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Learning on the Job and the Cost of Business Cycles*

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Abstract

We show that business cycles reduce welfare through a decrease in the average level of employment in a labor market search model with learning on-the-job and skill loss during unemployment. Empirically, unemployment and the job finding rate are negatively correlated. Since new jobs are the product of these two, business cycles imply that fewer news jobs are created and employment falls. Learning on-the-job implies that the decrease in employment reduces aggregate human capital. This reduces the incentives to post vacancies, further decreasing employment and human capital. We quantify this mechanism and find large output and welfare costs of business cycles.

Keywords: Search and matching, labor market, human capital, skill loss, stabilization policy.

JEL classification: E32, J64.

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1 Introduction

A major question in macroeconomics is how large the welfare costs of business cycles are. Since Lucas (1987), it has been well established that the cost of aggregate consumption fluctuations is negligible. Business cycles can induce welfare costs in other ways though, e.g., through their effect on the cross-sectional distribution of consumption (Imrohoroğlu, 1989, and many others). Furthermore, business cycles may affect welfare negatively by reducing the average level of output, a view that has been argued by DeLong and Summers (1989), Hassan and Mertens (2017) and Summers (2015). Another strand of the literature highlights the effect of human capital dynamics on macroeconomic fluctuations, see e.g., Kehoe, Midrigan and Pastorino (2015) and Krebs and Scheffel (2017).

Our paper adds to this literature by presenting a new mechanism that amplifies how business cycles reduce the level of output. We show that business cycles substantially reduce the level of employment, output and welfare in a labor market search model with human capital dynamics. There are two channels through which business cycles reduce employment, and they constitute the initial step in the main mechanism of this paper. The first channel is as follows: Empirically the job finding rate and the unemployment rate are strongly negatively correlated (see e.g. Shimer, 2005). Since new jobs are the product of these two, aggregate volatility implies that fewer new jobs are created and employment decreases, all else equal. At an intuitive level, this happens because the job finding rate in general is high when unemployment is low and vice versa. The second channel is specific to the search and matching framework and works through the job finding rate. Specifically, given some weak parameter restrictions, the job finding rate is a concave function of TFP in the textbook version of this type of model, which implies that business cycles reduce the average job finding rate and, in turn, further reduce employment.[1] In other words, worker congestion increase in booms, in the sense that the increase in the job finding rate slows down as TFP increases. In appendix A.1, we formally derive sufficient conditions for when business cycles reduce employment in the stylized model. In settings with learning on-the-job and skill loss during unemployment, any resulting fall in employment from these two initial channels implies that average human capital falls. This, in turn, reduces the incentives to post vacancies, further reducing employment and so on in a vicious circle, thereby amplifying the initial impact of aggregate volatility on employment. Thus, aggregate volatility substantially reduces employment, human capital and output. This process, including the amplification mechanism, is illustrated graphically in Figure 1. The size of the cost of business cycles generated by this mechanism

[1] More generally, any convex cost (or concave benefit or production function) in any cyclical variable tends to induce a negative relationship between aggregate volatility and average consumption or employment. Prominent examples are convex capital adjustment costs and convex vacancy posting costs, both of which are commonly used in the business cycle literature.

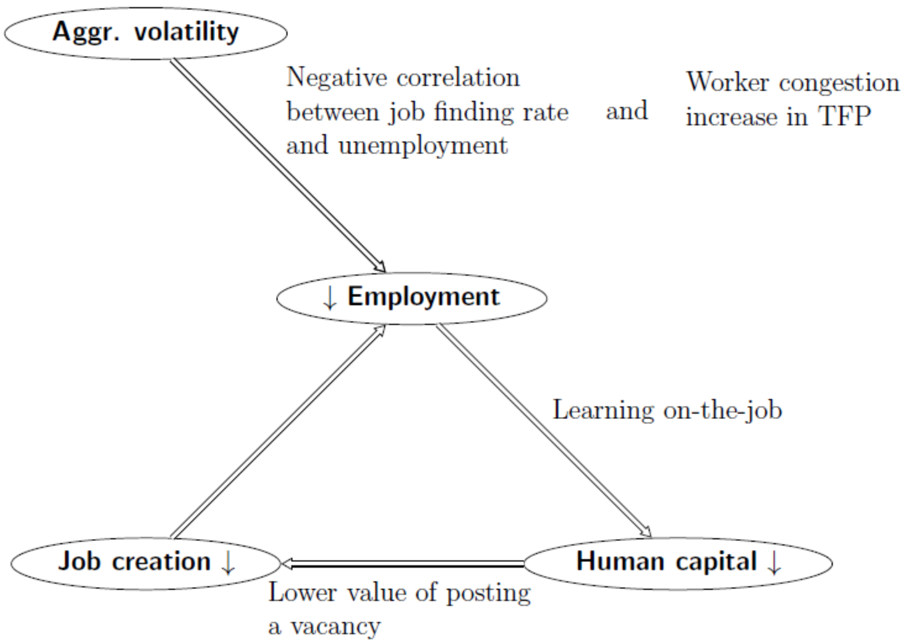

Figure 1: Illustration of main mechanism - how aggregate volatility reduces employment, human capital and thereby output.

is accordingly largely determined by how sensitive the human capital distribution is to changes in employment and how sensitive job creation is to changes in the human capital distribution. Since our mechanism works through the average level of consumption, it is fundamentally different from most of the cost of business cycles literature, which analyses the effects of business cycles on welfare through (aggregate or idiosyncratic) consumption volatility. Our amplification mechanism also extends beyond the cost of business cycles. For example, the effect of a change in taxation or unemployment benefits that affects average employment will over time be amplified by the human capital mechanism that we have outlined.

We capture the mechanism described above that relates business cycles and the average level of output using a search and matching framework with general human capital dynamics (learning on-the-job and skill loss during unemployment). As argued above, an important determinant of the size of the cost of business cycles is how sensitive job creation is to changes in the human capital distribution of both unemployed and employed workers. Thus, we allow for on-the-job search to capture the effect of employed workers' human capital on job creation. In addition, to allow for a flexible bargaining framework in a context with on-the-job search, we use the bargaining protocol from Cahuc, Postel-Vinay and Robin (2006), henceforth CPVR. In this framework workers can have positive bargaining power and receive the value of their outside option plus a share of the value of the match above the outside option. To allow for a positive bargaining power of workers is important since the level of bargaining power can have substantial effects on welfare in search and matching models. We are not aware of any previous model that uses the bargaining framework of CPVR in a setting with aggregate uncertainty using global solution methods. In this paper, we propose and implement an algorithm for solving models where workers with positive bargaining power that can search on-the-job meet firms with different levels of productivity. Thus, the paper also makes a methodological contribution. In our mind, our solution algorithm is useful for future research where heterogeneity in the labor market interacts with the business cycle.

The main purpose of our exercise is to provide a credible quantification of the cost of business cycles through the mechanism we have sketched above. One key determinant of this cost is the speed of human capital accumulation when employed compared to the skill loss during unemployment. We estimate the human capital gains when employed by matching the empirical "return to experience" (wage profile of employed workers) reported by Buchinsky et al. (2010). The model is calibrated by matching the return to experience and other relevant moments, including volatility of GDP and unemployment, standard worker flow moments and the degree of wage dispersion. We then compute the cost of business cycles by comparing the equilibrium for our full model to the equilibrium from the same model, but without aggregate volatility. We find that business cycles reduce steady state employment, GDP and welfare by substantial amounts. In particular, eliminating aggregate volatility increases welfare (GDP) by 0.70-1.68 percent (1.55 percent), depending on the interpretation of the flow value of unemployment. These are fairly large effects, relative to the cost of aggregate consumption volatility as in, e.g., Lucas (1987). Accounting for the transition dynamics, the welfare gains of eliminating business cycles are somewhat smaller, 0.37-1.28 percent. Human capital dynamics are pivotal for the results - if we disable them in our model, the implied employment, GDP and, in particular, welfare losses from business cycles are substantially smaller. Note that, since we assume risk neutral agents and hence abstract from, e.g., the direct welfare costs of consumption volatility, we do not capture the full welfare cost of business cycles and our results can accordingly be interpreted as a lower bound for these costs.

An important fact regarding the unemployment rate is that it varies across workers, where workers with low human capital tend to have higher unemployment rates. In our model, we are able to capture this fact using heterogeneity in match-specific productivity, which induces lower job finding rates and higher separation rates for workers with low human capital. This tends to worsen the composition of the unemployment pool and implies that a worsening of the human capital distribution has strong effects on job creation. Specifically, workers with low human capital can meet firms whose match productivity imply a negative surplus. In addition, matches that are formed when the worker has low human capital, face an elevated risk of separating in the next downturn. Hence, expected surplus (over match productivity) when hiring workers with low human capital is low. Furthermore, for workers with a higher level of human capital, fewer meetings have negative surplus and future separation rates are lower. Since the value of a new match depends on the human capital of workers, a reduction in the human capital among the unemployed have large effects on the incentives for firms to post vacancies. This leads to substantial effects on job creation, unemployment and welfare. In models with learning on-the-job but without match-specific productivity, a worsening of the human capital distribution has no effect on the job finding rates and job separations through variations in these across human capital, leading to substantially smaller effects on job creation, unemployment and welfare. Specifically, using a textbook search and matching model, Jung and Kuester (2011) find effects that are an order of magnitude smaller than in our paper.[2]

There is indicative empirical support for the relationship between aggregate volatility, unemployment and output implied by our model. Hairault et al. (2010) uses data for 20 OECD countries for the period 1982-2003 and find significant positive effects of TFP volatility on average unemployment. There is also ample evidence of a significant negative relationship between volatility of output and the average growth rate of output, see e.g., Ramey and Ramey (1995) and Luo et al. (2019). Direct evidence of human capital dynamics, in the form of effects on measurable skills, is documented by Edin and Gustavsson (2008). They find sizeable skill loss effects of unemployment. Additional indirect evidence is provided by Schmieder, von Wachter and Bender (2016) who estimate a substantial casual effect on the re-employment wage of an additional month of unemployment, also indicating considerable loss of human capital. There is also evidence that labor market conditions affect the future "employability" of workers. Yagan (2019) establishes a strong link between local shocks to employment growth during the Great Recession, 2007-2009, and the 2015 employment rates of workers exposed to these shocks and argues that this link is due to depreciation of general human capital during non-employment spells.

There are a number of papers analyzing related issues in a search and matching labor-market setting. Dupraz, Nakamura and Steinsson (2019) use a model with downward nominal wage rigidities to analyze the effects of varying the inflation target on unemployment, output and welfare in a business cycle setting. The effects of business cycles on average unemployment and output can be large if the inflation target is low, due to the inability of real wages to fall and thereby clear the market in response to contractionary shocks. Den Haan and Sedlacek (2014) quantify the cost of business cycles

[2]Hairault et al. (2010) also analyze a model with neither learning on-the-job nor match-specific productivity and find effects substantially smaller than ours.

in a setting where an agency problem generates inefficient job separations in downturns, thereby reducing average employment and GDP. Our framework does not include any such agency problem and is bilaterally efficient. Furthermore, our model shares mechanisms with a number of papers that analyze earnings losses from job displacement (Burdett, Carrillo-Tudela and Coles, 2020, Huckfeldt, 2016, Jarosch, 2015, Jung and Kuhn, 2019, and Krolikowski, 2017). Finally, Laureys (2014) analyzes the effects of skill loss in a business cycle setting using a linear framework.

The paper is outlined as follows. Section 2 presents the model, Section 3 documents the calibration and Section 4 provides the quantitative results. Finally, Section 5 concludes.

2 Model

We set up a business cycle model with a search and matching labor market and human capital dynamics. We allow for on-the-job search to capture the direct effect of employed workers' human capital on vacancy postings. The basic building blocks of our model are similar to Lise and Robin (2017), henceforth LR, except for the wage bargaining where we follow CPVR.[3] This wage setting framework implies that workers get the value of their outside option plus a share β, reflecting their bargaining strength, of the value of the match above the outside option. When a worker is hired out of unemployment the outside option is the value of unemployment. If instead an employed worker receives a poaching offer from another firm, the outside option is the value of the second-best match.

In terms of human capital dynamics, the model is in the tradition of Pissarides (1992) and Ljungqvist and Sargent (1998). As in these papers, we model general human capital as stemming from learning on-the-job and skill loss during unemployment. Worker human capital, denoted by x, follows a stochastic process and $\pi_{xe}(x, x')$ $(\pi_{xu}(x, x'))$ denote the Markov transition probability for the worker's human capital level while employed (unemployed).[4] Firm match-specific productivity is denoted by y.

To summarize the above aspects of our model, in any time period there is heterogeneity across employed workers in terms of human capital x, match-specific productivity y and wage w. Unemployed workers only differ in terms of their human capital.

[3] Compared to LR, the features we add are i) positive bargaining power of workers, and ii) learning on-the-job as well as skill loss during unemployment. A simplification compared to LR is that in our model the match-specific productivity y of a match is not known when a vacancy is posted.

[4] Our human capital dynamics differ slightly from Ljungqvist and Sargent (1998, 2008) and Jung and Kuester's (2011) extension with human capital in that we do not assume a sudden loss of general human capital when a worker separates from a job. These papers abstract from heterogeneity in match-specific productivity and therefore assume, as a short-cut, that part of the human capital loss occurs when a worker is separated from a job. This reduces the dependence of the human capital distribution on employment (or any endogenous variable in the model), especially if one only allows for exogenous separations.

Utility is linear in consumption and there is no physical capital. Each firm employs (at most) one worker, and output from a match is $p(x, y, z) = xyz$ where z is an aggregate TFP shock with Markov transition probability $\pi(z, z')$. Note that the assumption of risk neutral agents implies that we abstract from, e.g., the direct welfare costs of consumption volatility. Thus, we do not capture the full welfare cost of business cycles and our results only reflect one of several factors affecting these costs.

2.1 Timing

Let us start the detailed model description by providing an overview of the timing protocol. The sequence of events within a period are as follows. First, the aggregate productivity shock z and the idiosyncratic human capital shocks x are realized. Second, a fraction ν of workers die and are replaced by newborn unemployed workers with human capital at the lowest possible level, $\underline{x}$, as in Ljungqvist and Sargent (1998). Third, separations into unemployment occur. Then, firms post vacancies and workers search for jobs. Finally, new matches are formed, wages are set and production takes place.

2.2 Separations

The ability of recently separated workers to search for jobs within the period, makes it convenient to define match values and match surplus both before and after the search phase has occurred, i.e., at the separation stage and the matching stage. The surplus of a match at the separation stage is $S^s(x, y, z, \Gamma)$ where Γ denotes the endogenous aggregate state. Matches with $S^s(x, y, z, \Gamma) < 0$ are endogenously dissolved. In addition, a fraction δ of matches are exogenously destroyed every period.

The stock of unemployed workers after separations when the aggregate productivity evolves from z_{-1} to z is:

$$u^s(x, z) = \nu \mathbf{1}\{x = \underline{x}\} + (1 - \nu)\left[\sum_{x_{-1} \in X} u(x_{-1}, z_{-1})\, \pi_{xu}(x_{-1}, x) \right. \tag{1}$$

$$\left. + \sum_{y \in Y} \sum_{x_{-1} \in X} \left(\mathbf{1}\{S^s(x, y, z, \Gamma) < 0\} + \delta \mathbf{1}\{S^s(x, y, z, \Gamma) \geq 0\}\right) h(x_{-1}, y, z_{-1})\, \pi_{xe}(x_{-1}, x)\right]$$

where $\mathbf{1}\{\}$ is the indicator function, u (h) is the distribution of unemployed (employed) workers at the end of a period, X is the set of human capital states and Y is the set of match-specific productivities. Here, the first term is the newborn workers and the remaining terms captures the evolution of the surviving workers.

The stock of matches of type (x, y) at this point is:

$$h^s(x, y, z) = (1-\delta)(1-\nu) \sum_{x_{-1} \in X} \mathbf{1}\{S^s(x, y, z, \Gamma) \geq 0\} h(x_{-1}, y, z_{-1}) \pi_{xe}(x_{-1}, x). \quad (2)$$

2.3 Search and matching

An employed worker exerts search effort s_1. The search effort of unemployed workers is normalized to unity. Accordingly, the aggregate amount of search effort is:

$$L \equiv \sum_{x \in X} u^s(x, z) + s_1 \sum_{x \in X} \sum_{y \in Y} h^s(x, y, z). \quad (3)$$

Vacancy posting costs are linear and each vacancy posted incurs a cost of c_0. The free entry condition for vacancy creation therefore implies:

$$c_0 = qJ(z, \Gamma) \quad (4)$$

where q is the probability of a firm meeting a worker and J is the expected value of a new match for a firm, as defined below. Note that the match-specific productivity, y, is observed when the firm meets a worker after the vacancy has been posted.[5]

We assume the following Cobb-Douglas meeting function:

$$M \equiv \min\left\{\alpha L^{\omega} V^{1-\omega}, L, V\right\} \quad (5)$$

where V is the number of vacancies posted and ω is the matching function elasticity. The probability of a firm meeting a worker (assuming an interior solution) is:

$$q = \frac{M}{V} = \alpha \theta^{-\omega},$$

where $\theta \equiv \frac{V}{L}$ is labor market tightness. Together with the matching function (5), this implies that equilibrium vacancy postings are determined by:

$$V = L\left(\frac{\alpha J(z, \Gamma)}{c_0}\right)^{\frac{1}{\omega}}. \quad (6)$$

[5] This assumption substantially simplifies the computation of the equilibrium.

We can then write labor market tightness as a function of z and Γ:

$$\theta(z,\Gamma) = \left(\frac{\alpha J(z,\Gamma)}{c_0}\right)^{\frac{1}{\omega}}. \tag{7}$$

Finally, the probability that an unemployed worker meets a firm (the job meeting rate) is, assuming an interior solution:

$$f(z,\Gamma) = \frac{M}{L} = \alpha\theta(z,\Gamma)^{1-\omega}. \tag{8}$$

2.4 Values

A worker who is unemployed during the production phase receives a flow payoff of $b(x,z)$ representing unemployment insurance, utility of leisure and value of home production.[6] The value of unemployment at the matching stage is:

$$\begin{aligned} B(x,z,\Gamma) &= b(x,z) \\ &+\frac{1-\nu}{1+r}\sum_{x'\in X}\sum_{z'\in Z}[\sum_{y'\in Y} f(z',\Gamma')\left[B(x',z',\Gamma') + \beta\max\left\{S(x',y',z',\Gamma'),0\right\}\right] g(y') \\ &+\left(1-f(z',\Gamma')\right) B(x',z',\Gamma')] \times \pi_{xu}(x,x')\,\pi(z,z'), \end{aligned} \tag{9}$$

where r is the discount rate, Z is the set of aggregate productivity states, β is the bargaining strength of workers, S the surplus of a match (defined below) and $g(y)$ is the probability density function (pdf) of the productivity of newly created matches. Thus, B is the flow payoff b plus the job meeting rate $f(z',\Gamma')$ times the discounted value of a job tomorrow plus $(1-f(z',\Gamma'))$ times the discounted value of being unemployed tomorrow. The max operator ensures that only matches with positive surplus are formed. Note that while a worker is unemployed his human capital (weakly) decreases from x to x' with probability $\pi_{xu}(x,x')$.

The match value at the matching stage, using that the job meeting rate for employed workers is $s_1 f(z',\Gamma')$, can be written as follows:

$$\begin{aligned} P(x,y,z,\Gamma) &= p(x,y,z) + \frac{1-\nu}{1+r}\sum_{x'\in X}\sum_{z'\in Z}[(1-(1-\delta)I_{S\geq 0}) B^s(x',z',\Gamma') + (1-\delta) I_{S\geq 0} \\ &\times\{\sum_{\tilde{y}'\in Y} s_1 f(z',\Gamma')\left\{P(x',y,z',\Gamma') + \beta\max\left[P(x',\tilde{y}',z',\Gamma') - P(x',y,z',\Gamma'),0\right]\right\} g(\tilde{y}') \\ &+\left(1-s_1 f(z',\Gamma')\right) P(x',y,z',\Gamma')\}]\pi_{xe}(x,x')\,\pi(z,z') \end{aligned} \tag{10}$$

where $\tilde{y}'$ denotes the match quality of the poaching firm and where the indicator for non-separation

[6] Unemployment insurance is financed by lump-sum taxation on all workers.

is:

$$I_{S\geq 0}=\mathbf{1}\left\{S^{s}\left(x',y,z',\Gamma'\right)\geq 0\right\}.$$

Here, B^s is the value when unemployed and S^s is the surplus of the match at the separation stage as defined below. The first term in (10) is the flow output, the second term the value when the match separates tomorrow, the third term the value when receiving a poaching offer tomorrow and the last term the value when not receiving a poaching offer tomorrow. Also note that, regardless of what happens tomorrow, human capital while employed today increases from x to x' with probability $\pi_{xe}(x,x')$. Then, in term of surplus of a match, i.e., $S=P-B$,

$$S(x,y,z,\Gamma)=p(x,y,z)-b(x,z)+\frac{1-\nu}{1+r}\sum_{x'\in X}\sum_{z'\in Z}(1-\delta)\,I_{S\geq 0}S^{s}\left(x',y,z',\Gamma'\right)\pi_{xe}\left(x,x'\right)\pi\left(z,z'\right)$$
$$+\frac{1-\nu}{1+r}\sum_{x'\in X}\sum_{z'\in Z}B^{s}\left(x',z',\Gamma'\right)\left(\pi_{xe}\left(x,x'\right)-\pi_{xu}\left(x,x'\right)\right)\pi\left(z,z'\right). \tag{11}$$

Since we allow for a positive bargaining power of workers, the values at the separation stage differ from the values at the matching stage. In particular, at the separation stage, the value of search includes the share of the surplus received when hired at the matching stage. Accordingly, the value for an unemployed worker at the separation stage is:

$$\begin{aligned}B^{s}(x,z,\Gamma) &= (1-f(z,\Gamma))\,B(x,z,\Gamma) \\ &\quad+\sum_{\tilde{y}\in Y}f(z,\Gamma)\left[B(x,z,\Gamma)+\beta\max\left\{S(x,\tilde{y},z,\Gamma),0\right\}\right]g(\tilde{y}).\end{aligned} \tag{12}$$

The corresponding surplus of a match at the separation stage is:

$$S^{s}(x,y,z,\Gamma)=S(x,y,z,\Gamma)+\sum_{\tilde{y}\in Y}s_{1}f(z,\Gamma)\left[\beta\max\left\{S(x,\tilde{y},z,\Gamma)-S(x,y,z,\Gamma),0\right\}\right]g(\tilde{y}) \tag{13}$$
$$-\sum_{\tilde{y}\in Y}f(z,\Gamma)\left[\beta\max\left\{S(x,\tilde{y},z,\Gamma),0\right\}\right]g(\tilde{y}).$$

Recalling that workers receive a value corresponding to their outside option plus a share β of the surplus of the match, the expected value of a new match for a firm is:

$$\begin{aligned}J(z,\Gamma) &= \frac{1}{L}\sum_{x\in X}\sum_{y\in Y}u^{s}(x,z)\max\left\{(1-\beta)\,S(x,y,z,\Gamma),0\right\}g(y) \\ &\quad+\frac{1}{L}\sum_{x\in X}\sum_{y\in Y}\sum_{\tilde{y}\in Y}s_{1}h^{s}(x,\tilde{y},z)\max\left\{(1-\beta)\left(S(x,y,z,\Gamma)-S(x,\tilde{y},z,\Gamma)\right),0\right\}g(y).\end{aligned} \tag{14}$$

The first term in (14) refers to expected surplus from recruiting out of the pool of unemployed (u^s), and the second term refers to expected surplus from recruiting from the pool of employed workers (h^s).

In the classical search and matching model, an increase in (steady state) employment decreases the vacancy filling rate through the matching function and hence reduces vacancy posting. The same applies here; see (4). In our model, as can be seen from (14), there are two additional channels affecting job creation. First, an increase in employment leads to a larger fraction of new hires coming from other firms. For a given level of worker human capital, the surplus to the firm of poaching workers from other firms is lower than from hiring unemployed workers, and hence this mechanism also reduces the incentives to post vacancies. Second, and counteracting the first two effects, a higher employment level increases average human capital among both pools of workers the firms hires from, which leads to stronger incentives for vacancy posting. This last effect is the amplification mechanism sketched in Figure 1.

Let us here mention a computational aspect of the model. Solving the model is non-trivial because the surplus (11) depend on the probability of the worker receiving a job offer the next period. This, in turn, depends on the next period's labor market tightness. According to (7) next period's tightness is fully determined by the expected value of a new match to a firm in the next period, i.e., $J(z', \Gamma')$. As can be seen from (14), this depends on the distribution of unemployed workers across human capital and the distribution of matches over human capital and match-specific productivity. Hence, the endogenous aggregate state Γ can be written as a function of L and the two terms within the summations in (14). Thus, three moments fully capture the implications of this large-dimensional object. We then use a Krusell and Smith (1998)-like algorithm to let these three moments summarize and predict the labor market tightness, thereby enabling us to solve the model. For details on the solution algorithm, see Appendix A.3. In Appendix A.3.4, we in addition document the numerical accuracy of our algorithm and its implementation.

2.5 Distributional dynamics

For a new match to be formed, two conditions are required: the two parties must meet according to the meeting function (5) and the match must be an improvement over the status quo (the current match or unemployment). The unemployment distribution after matching accordingly is:

$$u(x,z) = u^s(x,z)\left(1 - \frac{M}{L}\sum_{y\in Y}\mathbf{1}\{S(x,y,z,\Gamma)\geq 0\}\,g(y)\right). \tag{15}$$

The corresponding expression for the distribution of matches is:

$$\begin{aligned} h(x,y,z) &= h^s(x,y,z) + \underbrace{u^s(x,z)\frac{M}{L}\mathbf{1}\{S(x,y,z,\Gamma)\geq 0\}g(y)}_{\text{mass hired from unemployment}} \\ &\quad \underbrace{-h^s(x,y,z)\,s_1\frac{M}{L}\sum_{\tilde{y}\in Y}\mathbf{1}\{S(x,\tilde{y},z,\Gamma)>S(x,y,z,\Gamma)\}g(\tilde{y})}_{\text{mass lost to more productive matches}} \\ &\quad \underbrace{+s_1\frac{M}{L}\sum_{\tilde{y}\in Y}h^s(x,\tilde{y},z)\mathbf{1}\{S(x,y,z,\Gamma)>S(x,\tilde{y},z,\Gamma)\}g(y)}_{\text{mass poached from less productive matches}}. \end{aligned} \tag{16}$$

Note that, from the term related to hiring from unemployment, the job finding rates tend to be higher for workers with higher human capital, x, as they tend to be employable (i.e., $S(x,y,z,\Gamma)\geq 0$) by a larger fraction of the potential employers. This is in line with the empirical evidence in Morchio (2020).

2.6 Wage determination and worker values

Let $W(w,x,y,z,\Gamma)$ denote the present value to a worker with human capital x in a match with productivity y, wage w and aggregate productivity z. These worker values are determined according to the bargaining protocol in CPVR and are detailed as follows. Denote the renegotiated wage by w'. Workers hired out of unemployment receive the wage w' such that their value is equal to the value of unemployment plus a share β of the match surplus:

$$W\left(w',x,y,z,\Gamma\right) = B(x,z,\Gamma) + \beta S(x,y,z,\Gamma). \tag{17}$$

For employed workers who have received a poaching offer, the bargaining protocol implies that these workers receive a present value $W(w',x,y,z,\Gamma)$ equal to the value of the second-best match that they have encountered during a spell of continuous employment plus a share β of the difference in surplus between the best and second-best match. Formally, if a worker of type x employed at a firm of type y meets a firm of type $\tilde{y}$ then, if $S(x,y,z,\Gamma)<S(x,\tilde{y},z,\Gamma)$, the worker switches to the new firm and gets the wage w' satisfying

$$W\left(w',x,\tilde{y},z,\Gamma\right) = P(x,y,z,\Gamma) + \beta\left[S(x,\tilde{y},z,\Gamma) - S(x,y,z,\Gamma)\right]. \tag{18}$$

If, instead, $S(x,y,z,\Gamma)\geq S(x,\tilde{y},z,\Gamma)$, the worker remains in his current match and gets a wage

w' that satisfies:

$$W\left(w', x, y, z, \Gamma\right) = \max\left\{P\left(x, \tilde{y}, z, \Gamma\right) + \beta\left[S\left(x, y, z, \Gamma\right) - S\left(x, \tilde{y}, z, \Gamma\right)\right], W\left(w, x, y, z, \Gamma\right)\right\}. \tag{19}$$

Note that, in case the value at the current wage is higher than the one implied by the outside option, the wage is unchanged.

Wages for workers who do not receive poaching offers can also be rebargained, as aggregate or idiosyncratic shocks might affect the various values. First, if the wage is such that it implies a worker value that is larger than the match value, then the match would break down unless there is renegotiation. Hence, the wage is then set so that $W\left(w', x, y, z, \Gamma\right) = P\left(x, y, z, \Gamma\right)$. Second, if the wage is such that the worker value is lower than $B\left(x, z, \Gamma\right) + \beta S\left(x, y, z, \Gamma\right)$, the worker can ask for a renegotiation with unemployment as the outside option. Hence, the wage is then set so that $W\left(w', x, y, z, \Gamma\right) = B\left(x, z, \Gamma\right) + \beta S\left(x, y, z, \Gamma\right)$. Finally, the current wage w is unchanged when the value W is in the bargaining set:

$$B\left(x, z, \Gamma\right) + \beta S\left(x, y, z, \Gamma\right) \leqslant W\left(w, x, y, z, \Gamma\right) \leqslant P\left(x, y, z, \Gamma\right). \tag{20}$$

To solve for wages, we compute the value for a worker earning w today, given that future values are (partially) determined by (17)-(20). An employed worker earning the wage w in the current period faces four possibilities in the next period: i) staying employed and not meeting any new firm, ii) staying employed and receiving a successful poaching offer and switching jobs, iii) staying employed and receiving an unsuccessful poaching offer (and staying in the old job) and iv) separating. Note that, if the worker becomes separated in the next period the worker still has a chance to find a new job within the period. Imposing an interior solution for M, $M = \alpha L^{\omega} V^{1-\omega}$ and using the definition of q, the probability of meeting a new firm for an employed worker is $s_1 f\left(z', \Gamma'\right)$. Then, given the wage, w, the worker value (at the matching stage) is:

$$\begin{aligned} W\left(w, x, y, z, \Gamma\right) = w + \frac{1-\nu}{1+r} \sum_{x' \in X} \sum_{z' \in Z} [&\left(1 - s'\right) \left\{\left(1 - s_1 f\left(z', \Gamma'\right)\right) W'_{np}\right. \\ &+ s_1 f\left(z', \Gamma'\right) \sum_{\tilde{y} \in Y} \left(I_{\tilde{y} > y} W'_{p, \tilde{y} > y} + \left(1 - I_{\tilde{y} > y}\right) W'_{p, \tilde{y} \leq y}\right) g\left(\tilde{y}\right)\} \\ &+ s' \left(B\left(x', z', \Gamma'\right) + f\left(z', \Gamma'\right) \sum_{y' \in Y} \beta S\left(x', y', z', \Gamma'\right) g\left(y'\right)\right)] \pi_{xe}\left(x, x'\right) \pi\left(z, z'\right), \end{aligned} \tag{21}$$

where

$$
\begin{aligned}
s' &= \left(\mathbf{1}\left\{S\left(x', y, z'\right) < 0\right\} + \delta \mathbf{1}\left\{S\left(x', y, z', \Gamma'\right) \geq 0\right\}\right) \\
W'_{np} &= \min\left\{P\left(x', y, z', \Gamma'\right), \max\left\{W\left(w, x', y, z', \Gamma'\right), B\left(x', z', \Gamma'\right) + \beta S\left(x', y, z', \Gamma'\right)\right\}\right\} \\
I_{\tilde{y}>y} &= \mathbf{1}\left\{S\left(x', \tilde{y}, z', \Gamma'\right) > S\left(x', y, z', \Gamma'\right)\right\} \\
W'_{p,\tilde{y}>y} &= P\left(x', y, z', \Gamma'\right) + \beta\left[S\left(x', \tilde{y}, z', \Gamma'\right) - S\left(x', y, z', \Gamma'\right)\right] \\
W'_{p,\tilde{y}\leq y} &= \max\left\{P\left(x', \tilde{y}, z', \Gamma'\right) + \beta\left[S\left(x', y, z', \Gamma'\right) - S\left(x', \tilde{y}, z', \Gamma'\right)\right], W\left(w, x', y, z', \Gamma'\right)\right\},
\end{aligned}
$$

where s' denotes separations, W'_{np} the value when not receiving a poaching offer, $I_{\tilde{y}>y}$ a successful poaching offer, $W'_{p,\tilde{y}>y}$ the value of a successful poaching offer and $W'_{p,\tilde{y}\leq y}$ the value of an unsuccessful poaching offer.

2.7 Wage distribution

When determining the wage distribution, it follows from the description of the wage setting above that the current wage of the worker is a state variable. The distribution of matches over w, x and y after separations is:

$$
h^{s,w}\left(w, x, y, z\right) = \left(1-\delta\right)\left(1-\nu\right)\sum_{x_{-1}\in X}\mathbf{1}\left\{S^{s}\left(x, y, z, \Gamma\right) \geq 0\right\} h^{w}\left(w, x_{-1}, y, z_{-1}\right)\pi_{xe}\left(x_{-1}, x\right). \quad (22)
$$

Analogously to (16) in section 2.5, we define $h^{w}\left(w, x, y, z\right)$, i.e., the distribution after matching and wage rebargaining; see Appendix A.2.

3 Calibration

3.1 Distributions and shock processes

The log of the exogenous part of TFP, z, follows an AR(1) process approximated by a Markov chain. The log of match productivity, $g\left(y\right)$, is normally distributed and its mean value is normalized to 0.5. The number of gridpoints for x, y and z are 10, 8 and 5, respectively.[7] The wage grid contains 15 points and is chosen separately for each parameter vector so as to only cover the relevant wage interval.[8] In constructing the grid for human capital, x, we, as e.g., Jarosch (2015), follow Ljungqvist and Sargent (1998, 2008) in using an equal-spaced grid and in setting the ratio between the maximum

[7] For z, we use Tauchen and Hussey's (1991) discretization of AR(1) processes with optimal weights from Flodén (2008). This algorithm has been shown by Flodén (2008) to also be accurate for processes with high persistence.

[8] The coarseness of the wage grid is less restrictive than it seems, as we map each "off-the-grid" wage to the two nearest grid points using the inverse of the distance to the grid point as weight. Furthermore, the wage grid has no impact on the allocations in the model.

and minimum value of x to 2.[9] The structure of the transition matrices $\pi_{xe}(x, x')$ and $\pi_{xu}(x, x')$ for human capital also closely follows Ljungqvist and Sargent. Abstracting from the bounds, the probability of an employed worker to increase his/her human capital by one gridpoint is x_{up} and the probability for an unemployed worker to decrease his/her human capital by one gridpoint is x_{dn}. With the reciprocal probabilities, the human capital of a worker is unchanged. Note that there is very little direct evidence on the shape of human capital dynamics. However, Edin and Gustavsson (2008) find that skill loss appears to be linear in time out-of-work, in line with the assumption above.

3.2 Calibration approach

The frequency of the model is monthly. We calibrate the model based on U.S. data. Parameters whose values are well established in the literature or can be set based on model-independent empirical evidence are set outside the model. Table 1 documents these parameter values and their sources.

Table 1: Parameters set outside the model

	Explanation	Value	Source
ω	Matching function elasticity	0.5	Pissarides (2009)
δ	Exogenous match separation rate	0.030	Fujita-Ramey (2009)
c_0	Vacancy posting cost	0.06375	Fujita-Ramey (2012)
ν	Retirement rate	$1/(40 * 12)$	40-year work-life
ρ	TFP shock persistence	0.960	Hagedorn-Manovskii
r	Interest rate	$1.05^{1/12} - 1$	Annual r of 5%

The meeting function elasticity, ω, is set in line with the convention in the literature. The exogenous match separation rate, δ, is set equal to the mean E2U transition rate reported by Fujita and Ramey (2009), adjusted for workers finding a new job the same month as they lost the old job.[10] This adjustment implies that the separation rate exceeds the E2U rate by a factor of 1/(1-job finding rate). By using Fujita and Ramey's number for E2U transitions, which is 0.020, we control for the fact that empirically, but not in our model, workers flow in and out of the labor force. We set the vacancy posting cost c_0 along the lines for Fujita and Ramey (2012) who refer to evidence that vacancy costs are 6.7 hours per week posted.[11] We set the retirement (or death) rate to match an average work-life of 40 years, as e.g. Huckfeldt (2016). To compute the persistence of the AR process for TFP, we follow along the lines of Hagedorn and Manovskii (2008). Specifically, we simulate a monthly Markov chain to match a quarterly autocorrelation of (HP-filtered) log labor productivity of 0.765. Finally,

[9] The range of x-values is between 0.5 and 1. We explore a wider support for the values of x in a robustness exercise documented in section 4.3.3.

[10] This calibration approach for δ assumes that the average endogenous separation rate in our model is negligible. We confirm this ex post - it is merely 0.0036 at the monthly frequency, i.e., 10% of the total separation rate.

[11] Fujita-Ramey note that 6.7 hours per week is equivalent to 0.17 of a work week. Considering a monthly frequency, and assuming that vacancy posting costs are proportional to the time the vacancy is kept posted, this implies $c_0 = 0.17E(xyz) \approx 0.17\bar{x}\bar{y}\bar{z} = 0.06375$ where $\bar{x}$, $\bar{y}$ and $\bar{z} = 1$ are the midpoints of the grids over x, y and z, respectively.

we set r to yield an annualized interest rate of 5% as in LR. For simplicity, and in line with most of the literature, the flow payoff from unemployment is $b(x,z) = b_0$ in our baseline calibration, i.e., invariant of aggregate productivity and human capital.

Table 2: Parameters obtained by moment-matching

Parameter	Explanation	Value	Main identifying moment
α	Matching function productivity	0.474	U2E transition rate, mean
s_1	Relative search intensity of employed	0.156	J2J transition rate, mean
x_{up}	Human capital gain, probability	0.0315	Return to experience
b_0	Unemployment payoff	0.321	Unemployment, std.dev.
β	Bargaining strength of workers	0.733	Wage elasticity wrt prod.
σ_y	Match-specific productivity dispersion	0.122	Wage disp: Mean-min ratio
$100\sigma_z$	TFP shock std.dev.	0.670	GDP, std.dev.

The remaining parameters of our model are calibrated jointly to match key first and second moments. Table 2 documents the 7 calibrated parameters and the 7 moments matched, including the main identifying moment for each parameter. We minimize the squared percentage deviation between model and data moments. Let us now motivate the choice of moments. Note first, that since we are interested in the cost of business cycles from a mechanism driven by unemployment volatility, it is important to match GDP and unemployment volatility. Turning to identification, the model parameters are jointly estimated, but some moments are more informative about certain parameters. The mean transition rate from unemployment to employment is informative about the matching function productivity α. The job-to-job transition rate is informative about the relative search intensity of employed workers s_1. Return to experience, measured as the average percentage wage increase while employed, is informative about on-the-job accumulation of human capital, x_{up}.[12] Unemployment volatility is informative about the unemployment payoff parameter, b_0. As pointed out by Hagedorn and Manovskii (2008), wage elasticity with respect to labor productivity is informative regarding worker bargaining strength, β. Wage dispersion is informative about the dispersion of match-specific productivity of new vacancies, σ_y. Finally, the volatility of GDP and unemployment are both informative about the standard deviation of the aggregate productivity process.

Let us comment on the cross-sectional data we use. The relevant measure of wage dispersion for our model is "residual" wage dispersion, i.e., controlling for heterogeneity not present in the model,

[12] As in Jarosch (2015), we impose a relationship between x_{up} and x_{dn} such that the number of increases in human capital roughly equals the number of decreases to minimize bunching at end-points of the human capital grid X. In particular, letting u^{tot} denote the (implicitly, through the mean values of E2U and U2E) targeted value of unemployment, we impose $(1-\nu)\, x_{up}\left(1-u^{tot}\right)\Delta x = (1-\nu)\, x_{dn} u^{tot}\Delta x + \nu\left(\bar{x}-\underline{x}\right)$ where Δx is the distance between two gridpoints and $\bar{x}$ represents average human capital for dying workers. For computational reasons, we set $\bar{x}$ to the midpoint of the grid. Furthermore $\underline{x}$ is the lower bound of the grid, representing the human capital of newly born workers. This implies $x_{dn} = \left(x_{up} - \frac{\nu}{1-\nu}\frac{[\bar{x}-\underline{x}]}{(1-u^{tot})\Delta x}\right)\frac{1-u^{tot}}{u^{tot}}$.

Table 3: Data moments and matched model moments

Moment	Data source	Target value (data)	Model value
U2E transition rate, mean	Fujita-Ramey (2009)	0.340	0.348
J2J transition rate, mean	Moscarini-Thompson	0.0320	0.0351
Unemployment, std.dev.	BLS 1980-2010	0.107	0.117
GDP, std.dev.	BEA 1980-2010	0.0136	0.0141
Wage disp: Mean-min ratio	Hornstein et al.	1.50	1.59
Wage elasticity wrt productivity	Hagedorn-Manovskii	0.449	0.454
Return to experience	Buchinsky et al.	0.0548	0.0483

Notes: U2E and J2J transition rates are at a monthly frequency. Unemployment is a quarterly mean of a monthly series. This variable, as well as GDP, labor productivity and aggregate wages (at the quarterly frequency), have been logged and HP-filtered with $\lambda = 1,600$, both in the data and the model.

such as education, sex, race etc. We take the mean-min ratio (capturing the minimum by the 10th wage percentile) from Hornstein, Krusell and Violante (2007) as our measure of wage dispersion. We use their preferred measure of 1.50, which is an average of their ratios from census, OES and PSID data. Similarly to Kehoe et al. (2015) we use estimates from Buchinsky et al. (2010) to obtain the "return to experience". Specifically, from Buchinsky's estimated coefficients we obtain the marginal return to experience of a worker in his third year of employment. We then match that to the wage increase of workers in the model who works for three years for the same employer. We can thereby keep the match-specific productivity fixed and obtain a clean measure of the effect of human capital on wages. We believe that their estimate of return to experience captures general human capital and not firm-specific human capital since Buchinsky et al. (2010) control for firm-specific seniority.

4 Results

4.1 Targeted moments and the parameter estimates

The moment-matching exercise can be evaluated by comparing the last two columns in Table 3. The model is able to fit most of these moments well, with less than 10 percent deviation for all but one moment, wage dispersion.

It might appear surprising that we need to calibrate the volatility of (the exogenous part of) TFP, but this is necessary since the model has internal amplification and propagation of the exogenous TFP shocks, as the distribution of human capital of workers, the productivity of matches and sorting between workers and jobs varies over the cycle. All of this implies that measured TFP in our model is a combination of exogenous TFP and endogenous propagation.[13]

[13] One could potentially also calibrate the persistence of exogenous TFP jointly with the 7 parameters in Table 2 to match e.g., the persistence of GDP. However, to reduce computational complexity we calibrate this parameter as outlined

The above moment-matching exercise determines the 7 parameters in Table 2. The value for s_1 in Table 2 indicates that employed workers meet prospective employers slightly less than $1/6^{th}$ as often as unemployed workers. We follow LR and report the replacement ratio for unemployed workers as a fraction of the output of the best possible match. The value of b_0 implies that this ratio is 0.643, averaged over the human capital values. Given this low value relative to e.g., Hagedorn and Manovskii (2008), one might ask how our model is able to generate unemployment volatility that is in line with the data. One reason is that due to heterogeneity in human capital and noting that the unemployment payoff is invariant to the individual worker's human capital, many workers that are hired from unemployment have a relatively low productivity and hence a much higher replacement rate than the average value of 0.643. Profits for hiring firms therefore tend to be low and hence sensitive to variations in aggregate productivity. Thus, in settings with worker heterogeneity, a low b_0 can generate sufficient volatility in unemployment; a point also noted by Lise and Robin (2017). Moreover, we find that worker bargaining strength is fairly high, 0.733, which is substantially above Hagedorn and Manovskii (2008). Note that in our bargaining setup, wages in ongoing matches do not change when they remain in the bargaining set. Thus, our model has wage rigidity in the spirit of Hall (2005), which tends to drive the wage elasticity down, thereby yielding a higher estimate of β. Finally, in light of Hornstein, Krusell and Violante (2007), it may be surprising that we are able to match wage dispersion. However, in contrast to their model, we allow for heterogeneity in both human capital and match productivity, which enables us to match this moment well; see also Krolikowski (2019).

Given the centrality of human capital dynamics for our mechanism, we report and comment in more detail on our estimates of the related parameters. The estimated Markov transition probability ($x_{up} = 0.0315$) imply that the expected monthly human capital increase for an employed worker is 0.164 percent, while the expected decrease when unemployed is 0.931 percent (for $x_{dn} = 0.366$).[14]

We know of only one study with a direct measure in the literature of general human capital loss while non-employed: Edin and Gustavsson (2008). They use a Swedish panel of individual level data that includes test results on labor market-relevant general skills and information about employment status between test dates. First, they find that time-out-of-work (compared to employment) implies skill loss, significant at the 1% level. Second, this skill loss appears to be linear in time out-of-work. Third, the speed of skill loss is substantial; being out-of-work for a year implies losing skills equivalent to 0.7 years of schooling.

Our values for human capital dynamics can be compared to estimates in models broadly similar to

above. Moreover, the persistence of GDP turns out to be fairly well matched in our calibration.

[14] These values take into account the distribution of employed and unemployed workers across the human capital grid, including the effects of the bounds of the human capital grid.

ours.[15] Huckfeldt (2016) reports a 0.330 percent expected monthly human capital increase for workers in skill-intensive jobs (0.220 percent in skill-neutral jobs). For unemployed workers Huckfeldt obtains a gradual human capital decrease of 1.13 percent per month. Jarosch (2015) reports only the monthly human capital Markov transitions probabilities: 0.0141 for employed and 0.131 for unemployed. In Jarosch (2015), for an employed worker with the mid-point of human capital, this implies an expected increase of 0.134 percent, and for the unemployed worker with the mid-point of human capital, it implies a 1.25 percent decrease. To sum up this comparison to the literature, our human capital accumulation for employed workers is in between the estimates of Huckfeldt (2016) and Jarosch (2015), while for unemployed workers our value is slightly below their estimates.

4.2 Welfare measure

As is standard in the cost of business cycle literature since Lucas (1987), we report the fraction of expected consumption agents are willing to forego to eliminate business cycles. In our model, the linearity of utility in consumption makes welfare calculations straightforward, since then the flow of aggregate welfare is proportional to aggregate consumption.

To compute market consumption, we deduct vacancy posting costs from GDP. Note that one may interpret the unemployment payoff, b, in two ways, which has different welfare implications. In the first interpretation, b is home production (or equivalently, from a welfare perspective, utility of leisure) in which case the welfare relevant quantity is the sum of market consumption and the unemployment payoff. In the second interpretation, b is a pecuniary transfer with no direct effect on aggregate utility. We report results for both interpretations.[16]

4.3 Results for cost of business cycles

Our main exercise is to compute the consequences for welfare, GDP and employment of eliminating aggregate volatility.[17] As documented in Table 4, we find that in our model the elimination of ag-

[15] First, there is an older empirical literature that attributes all wage loss when re-employed after an unemployment spell to human capital loss and furthermore assumes that the wage equals marginal product of labor. This is not consistent with our model so we can not use that literature for calibration or straight comparison. Second, some papers look at the effect on wages of an additional month of unemployment. The estimates in Neal (1995) imply that an additional month of unemployment reduces the re-employment wage by 1.5%, which, under the assumption that the wage equals marginal product of labor, is very much in line with the results here. Recent results by Schmieder et al. (2016) shows that re-employment wages decrease by 0.8% per (additional) month unemployed. This is somewhat lower than our result, but reasonably well in line if we think that there is some surplus sharing so that wages decrease less than human capital for an additional month of unemployment. Under the assumption that the wage changes roughly in proportion to the marginal product of labor, these two empirical studies bracket our results where the difference in the change in human capital for employed and unemployed workers is $0.931\% + 0.164\% = 1.095\%$.

[16] There is also an intermediate case where b consists of both home production and transfers. The welfare gain of eliminating aggregate volatility generated by our mechanism will then fall between these two cases.

[17] We do this by setting exogenous productivity z constant and equal to the average in the stochastic simulation.

gregate volatility increases steady state GDP by a substantial amount, 1.55 percent.[18] This also has consequences for steady state consumption and welfare, which increase by 0.70-1.68 percent depending on the interpretation of the unemployment payoff. As we will document below, these fairly large effects are due to the positive relationship between employment and human capital accumulation. Another way to describe the consequences of removing aggregate volatility is through the effects on the unemployment rate which falls from 5.78 percentage points to 4.59 percentage points, corresponding to a 21 percent decrease.

Note that the assumption of risk neutral agents implies that only changes in levels of consumption and employment matter for welfare. We thus abstract from the welfare costs of consumption volatility. Our results capture only one of several factors that account for the total cost of business cycles and can be viewed as a lower bound of this cost.

From an accounting perspective, the increase in GDP can be decomposed into the increase in employment and the change in the average level of human capital of employed workers[19];

$$E\left(x \times h\left(\cdot\right)\right) = \frac{1}{\sum_t^T h\left(x, y, z_t\right)} \sum_t^T \sum_{x \in X} \sum_{y \in Y} x h\left(x, y, z_t\right).$$

As can be seen from Table 4, the increase in employment accounts for the bulk of the effect on GDP. To understand the effects of human capital on employment, recall from (14) that job creation is affected by the human capital of both employed and unemployed workers. We find that the effects through the unemployed dominates. This is partly due to that the average levels of human capital for the unemployed changes more;

$$E\left(x \times u\left(\cdot\right)\right) = \frac{1}{\sum_t^T u\left(x, z_t\right)} \sum_t^T \sum_{x \in X} x u\left(x, z_t\right)$$

increases by 3.69 percent while $E\left(x \times h\left(\cdot\right)\right)$ increases by 0.30 percent. In addition, job creation is much more sensitive to changes in human capital of the unemployed. Specifically, the elasticity of $J\left(z, \Gamma\right)$ with respect to $E\left(x \times u\left(\cdot\right)\right)$ is 1.49 while the elasticity of $J\left(z, \Gamma\right)$ with respect to $E\left(x \times h\left(\cdot\right)\right)$ is 0.27. It may be surprising that the change in $E\left(x \times h\left(\cdot\right)\right)$ is so moderate. However, the reason is that the composition of the employed workers is affected by the elimination of business cycles. In

[18] This indicates that the Oi-Hartman-Abel effect, where higher aggregate volatility increases output and employment, is relatively unimportant; see Bloom et al. (2018). Moreover, the counteracting effect emphasized in Laureys (2014) working through compositional effects on job creation does not seem to be important here.

[19] Although negligible for our exercise, there are other factors than human capital affecting average productivity. Examples include the change in the average level of match-specific productivity, $E\left(y \times h\left(\cdot\right)\right)$, and the changed degree of sorting between workers and firms (as well as the covariation between any of these objects with the cycle).

particular, the positive effect that higher employment has on human capital is counteracted by the tendency that, in the absence of aggregate volatility, firms tend to accept workers with lower human capital.

Table 4: Steady state effects of eliminating business cycles (in percent)

	Baseline	No human capital dynamics
Welfare, b transfer, (GDP-vacancy cost)	1.68	0.56
Welfare, b home prod, (GDP-vacancy costs+$b*u$)	0.70	0.03
GDP	1.55	0.53
Employment	1.26	0.71
$E\left(x \times u\left(\cdot\right)\right)$	3.69	-
$E\left(x \times h\left(\cdot\right)\right)$	0.29	-

In contrast to the simple model discussed in the introduction and Appendix A.1, both the job creation margin and the separation margin can contribute to the cost of business cycles. One way of quantifying their relative importance is to turn off the job creation channel by counterfactually fixing the job finding rate to the value in the economy without aggregate volatility. The welfare gain of eliminating business cycles is then 0.8 percent, which indicates that the job creation and separation margins contribute roughly equally to the cost of business cycles.

4.3.1 The importance of human capital dynamics

Let us now quantify the importance of the change in the human capital distribution for the cost of business cycles. To do this we perform a counterfactual exercise where we keep the human capital distribution of the population (i.e., combining employed and unemployed workers) fixed when we remove the aggregate volatility, thus shutting down the amplification mechanism discussed in Figure 1 and in conjunction with equation (14). All other aspects of the computation is the same as in the baseline exercise.[20] The last column of Table 4 confirms the importance of learning on-the-job, as the version of our model without human capital dynamics implies that aggregate fluctuations have substantially smaller effect on welfare, GDP and employment. In particular, human capital is very important for the welfare effects of removing business cycles.

4.3.2 Accounting for the transition

We now compute the welfare consequences of eliminating aggregate volatility taking the transition dynamics into account. As reported in Table 5, we find that in our model the elimination of aggregate volatility when taking the transition into account increases welfare by 0.37-1.28 percent depending

[20] We fix the human capital distribution by setting $x_{up} = x_{dn} = \nu = 0$ and assume that it is given by the average distribution in the baseline calibration with aggregate volatility. We also keep the incentives for job creation and destruction unchanged, i.e., S and B are computed with the baseline human capital parameters.

on the interpretation of the unemployment payoff.[21] We note that the welfare gains from removing business cycles are lower when accounting for the transition than when simply comparing steady states. The gains when accounting for the transition are lower for two reasons: discounting of the increased future consumption and the extra vacancy posting costs related to the increase in employment along the transition path. Note also that the transition to the non-stochastic steady state is reasonably fast; the half-time of the transition of GDP is 3.8 years.

Table 5: Welfare effects of eliminating business cycles (in percent)

Welfare, b transfer	1.28
Welfare, b home prod	0.37

4.3.3 Robustness

Two key determinants of the cost of business cycles in our model are i) how sensitive the human capital distribution is to the change in (un)employment, and ii) how sensitive job creation is to changes in the human capital distribution of unemployed and employed workers. An important factor affecting the sensitivity of the human capital distribution is the range of values that human capital can take and two important factors affecting the sensitivity of job creation to human capital is to what degree the unemployment payoff depends on human capital and the bargaining strength of workers.

Thus, to judge the robustness of the results we re-calibrate it under alternative assumptions and report the steady state welfare, GDP and employment cost of business cycles in Table 6. First, we document what the cost of business cycles is when allowing for a wider range of values for human capital. Recall that in our main calibration we have followed Ljungqvist and Sargent (1998, 2008) and assumed that the ratio between the highest and the lowest human capital value is 2. We illustrate the effects of increasing this ratio by 20% to 2.4. We then re-calibrate the model by matching the same moments as above in Table 3. We find that eliminating aggregate volatility leads to an increase of welfare and GDP of 0.47-1.02 and 0.94 percent, respectively. In other words, compared to our baseline calibration the cost of business cycles decrease somewhat, but the results seem not to be very sensitive to the range.

Second, we vary the unemployment payoff by setting $b(x,z) = b_0 + b_1 x$. In our main calibration we have chosen to impose $b_1 = 0$ and calibrate b_0 internally. One might wonder to what degree

[21] We compute welfare when taking the transition into account in the following way. First, we simulate the economy with aggregate volatility for several thousand periods. We then draw 1000 starting points for the transition from this simulation and compute welfare in each of these starting points, given that productivity is constant at its mean value for all future periods. Finally, we calculate the mean across the 1000 transitions.

our results are sensitive to this assumption, and whether a high b_1, i.e., a strong dependence of the unemployment payoff on human capital, would substantially reduce the effect of changes in human capital on job creation. In this robustness exercise, we set $b_1 = 0.9$ and otherwise re-calibrate our model in the same way as in the baseline. The cost of business cycles are reduced by more than a factor two and are reported in the third line of Table 6. We elaborate on this result in section 4.3.4, but the basic intuition is that imposing $b_1 = 0.9$ implies that the surplus of a match depends much less on the human capital level, x, and job creation (and thereby employment) accordingly do not fall as much in response to the fall in the work force's human capital.

Finally, we explore the sensitivity of our results to the bargaining strength of workers. In particular, we fix the bargaining power at 0.50, as is commonly done in the literature that, differently from our setup, considers Nash bargaining with unemployment as the (only) outside option of the worker. We then re-calibrate the model by matching the same moments as above in Table 3, except the elasticity of wages, that was used to identify bargaining power in the baseline calibration. We find that when $\beta = 0.50$, the elimination of business cycles have somewhat smaller effects on all variables compared to our baseline calibration.

Table 6: Steady state effects of eliminating business cycles under alternative assumptions (in percent)

Model version	Welfare, b transfer	Welfare, b home prod	GDP	Employment
Baseline	1.68	0.70	1.55	1.26
Wider human cap. range	1.02	0.47	0.94	0.76
Unemp. payoff incr. in x	0.62	0.26	0.45	0.43
$\beta = 0.50$	1.30	0.65	1.41	0.81

4.3.4 Comparison with Jung and Kuester

Jung and Kuester (2011) analyze the welfare cost of business cycles in a simpler setting than ours, using a solution method of local second-order approximations. In their extension that includes human capital dynamics and where workers are risk neutral, they find that eliminating business cycles increases employment by 0.11 percent and welfare by 0.16 percent.

Our results for the cost of business cycles are roughly an order of magnitude larger than in Jung and Kuester (2011). The reason is that they abstract from match-specific productivity and assume that the unemployment payoff is proportional to human capital, x. In terms of exposition, Jung and Kuester (2011) do not describe the amplification mechanism that we outline in Figure 1, i.e., how the reduction in human capital feeds back to job creation and further reduces employment and thereby human capital.

Let us start by understanding why these two assumptions yield the low sensitivity of job creation to human capital obtained in Jung and Kuester (2011). In their model, wages are determined in bargaining over flow surpluses. The wage is (in their baseline without capital) $w = \beta xz + (1-\beta)\, b_1 x$ and firm flow surplus is $xz - w = (1-\beta)\,(z - b_1)\, x$, i.e., proportional to x.[22] The value of a new job for the firm is a sum (appropriately discounted) over current and future flow surpluses $(1-\beta)\,(z-b_1)\, x$ and hence, since human capital can increase by at most a factor 2, the value of a job can also increase by at most a factor 2. Accordingly, job creation is not very sensitive to changes in human capital in this setting.

If we relax the assumption that the unemployment payoff is proportional to x in their model, things change. To see this, assume that the unemployment payoff now is $b_0 + b_1 x$. Then firm flow surplus is $(1-\beta)\,[(z-b_1)\,x - b_0]$. This surplus can be made arbitrarily small for the lowest level of human capital $\underline{x}$, by setting $b_0 = (z-b_1)\,\underline{x} - \varepsilon$ for ε small. Then the percentage increase in firm surplus from an increase in human capital can be much larger than in the proportional case and hence job creation can be much more sensitive to changes in human capital.

As we know from our robustness exercise, in our model the qualitative results are invariant to the details of the unemployment payoff; see Table 6. Instead, match-specific productivity is important for the sensitivity of job creation to human capital. Specifically, workers with low human capital can meet firms whose match productivity y imply a negative surplus. Moreover, matches that are formed when the worker has low human capital, face a substantial probability of separating in a future downturn. Hence, average surplus (over match productivity) for workers with low human capital is low. Furthermore, for workers with a higher level of human capital, fewer meetings have negative surplus and future separation rates are lower. In contrast to the framework in Jung and Kuester (2011), this implies a substantially higher average surplus, compared with matches of workers with low human capital. Specifically, in our baseline calibration, when human capital increases from the lowest to the highest value, i.e., by a factor 2 as in Jung and Kuester (2011), the average surplus for an unemployed worker finding a job increases by a factor 5.7. Hence, in our model, an upward shift in the human capital distribution has dramatic effects on job creation.

5 Conclusions

A central question in macroeconomics is how large the welfare costs of business cycles are. We document that human capital dynamics yield a new mechanism that amplifies how business cycles reduce

[22] These expressions use the assumption in Jung and Kuester (2011) that $b_0 = 0$. They also set $b_1 = 0.9$ as in our robustness exercise above.

the level of employment, output and welfare. There are two channels through which business cycles reduce employment, and they constitute the initial step in the main mechanism of this paper. In settings with learning on-the-job and skill loss during unemployment, any resulting fall in employment from these two initial channels implies that average human capital falls. This, in turn, reduces the incentives to post vacancies, further decrease employment and so on in a vicious circle, thereby amplifying the initial impact of aggregate volatility on employment. In our calibration, we find that the steady state output and welfare gains from eliminating business cycles are large - they amount to 1.55 percent and 0.70-1.68 percent, respectively. The alternative parameter assumptions explored indicate that the cost of business cycles are only mildly affected, except in the case when unemployment benefits are strongly increasing in human capital. We also show that human capital dynamics are central for the results - if we disable this mechanism in our model, the implied gains in output from eliminating business cycles are substantially smaller and the welfare gains might even be negligible.

To conclude, let us briefly discuss some broader implications of our results. In our model, there is only one type of aggregate shock. If we view this shock as a "catch-all" for any variation in firm profitability including effects of fiscal and monetary policy, we can draw interesting policy conclusions. In particular, a policy that successfully stabilizes unemployment (or job finding rates) raises the average level of output. For this reason, our paper rationalizes an unemployment stabilization mandate for monetary and fiscal policy. In this sense we reach the same conclusion as Berger et al. (2016) and Galí (2016) but for a very different reason. Berger et al.'s argument is about unemployment stabilization reducing idiosyncratic risk related to layoffs, while Galí's mechanism is about hysteresis due to insider-outsider dynamics. Our mechanism is about unemployment stabilization leading to a higher average level of output, thereby more closely related to the argument by Summers (2015) that stabilization policy can have major effects on average levels of output over periods of decades. Fatás and Summers (2018) provide empirical evidence of this phenomenon. In particular, they document the negative long-term effects on output of fiscal austerity measures in the recovery phase of the business cycle.

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A Appendix

A.1 Why business cycles decrease employment - the initial step in our main mechanism

A.1.1 Preliminaries

In models where employment is a state variable the law of motion for employment can be written as

$$e_t = (1-\delta)\, e_{t-1} + f_{t-1} u_{t-1} \tag{23}$$

where e denotes employment, δ the separation rate, f the job finding rate and u unemployment. We assume covariance stationarity in all variables in (23). Let $Cov\,(f,u)$ denote the unconditional covariance between the job finding rate and unemployment.

In the classical Mortensen Pissarides search and matching model, the firm value of a match is

$$J_t = z_t - w_t + \frac{1-\delta}{1+r} E_t J_{t+1} \tag{24}$$

where z is the value of the flow output of the match, w is the wage and $z_{t+1} = \rho_z z_t + \varepsilon^z_{t+1}$. Furthermore, r denotes the discount rate. The worker value when employed is

$$W_t = w_t + \frac{1}{1+r} E_t \left((1-\delta)\, W_{t+1} + \delta U_{t+1} \right) \tag{25}$$

and when unemployed

$$U_t = b + \frac{1}{1+r} E_t \left(f\,(\theta_t)\, W_{t+1} + (1 - f\,(\theta_t))\, U_{t+1} \right) \tag{26}$$

where $f\,(\theta_t)$ is the job finding rate, which is $f\,(\theta_t) = \theta_t^{1-\omega}$, where ω is the matching function elasticity with respect to unemployment, under the Cobb-Douglas matching function $m_t = v_t^{1-\omega} u_t^{\omega}$ noting that tightness is defined as $\theta_t = \frac{v_t}{u_t}$.

The job creation condition is

$$c_0 = q\,(\theta_t)\, \frac{1}{1+r} E_t J_{t+1} \tag{27}$$

where q is the vacancy filling rate, which is $q\,(\theta_t) = \theta_t^{-\omega}$ under a Cobb-Douglas matching function.

Wages are determined by Nash Bargaining:

$$\beta J_t = (1-\beta)\, (W_t - U_t) \tag{28}$$

where β denotes the bargaining strength of workers. Hence, using (24), (25), (26) and (27), the wage is

$$w_t = \beta z_t + (1-\beta)\left(b + \theta_t \frac{\beta}{1-\beta} c_0\right). \tag{29}$$

We can then find θ by combining the solution for the wage in (29) with job creation (27) and the firm value (24). We restrict attention to the case when the solution for θ (as a function of z) is C^2 with θ'' bounded.

Finally, let us point out that the labor force participation rate is taken as given. Accordingly, an increase in unemployment implies an equal size reduction in employment.

A.1.2 Proposition and proof

This paper analyzes the cost of business cycles by comparing the outcomes in a model with and without aggregate volatility, respectively. The following proposition establishes sufficient conditions for when aggregate volatility leads to an increase in unemployment.

Proposition 1 *If $Cov(f_{t-1}, u_{t-1}) < 0$ and $1 - \delta - \frac{\beta}{\omega} f_t > 0$, then, for any matching function elasticity $\omega > \tilde{\omega}$ where $\tilde{\omega} < \frac{1}{2}$ ($\beta > 0$) and $\omega \geq \frac{1}{2}$ ($\beta = 0$), aggregate volatility increases average unemployment.*

Proof:

Let $E(f_t)$ denote the unconditional average of the job finding rate in the stochastic economy and let $\bar{f}$ denote the job finding rate in a non-stochastic economy. We first show that if $1-\delta-\frac{\beta}{\omega} f_t > 0$ then, for any $\omega > \tilde{\omega}$ where $\tilde{\omega} < \frac{1}{2}$, we have $E(f_t) < \bar{f}$. Second, we establish that, if $Cov(f_{t-1}, u_{t-1}) < 0$ and $E(f_t) \leq \bar{f}$, aggregate volatility increases average unemployment.

Step 1. Showing $E(f_t) \leq \bar{f}$. **Case 1.** $\beta > 0$ and $\omega < 1$. Proving that if $1 - \delta - \frac{\beta}{\omega} f_t > 0$ then, for any $\omega > \tilde{\omega}$ where $\tilde{\omega} < \frac{1}{2}$, we have $E(f_t) < \bar{f}$.

Using that $q(\theta_t) = \theta_t^{-\omega}$, we can rewrite job creation as

$$\theta_t^{\omega} = \frac{1-\beta}{1+r} \frac{\rho_z z_t - b}{c_0} - E_t \theta_{t+1} \frac{\beta}{1+r} + \frac{1-\delta}{1+r} E_t \theta_{t+1}^{\omega}. \tag{30}$$

Define $h(z_t) = \theta_t^{\omega}$. Then we have

$$h(z_t) = \frac{1-\beta}{1+r} \frac{\rho_z z_t - b}{c_0} - E_t (h(z_{t+1}))^{\frac{1}{\omega}} \frac{\beta}{1+r} + \frac{1-\delta}{1+r} E_t (h(z_{t+1})).$$

Differentiating with respect to z_t, using that $z_{t+1} = \rho_z z_t + \varepsilon^z_{t+1}$,

$$h'(z_t) = \frac{1-\beta}{1+r}\frac{\rho_z}{c_0} - E_t\frac{1}{\omega}\left(h\left(\rho_z z_t + \varepsilon^z_{t+1}\right)\right)^{\frac{1}{\omega}-1} h'\left(\rho_z z_t + \varepsilon^z_{t+1}\right)\rho_z\frac{\beta}{1+r}$$
$$+\frac{1-\delta}{1+r}E_t\left(h'\left(\rho_z z_t + \varepsilon^z_{t+1}\right)\right)\rho_z. \tag{31}$$

Note that, using that $h(z_t) = \theta^{\omega}_t$ and hence $h(z_t)^{\frac{1-\omega}{\omega}} = \theta^{1-\omega}_t = f(\theta_t)$, we have $h'(z_t) > 0$.

Differentiating again gives

$$h''(z_t) = -E_t\frac{1}{\omega}\left(\frac{1}{\omega}-1\right)\left(h\left(\rho_z z_t + \varepsilon^z_{t+1}\right)\right)^{\frac{1}{\omega}-2}\left(h'\left(\rho_z z_t + \varepsilon^z_{t+1}\right)\right)^2\rho_z^2\frac{\beta}{1+r}$$
$$-E_t\frac{1}{\omega}\left(h\left(\rho_z z_t + \varepsilon^z_{t+1}\right)\right)^{\frac{1}{\omega}-1} h''\left(\rho_z z_t + \varepsilon^z_{t+1}\right)\rho_z^2\frac{\beta}{1+r} + \frac{1-\delta}{1+r}E_t\left(h''\left(\rho_z z_t + \varepsilon^z_{t+1}\right)\right)\rho_z^2.$$

Since θ is C^2 with θ'' bounded it follows that h is C^2 and that h'' is bounded with some upper bound $\bar{h}''$. Suppose $\bar{h}''$ is non-negative. Then there is some $\hat{z}_t$ such that $\bar{h}'' = h''(\hat{z}_t)$ that satisfies the expression above.

If $E_t\frac{1}{\omega}\left(h\left(\rho_z \hat{z}_t + \varepsilon^z_{t+1}\right)\right)^{\frac{1}{\omega}-1} h''\left(\rho_z \hat{z}_t + \varepsilon^z_{t+1}\right) \geq 0$ then, noting that $h(\hat{z}_t) = \theta^{\omega}_t > 0$ and $h''\left(\rho_z \hat{z}_t + \varepsilon^z_{t+1}\right) \leq \bar{h}''$

$$\bar{h}'' \leq -E_t\frac{1}{\omega}\left(\frac{1}{\omega}-1\right)\left(h\left(\rho_z \hat{z}_t + \varepsilon^z_{t+1}\right)\right)^{\frac{1}{\omega}-2}\left(h'\left(\rho_z \hat{z}_t + \varepsilon^z_{t+1}\right)\right)^2\rho_z^2\frac{\beta}{1+r} + \frac{1-\delta}{1+r}\bar{h}''\rho_z^2.$$

Since $\frac{1-\delta}{1+r}\rho_z^2 < 1$ we must have $\bar{h}'' < 0$ and we have a contradiction.

If $E_t\frac{1}{\omega}\left(h\left(\rho_z \hat{z}_t + \varepsilon^z_{t+1}\right)\right)^{\frac{1}{\omega}-1} h''\left(\rho_z \hat{z}_t + \varepsilon^z_{t+1}\right) < 0$ then, using that $h(z_t)^{\frac{1-\omega}{\omega}} = f(\theta_t)$,

$$h''(\hat{z}_t) = -E_t\frac{1}{\omega}\left(\frac{1}{\omega}-1\right)\left(h\left(\rho_z \hat{z}_t + \varepsilon^z_{t+1}\right)\right)^{\frac{1}{\omega}-2}\left(h'\left(\rho_z \hat{z}_t + \varepsilon^z_{t+1}\right)\right)^2\rho_z^2\frac{\beta}{1+r}$$
$$+\frac{1}{1+r}E_t\left(\left(1-\delta-\frac{\beta}{\omega}f(\theta_{t+1})\right)h''\left(\rho_z \hat{z}_t + \varepsilon^z_{t+1}\right)\right)\rho_z^2.$$

Since $1-\delta-\frac{\beta}{\omega}f(\theta_{t+1}) \in (0,1)$, noting that $h(\hat{z}_t) = \theta^{\omega}_t > 0$ and $h''\left(\rho_z \hat{z}_t + \varepsilon^z_{t+1}\right) \leq \bar{h}''$, we have

$$\bar{h}'' \leq -E_t\frac{1}{\omega}\left(\frac{1}{\omega}-1\right)\left(h\left(\rho_z \hat{z}_t + \varepsilon^z_{t+1}\right)\right)^{\frac{1}{\omega}-2}\left(h'\left(\rho_z \hat{z}_t + \varepsilon^z_{t+1}\right)\right)^2\rho_z^2\frac{\beta}{1+r} + \frac{1}{1+r}\left(\bar{h}''\right)\rho_z^2.$$

Since $\frac{\rho_z^2}{1+r} < 1$ we have $\bar{h}'' < 0$, a contradiction. Hence, $h''(z) < 0$ for all z.

Since h is strictly concave and $f(\theta(z)) = h(z_t)^{\frac{1-\omega}{\omega}}$, it follows that $\frac{d^2 f(\theta(z))}{dz^2} < 0$ when $\omega \geq \frac{1}{2}$. Since h'' is C^2 and bounded, it follows that f'' is C^2 and bounded. Then there is some $\tilde{\omega} < \frac{1}{2}$ so that f is strictly concave for all $\omega > \tilde{\omega}$. Due to Jensen's inequality, the concavity of $f(z)$ implies that $E(f_t) < \bar{f}$.

Case 2. Now suppose $\beta = 0$ or $\omega = 1$. Then we can write

$$h(z_t) = \frac{1-\beta}{1+r}\frac{\rho_z z_t - b}{c_0} + kE_t(h(z_{t+1}))$$

where $k = \frac{1-\delta-\beta}{1+r}$. Guess that $h(z) = \gamma_z z + \gamma_0$. Using that $z_{t+1} = \rho_z z_t + \varepsilon_{t+1}^z$ gives

$$\begin{aligned} \gamma_z &= \frac{1}{1-k\rho_z}\frac{1-\beta}{1+r}\frac{\rho_z}{c_0} \\ \gamma_0 &= -\frac{1}{1-k}\frac{1-\beta}{1+r}\frac{1}{c_0}b. \end{aligned}$$

If $\omega = 1$ then $f(\theta(z)) = h(z_t)^{\frac{1-\omega}{\omega}}$ is constant and we have $E(f_t) \leq \bar{f}$. If $\beta = 0$ then $\frac{d^2 f(\theta(z))}{dz^2} = \frac{1-\omega}{\omega}\left(\frac{1-\omega}{\omega} - 1\right) h(z_t)^{\frac{1-\omega}{\omega}-2}(h'(z_t))^2 \leq 0$ when $\omega \geq \frac{1}{2}$. Then $E(f_t) \leq \bar{f}$ for $\omega \geq \frac{1}{2}$.

Step 2. Proving that, whenever $Cov(f_{t-1}, u_{t-1}) < 0$ and $E(f_t) \leq \bar{f}$, aggregate volatility increases average unemployment. The proof of this step follows Jung and Kuester (2011).

Covariance stationarity of equation (23) implies

$$\delta E(e_t) = E(f_{t-1}u_{t-1}).$$

Substituting in $e_t = 1 - u_t$ yields:

$$\delta E(1-u_t) = E(f_{t-1})E(u_{t-1}) + Cov(f_{t-1}, u_{t-1}).$$

Note that $E(u_t) = E(u_{t-1})$, $E(f_t)E(u_t) = E(f_{t-1})E(u_{t-1})$ by covariance stationarity. Subtracting the steady state version of equation (23):

$$-\delta(E(u_t) - \bar{u}) = \left[E(f_t) - \bar{f}\right]E(u_t) + \bar{f}\left[E(u_t) - \bar{u}\right] + Cov(f_{t-1}, u_{t-1}).$$

Then, collecting terms and dividing through:

$$E(u_t) - \bar{u} = -\frac{\left[E(f_t) - \bar{f}\right]E(u_t) + Cov(f_{t-1}, u_{t-1})}{\delta + \bar{f}} > 0 \qquad (32)$$

where we have made use of the observation that $E(u_t) > 0$.

Step 3. Combining Step 1 and 2 establishes that unemployment increases when the conditions in the proposition are satisfied.■

Let us elaborate on the empirical relevance of the conditions in the proposition. The condition $Cov(f_{t-1}, u_{t-1}) < 0$ clearly holds in the data. For example, Jung and Kuester (2011) reports a value of -5.06. An indication of the strength of this relationship is that the correlation is strongly negative, e.g., Shimer (2005) reports -0.949. The remaining two conditions are also supported in the data. First, empirical estimates of ω lie robustly above the cutoff $\tilde{\omega} < 0.5$ (or $\omega \geq 0.5$ when $\beta = 0,$); see e.g. Shimer (2005) who estimates ω to be 0.72. More recently, Barnichon (2012) estimates ω to be 0.59. Second, the condition $1 - \delta - \frac{\beta}{\omega} f_t > 0$ also holds for empirical values of δ, f_t and ω. Using $\delta = 0.03$ and $f_t = 0.340$ that are reported in Table 1 and Table 3 above together with the value of ω from Barnichon (2012), we get $1 - \delta - \frac{\beta}{\omega} f_t = 0.97 - \frac{\beta}{0.59} 0.340$. This expression is positive for any bargaining power $\beta \in [0, 1]$.

Now, let us focus on the intuition of the Proposition. First, the reason that a negative covariance between the job finding rate and the unemployment rate reduce employment is that new jobs are the product of these two. Aggregate volatility then implies that fewer new jobs are created and employment decreases, all else equal. Intuitively this happens because the job finding rate tend to be high when unemployment is low and vice versa. Second, to see why the conditions $1 - \delta - \frac{\beta}{\omega} f_t > 0$ and $\omega > \tilde{\omega}$ ($\omega \geq 0.5$ when $\beta = 0$) leads to a reduction in employment might seem less straightforward. However, in the proof we establish that the two conditions imply that f_t is concave in TFP, i.e., $E(f_t) \leq \bar{f}$. This, in turn, implies that congestion effects for workers increases "enough" during a boom. Note, however, that the conditions in the proposition are sufficient and, given the fact that $Cov(f_{t-1}, u_{t-1})$ is strongly negative in the data, we may actually have $E(f_t) > \bar{f}$ and still have business cycles leading to an increase in unemployment, i.e., $E(u_t) > \bar{u}$, see expression (32) in step 2 in the proof of the proposition.

Finally, note that the second step in the proof does not require newly created jobs to necessarily come from a search and matching framework. In any model where separations are constant and new jobs are given by $f_t u_t$ where $Cov(f_{t-1}, u_{t-1}) < 0$ and $E(f_t) \leq \bar{f}$, aggregate volatility leads to an increase in unemployment.

The model presented in this paper is richer than the canonical search and matching model. However, Figure 2 indicates concavity of the job finding rate in z also in our richer framework. Moreover, in the baseline calibration, the stochastic mean job finding rate is 0.3475 and the ergodic job finding rate is 0.3729, establishing that $E(f_t) < \bar{f}$ in our richer model. We also provide empirical evidence based on (HP detrended) TFP and job finding rates in levels, detrending data with the standard HP parameter 1600. Our baseline sample is 1977Q4-2012Q3 and we run the following regression: $f_t = \beta_0 + \beta_1 z_t + \beta_2 z_t^2 + \varepsilon_t$. The results are reported in Table 7.

From the table, note that the coefficient β_2 is negative, indicating concavity. The Great Recession seems to mute the relationship. When restricting the estimation to data before the Great Recession (the last observation is then 2008Q2) both β_1 and β_2 are strongly significant with $\beta_2 < 0$. We also document the data and the fitted values from the regression for the baseline sample in Figure 3, where

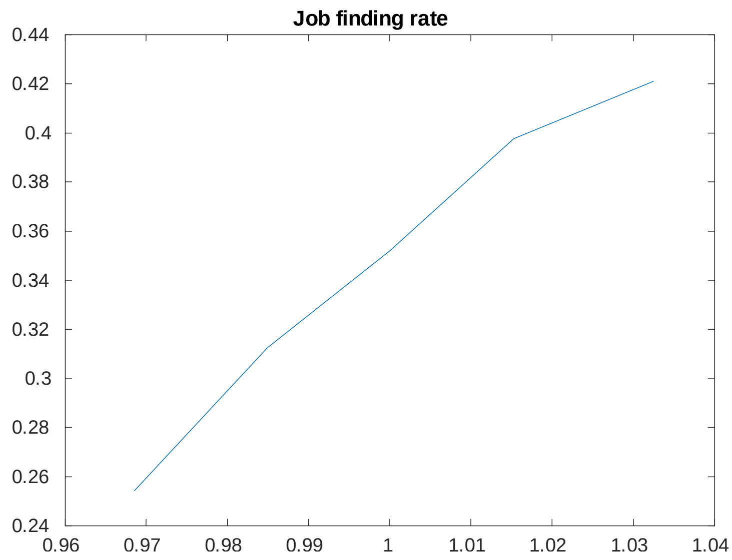

Figure 2: The job finding rate for different values of TFP

Table 7: Coefficients and t-values in regression of the job finding rate on TFP in US data.

	Baseline sample		Excluding Great Recession	
	Value	t-statistic	Value	t-statistic
β_0	0.001	0.79	0.003	2.16
β_1	0.558	4.52	0.439	3.55
β_2	-7.92	-1.50	-28.33	-3.48

Note: We base our analysis regarding the job finding rate on Fujita and Nakajima (2016) and for TFP we use (updated) data from Fernald (2012). The data for the job finding rate has gratiously been extended by Shigeru Fujita to cover our longer sample.

the concavity in TFP is visible.

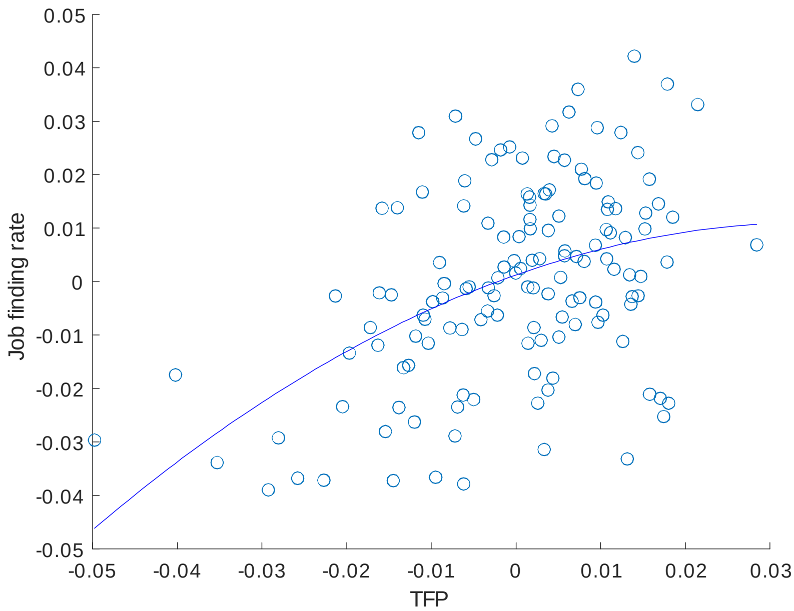

Figure 3: Empirical relationship between job finding rate and TFP, documented using a scatter plot and a fitted line with a quadratic term. Data has been HP-detrended with parameter 1600.

A.2 Employment transitions

When accounting for the wage distribution, the employment transition follows:

$$\begin{aligned}
&h^{w}\left(w^{*},x,y,z\right)= \\
&h^{s,w}\left(w^{*},x,y,z\right)-\underbrace{h^{s,w}\left(w^{*},x,y,z\right)s_{1}\frac{M}{L}\sum_{\tilde{y}\in Y}I_{\tilde{y}>y}g\left(\tilde{y}\right)}_{\text{mass lost to more productive matches}} \\
&\underbrace{-h^{s,w}\left(w^{*},x,y,z\right)s_{1}\frac{M}{L}\sum_{\tilde{y}\in Y}\mathbf{1}\left\{P\beta\left(x,\tilde{y},y,z,\Gamma\right)>W\left(w^{*},x,y,z,\Gamma\right)\right\}\left(1-I_{\tilde{y}>y}\right)g\left(\tilde{y}\right)}_{\text{mass lost to higher wage offers from less productive matches}} \\
&\underbrace{+s_{1}\frac{M}{L}\sum_{\tilde{y}\in Y}\sum_{\tilde{w}\in W^{grid}}h^{s,w}\left(\tilde{w},x,y,z\right)\mathbf{1}\left\{w\left(\tilde{w},x,y,z,\Gamma\right)=w^{*}\right\}\left(1-I_{\tilde{y}>y}\right)g\left(\tilde{y}\right)}_{\text{mass gained from increased wage due to offers from less productive matches}} \\
&\underbrace{+s_{1}\frac{M}{L}g\left(y\right)\sum_{\tilde{y}\in Y}h^{s}\left(x,\tilde{y}\right)\mathbf{1}\left\{W\left(w^{*},x,y,z,\Gamma\right)=P\left(x,\tilde{y},z,\Gamma\right)+\beta\left[S\left(x,y,z,\Gamma\right)-S\left(x,\tilde{y},z,\Gamma\right)\right]\right\}I_{y>\tilde{y}}}_{\text{mass poached from less productive matches}} \\
&\underbrace{-h^{s,w}\left(w^{*},x,y,z\right)\mathbf{1}\left\{W\left(w^{*},x,y,z,\Gamma\right)\notin BS\left(x,y,z,\Gamma\right)\right\}}_{\text{mass lost due to being outside bargaining set}} \\
&\underbrace{+\sum_{\tilde{w}\in W^{grid}}h^{s,w}\left(\tilde{w},x,y,z\right)\mathbf{1}\left\{w\left(\tilde{w},x,y,z,\Gamma\right)=w^{*}\right\}\mathbf{1}\left\{W\left(\tilde{w},x,y,z,\Gamma\right)\notin BS\left(x,y,z,\Gamma\right)\right\}}_{\text{mass gained from other wages being outside bargaining set}} \\
&\underbrace{+\frac{M}{L}u^{s}\left(x\right)g\left(y\right)S_{xyz}\mathbf{1}\left\{W\left(w^{*},x,y,z,\Gamma\right)=B\left(x,z,\Gamma\right)+\beta S\left(x,y,z,\Gamma\right)\right\}}_{\text{mass hired from unemployment}}
\end{aligned} \tag{33}$$

where W^{grid} is the wage grid and

$$\begin{aligned}
I_{\tilde{y}>y} &\equiv \mathbf{1}\left\{P\left(x,\tilde{y},z,\Gamma\right)>P\left(x,y,z,\Gamma\right)\right\} \\
P\beta\left(x,\tilde{y},y,z,\Gamma\right) &= P\left(x,\tilde{y},z,\Gamma\right)+\beta\left[S\left(x,y,z,\Gamma\right)-S\left(x,\tilde{y},z,\Gamma\right)\right] \\
I_{y>\tilde{y}} &\equiv \mathbf{1}\left\{P\left(x,y,z,\Gamma\right)>P\left(x,\tilde{y},z,\Gamma\right)\right\} \\
BS\left(x,y,z,\Gamma\right) &= \left[B\left(x,z,\Gamma\right)+\beta S\left(x,y,z,\Gamma\right),P\left(x,y,z,\Gamma\right)\right] \\
S_{xyz} &\equiv \mathbf{1}\left\{S\left(x,y,z,\Gamma\right)\geq 0\right\}.
\end{aligned}$$

A.3 Solution algorithm

A.3.1 Preliminaries

As can be seen from (9) and (10), the values B and P depend on Γ' through the job finding rate, and thereby the entire expected next period distribution of matches across x and y and unemployed workers distribution over x. The challenge is to reduce the dimensionality of the distributions Γ' to something manageable. The key to our algorithm is to note that all influence of the endogenous distributions goes through the next period labor market tightness, θ'. In addition, according to (7) labor market tightness is only a function of J in (14). Hence, we can write θ as a function of the three moments that make up (14); $\theta=\Theta\left(m_1,m_2,m_3,z\right)$. In particular, noting that $\sum_{x\in X}\sum_{y\in Y}h^s\left(x,y,z\right)=1-\sum_{x\in X}u^s\left(x,z\right)$ and accordingly $L_t\equiv\sum_{x\in X}u^s\left(x,z\right)+s_1\left(1-\sum_{x\in X}u^s\left(x,z\right)\right)$ we set

$$m_1=\sum_{x\in X}u^s\left(x,z\right). \tag{34}$$

Given that L_t can be computed using m_1, equation (14) implies that J is fully determined by the parameters β, s_1, the moment m_1, and the following two terms:

$$m_2=\frac{\sum_{x\in X}\sum_{y\in Y}u^s\left(x,z\right)\max\left\{S\left(x,y,z,\Gamma\right),0\right\}g\left(y\right)}{\sum_{x\in X}u^s\left(x,z\right)} \tag{35}$$

and

$$m_3=\sum_{x\in X}\sum_{y\in Y}\sum_{\tilde{y}\in Y}h^s\left(x,\tilde{y},z\right)\max\left\{S\left(x,y,z,\Gamma\right)-S\left(x,\tilde{y},z,\Gamma\right),0\right\}g\left(y\right). \tag{36}$$

To compute next period values of these moments we assume a linear-quadratic relationship to today's moments.[23] Thus, we write

$$m_i'=H_i\left(m_1,m_2,m_3,z,z'\right)\ \text{for } i\in\{1,2,3\}. \tag{37}$$

[23]In practice we only use the non-linear term m_1m_2.

Note that, similarly to LR, we can compute the evolution of the distributions u^s and h^s and θ without solving for wages and worker values. However, in contrast to LR, match surpluses and the value of unemployment is jointly determined with (tomorrow's) labor market tightness. Therefore we guess linear functions Θ and H_i for labor market tightness and the evolution of moments. We can then compute match values. Given the solution for match values we can compute the allocation for a sequence of aggregate productivity shocks and then update the guesses for Θ and H_i using standard estimation methods and iterate until convergence (see Krusell and Smith (1998)).[24] Given the above arguments it is unsurprising that the R^2 of the function $\Theta(m_1, m_2, m_3, z)$ is approximately unity (≥ 0.9997). It turns out that $H_i(m_1, m_2, m_3, z, z')$ also has a high R^2. Below in section A.3.4, we report accuracy test following the method in Den Haan (2010). In the end, we can replace the distributions in Γ' by (m_1, m_2, m_3) so that instead of (w, x, y, z, Γ) the final state vector is $(w, x, y, z; m_1, m_2, m_3)$. We discretize m_i on a grid. We choose fewer gridpoints for m_i (3 gridpoints) than for z as m_i is quantitatively less important. With the functions Θ and H_i at hand, we solve for values B and S and then residually compute P.

A.3.2 Detailed algorithm

Equilibrium without aggregate volatility Obtain the equilibrium without aggregate volatility (for a fixed $z = \bar{z}$) by the following steps:

Step 1. Guess the ergodic job finding rate f.

Step 2. Use value function iteration to solve for ergodic B and P jointly. Note that the ergodic versions of B and P corresponding to expressions (9) and (10) can be written as a function of x, y, $\bar{z}$ and f only. Then compute ergodic S along the lines of (11), i.e., as $P - B$.

Step 3. Compute the ergodic distributions for $u(x)$ and $h(x, y)$ (see below for details).

Step 4. Compute the equilibrium job finding rate f'. If f' is close to f then we are done. Otherwise set $f = df' + (1-d) f$ (where $d \in [0, 1]$ is a dampening parameter) and return to Step 2.

To obtain the ergodic distributions for $u_{t+1}(x)$ and $h_{t+1}(x, y)$ simulate above until convergence in these distributions.

Equilibrium with aggregate volatility Obtain the equilibrium with aggregate volatility by the following steps:

Step 1. Draw a sequence $\{z_t\}_{t=0 \ldots T}$ and guess functions Θ and H_i.

Step 2. Use value function iteration to solve for $B(x, z, \Gamma)$ in (9) and $S(x, y, z, \Gamma)$ in (11) jointly, interpolating next period values over next period moments.

[24] Specifically, we use $m_i' = H_i(m_1, m_2, m_3, z, z')$ to obtain next period moments in $\theta' = \Theta(m_1', m_2', m_3', z')$.

Step 3. For each t, guess current moments (m_1, m_2, m_3).

i) Interpolate S on the moments.

ii) Given interpolated S, we can solve for the allocation objects we are interested in:

iii) Calculate $u_t^s(x)$ and $h_t^s(x, y)$ using (1) and (2)

iv) Calculate L_t by aggregating over $u_t^s(x)$ and $h_t^s(x, y)$

v) Calculate J_t using (14).

vi) Calculate θ_t using (7)

vii) Calculate V_t using (6)

viii) Calculate $u_{t+1}(x)$ and $h_{t+1}(x, y)$ using (15) and employment transition (16)

ix) Compute updated moments $(m_1^{new}, m_2^{new}, m_3^{new})$

x) If $(m_1^{new}, m_2^{new}, m_3^{new})$ is close to (m_1, m_2, m_3) we are done. Otherwise, return to i).

Step 4. Update the functions Θ' and H_i' using the regressions described in A.3.1 with the time series for m_1, m_2 and m_3 and tightness θ. If Θ' is close to Θ we are done. Otherwise, return to Step 2 with the new guess.

Given the sequence based on $\{z_t\}_{t=0...T}$ above, we use the resulting sequence of θ (after removing an initial burn-in period) to compute allocations and wages and then the sequence of h_{t+1}^w to compute relevant moments of the wage distribution along the sequence where we have followed the algorithm described in section A.3.3 to compute worker values $W(w, x, y, z, \Gamma)$ and wages $w(w, x, y, z, \Gamma)$.

A.3.3 Algorithm for determination of W and w

With the functions Θ and H_i obtained in section A.3.2, we solve for worker values W, noting that the state vector is $(w, x, y, z; m_1, m_2, m_3)$. The solution is obtained by value function iteration, interpolating next period values over next period moments.

Once we know the worker values W we can solve for wages w residually. This amounts to rewriting equation (21) to find the wage that yields the right value of W for the current state vector $(w, x, y, z; m_1, m_2, m_3)$ given the expected future values for the worker. In all computations related to wages we interpolate linearly over the moments.

A.3.4 Accuracy tests

In spite of its limitations as an accuracy measure we start by reporting the R^2 of the perceived law of motion. The R^2 of Θ is 0.99995 while the prediction regressions H for m_1, m_2 and m_3 have the following R^2 : 0.99751, 0.99536 and 0.99948, respectively.

Both Krusell and Smith (1998) and Den Haan (2010) discuss the limitation of one-period ahead R^2. Evaluation of long-run forecasts is useful because errors can potentially cascade leading to divergence of the perceived law of motion. We accordingly report both the one-period ahead and the long-run (100-period ahead) difference between actual and perceived law of motion for θ in Table 8. We report both the mean, mean-absolute and max error between the perceived and true law of motion. Let $\hat{u} = \hat{\theta}_{t+1} - \theta_{t+1}$ and $|\hat{u}_t| =| \hat{\theta}_{t+1} - \theta_{t+1} |$. Furthermore, let $\hat{u}_{100}^{\max} = \max_t u_{t,t+100}$ where $u_{t,t+100} = \theta_{t,t+100} - \tilde{\theta}_{t,t+100}$ where $\theta_{t,t+100}$ is the variable computed using the true law of motion and $\tilde{\theta}_{t,t+100}$ the perceived law of motion, i.e., using (1), (2), (7), (15) and (16) for the true law of motion and the functions $H_i(m_1, m_2, m_3, z, z')$ and $\Theta(m_1, m_2, m_3, z)$ for the perceived law of motion.

There are four key takeaways from the accuracy results reported in Table 8. First, the maximum errors are reasonably small. Second, the average errors, which are what matters for the cost of business cycles, are negligible, less than 0.005 percent (<0.015 percent for the 100 period horizon). Third, the mean absolute errors $|\hat{u}|^{avg}$ one period ahead are small. Finally, and importantly, we note that the maximum errors and the average errors tend to not increase in the horizon. The mean absolute errors are increasing in the horizon, but to a moderate degree. Comparisons across models are merely indicative, but we note that $\hat{u}^{\max}(\theta)$ and $|\hat{u}|^{avg}(\theta)$ are below the corresponding $\hat{u}^{\max}$ and $|\hat{u}|^{avg}$ for the law of motion of capital for the Krusell and Smith (1998) model reported by Den Haan (2010).

Table 8: Accuracy test results for θ using 200 replications with $T = 6000$.

Horizon	1	100
$\hat{u}^{\max}$ (%)	1.43	1.43
$\hat{u}^{avg}$ (%)	0.00	-0.01
$\|\hat{u}\|^{avg}$ (%)	0.10	0.58

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