = \left(\mathring{p}_t^*\right)^{\frac{\lambda_t^*}{\lambda_t^*-1}}\left[\epsilon_t^*\left(\frac{k_t^*}{\mu_{z^+,*,t}\mu_{\Psi^*,t}}\right)^{\alpha^*}\left(h_t^*\left(\mathring{w}_t^*\right)^{-\frac{\lambda_t^{w,*}}{1-\lambda_t^{w,*}}}\right)^{1-\alpha^*} - \phi^*\right]
$$

$$
\begin{aligned}
\hat{y}_t^* \quad = \quad & \frac{\lambda^*}{1-\lambda^*}\widehat{\mathring{p}}_t^* + \frac{1}{y^*}\left(\frac{k^*}{\mu_{z^+,*}\mu_{\Psi^*}}\right)^{\alpha^*}\left(h^*\right)^{1-\alpha^*}\times \\
&\times\left[\hat{\epsilon}_t^* + \alpha^*\left(\hat{k}_t^* - \hat{\mu}_{z^+,*,t} - \hat{\mu}_{\Psi^*,t}\right) + \left(1-\alpha^*\right)\left(\hat{h}_t^* - \frac{\lambda^{w,*}}{1-\lambda^{w,*}}\widehat{\mathring{w}}_t^*\right)\right],
\end{aligned}
\tag{15.26}
$$

and aggregate household hours in terms of aggregate homogeneous hours worked

$$
\begin{aligned}
h_t^* &= H_t^*\left(\mathring{w}_t^*\right)^{\frac{\lambda_t^{w,*}}{1-\lambda_t^{w,*}}}, \\
\hat{h}_t^* &= \hat{H}_t^* + \frac{\lambda^{w,*}}{1-\lambda^{w,*}}\widehat{\mathring{w}}_t^*.
\end{aligned}
\tag{15.27}
$$

Finally, we now assume that the foreign monetary policy is conducted according to the following rule:

$$
\begin{aligned}
\log\left(\frac{R_t^*}{R^*}\right) \quad = \quad & \rho_{R^*}\log\left(\frac{R_{t-1}^*}{R^*}\right) + \left(1-\rho_{R^*}\right)\left[\log\left(\frac{\bar{\pi}_t^*}{\bar{\pi}^*}\right) + r_{\pi^*}\log\left(\frac{\pi_{t-1}^{c,*}}{\bar{\pi}_t^*}\right)\right. \\
&\left.+r_{y^*}\log\left(\frac{h_{t-1}^*}{h^*}\right)\right] + r_{\Delta\pi^*}\Delta\log\left(\frac{\pi_t^{c,*}}{\pi^{c,*}}\right) + r_{\Delta y^*}\Delta\log\left(\frac{h_t^*}{h^*}\right) + \varepsilon_{R^*,t}
\end{aligned}
$$

$$
\hat{R}_t^* = \rho_{R^*}\hat{R}_{t-1}^* + \left(1-\rho_{R^*}\right)\left[\widehat{\bar{\pi}}_t^* + r_{\pi^*}\left(\hat{\pi}_{t-1}^{c,*} - \widehat{\bar{\pi}}_t^*\right) + r_{y^*}\hat{h}_{t-1}^*\right] + r_{\Delta\pi^*}\Delta\hat{\pi}_t^{c,*} + r_{\Delta y^*}\Delta\hat{h}_t^* + \varepsilon_{R^*,t}. \tag{15.28}
$$

References

ADOLFSON, M., S. LASÉEN, L. CHRISTIANO, M. TRABANDT, AND K. WALENTIN (2013): “Ramses II – Model Description,” *Sveriges Riksbank Occasional Paper Series*, 12.

ADOLFSON, M., S. LASÉEN, J. LINDÉ, AND M. VILLANI (2005): “Bayesian Estimation of an Open Economy DSGE Model with Incomplete Pass-Through,” *Sveriges Riksbank Working Paper*, 179.

——— (2007): “Bayesian estimation of an open economy DSGE model with incomplete pass-through,” *Journal of International Economics*, 72(2), 481–511.

——— (2008): “Evaluating an estimated new Keynesian small open economy model,” *Journal of Economic Dynamics and Control*, 32, 2690–2721.

BERNANKE, B. S., M. GERTLER, AND S. GILCHRIST (1999): “The financial accelerator in a quantitative business cycle framework,” *Handbook of macroeconomics*, 1, 1341–1393.

CALVO, G. A. (1983): “Staggered prices in a utility-maximizing framework,” *Journal of monetary Economics*, 12(3), 383–398.

CHRISTIANO, L. J., M. EICHENBAUM, AND C. L. EVANS (2005): “Nominal rigidities and the dynamic effects of a shock to monetary policy,” *Journal of Political Economy*, 113(1), 1–45.

CHRISTIANO, L. J., M. TRABANDT, AND K. WALENTIN (2011): “Introducing financial frictions and unemployment into a small open economy model,” *Journal of Economic Dynamics and Control*, 35(12), 1999–2041.

CORBO, V., AND I. STRID (2020): “MAJA: A two-region DSGE model for Sweden and its main trading partners,” *Riksbank Working Paper Series*, 391.

ERCEG, C. J., D. W. HENDERSON, AND A. T. LEVIN (2000): “Optimal monetary policy with staggered wage and price contracts,” *Journal of Monetary Economics*, 46(2), 281–313.

FISHER, J. D. (2015): “On the Structural Interpretation of the Smets–Wouters “Risk Premium” Shock,” *Journal of Money, Credit and Banking*, 47(2-3), 511–516.

GALÍ, J. (2011a): “The return of the wage Phillips curve,” *Journal of the European Economic Association*, 9(3), 436–461.

——— (2011b): *Unemployment fluctuations and stabilization policies: a new Keynesian perspective.* MIT press.

GALÍ, J., F. SMETS, AND R. WOUTERS (2012): “Unemployment in an estimated new keynesian model,” *NBER macroeconomics annual*, 26(1), 329–360.

HOLSTON, K., T. LAUBACH, AND J. WILLIAMS (2016): “Measuring the natural rate of interest: International Trends and Determinants,” *Federal Reserve Bank of San Franscisco, Working Paper*, 11.

KING, R. G., C. I. PLOSSER, AND S. T. REBELO (1988): “Production, growth and business cycles: I. The basic neoclassical model,” *Journal of Monetary Economics*, 21(2-3), 195–232.

LAUBACH, T., AND J. C. WILLIAMS (2015): “Measuring the Natural Rate of Interest Redux,” *Federal Reserve Bank of San Franscisco Working Paper*, 16.

LINDÉ, J., J. MAIH, AND R. WOUTERS (2017): “Estimation of Operational Macromodels at the Zero Lower Bound,” Discussion paper, Mimeo.

MERZ, M. (1995): “Search in the labor market and the real business cycle,” *Journal of Monetary Economics*, 36(2), 269–300.

SALA, L., U. SÖDERSTRÖM, AND A. TRIGARI (2010): “Potential Output, the Output Gap, and the Labor Wedge,” .

Smets, F., and R. Wouters (2003): “An estimated dynamic stochastic general equilibrium model of the euro area,” *Journal of the European Economic Association*, 1(5), 1123–1175.

Smets, F., and R. Wouters (2007): “Shocks and frictions in US business cycles: A Bayesian DSGE approach,” *American Economic Review*, 97(3), 586–606.