SVERIGES RIKSBANK

Staff memo

Common features in short maturity interest rate forecasts

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A staff memo provides members of the Riksbank's staff with the opportunity to publish slightly longer qualified analyses of relevant issues. It is a publication for civil servants that is free of policy conclusions and individual standpoints on current policy issues. Staff memos are approved by the appropriate Head of Department.

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Summary

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In this memo, we summarize the results of forecasting comparisons of central bank policy rates and closely related short maturity interest rates. We analyse interest rate forecasts by the central banks in Sweden, Norway and New Zealand, market-based forecasts and forecasts by other forecasters in Sweden. We also look at the forecasts made by the Czech National Bank, market-based forecasts in the US and the dot plots of the FOMC.

The results show that everyone (different forecasters in different periods and in different countries) has overestimated the actual policy rate (or a closely related short maturity interest rate) on average since 2007. The overestimation of the interest rate is similar, roughly around 0.5 percentage points at the one-year horizon and between 1.5 and 2 percentage points at the two-year horizon. Relatively few forecasters publish forecasts over a three-year horizon, but for those that do, the overestimation is between 2 and 3 percentage points on average. Generally, the forecasts are rather similar but the forecast errors by the Riksbank are somewhat larger, even compared to other central banks.

A likely cause for the general overestimation is the fall in global real (risk-free) interest rates since the last two decades. This phenomenon has happened over a prolonged period and the successive change has been difficult to forecast. Also, since the financial crisis, negative surprises in the world economy have been more common than positive ones. This has most likely contributed to the overestimation of interest rates.

We show that simple time series models can produce forecasts with relatively low overestimation (low bias). The reason for this is that recursive estimates of the models to some extent can address the declining trend in interest rates. However, this does not mean that these methods will produce accurate forecasts going forward.

[1] The author wishes particularly to thank Ulf Söderström, Gabriela Guibourg, André Reslow, Rafael Barros De Rezende, Marie Hesselman and Gary Watson. The opinions expressed in this memo are those of the author and are not necessarily shared by the Riksbank.

Common features in short maturity interest rate forecasts

Introduction

Since 2007, the Riksbank has published forecasts of the future repo rate, the repo rate path. After a decade, it is natural to evaluate some of the experiences of forecasting interest rates, and in particular the Riksbank experience. In this memo, we analyse the forecast errors made by the Riksbank and other central banks. We also look at the short maturity interest rate forecasts made by other analysts. We conclude that both the central banks and other forecasters have overestimated the actual interest rates and discuss that a likely explanation for this is the gradual decline of the real (risk-free) interest rate. Also, the predominantly negative shocks to the world economy since 2008 have most likely contributed to the general tendency of overestimating interest rates. Using simple models, we show that forecasts with relatively small forecast errors can be produced. We discuss the reasons behind this and briefly touch upon what lessons that could be learnt from these simple models.

Forecasts by central banks and others

In this section, we analyse forecasts of central bank policy rates and other, closely related short maturity interest rates since 2007. It turns out that a wide range of forecasters have very similar forecast errors. There is a pronounced tendency towards overestimating the interest rate on average, and towards the bias also growing over the forecast horizon. There are differences between forecasters, but the similar traits in the forecast errors (even over different time periods) are perhaps what stands out the most.

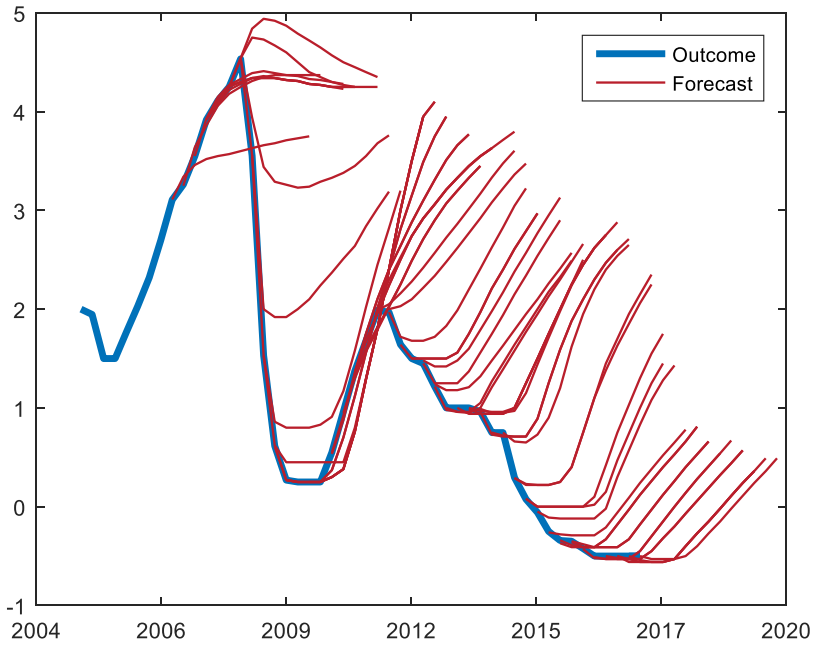
The repo rate forecasts by the Riksbank

Since 2007, the Riksbank has published forecasts of the future repo rate. Relatively few central banks have chosen to publish forecasts for the policy rate (or a closely related short maturity interest rate). Besides the Riksbank, Norges Bank and the Reserve Bank of New Zealand (RBNZ) regularly publish forecasts for the policy rate or a closely related short maturity interest rate.[2] The Czech National Bank (CNB) publishes staff forecasts of a short maturity interest rate since 2008.

Figure 1 shows the forecasts of the repo rate by the Riksbank since 2007. The figure clearly illustrates the tendency of the forecast to overestimate the outcome. The forecasts made before the financial crisis markedly overestimated the repo rate. During the period of interest rate increases in 2010, the forecasts did accurately capture the development on shorter horizons, but for the longer horizons, the forecasts overestimated the outcome. The forecasts during the period of interest rate decreases, starting in 2012, have generally overestimated the outcome.

[2] The RBNZ has previously published forecasts for a three-month interbank rate, but has recently started forecasting the policy rate.

Figure 1: Repo rate forecasts by the Riksbank since 2007
Percent



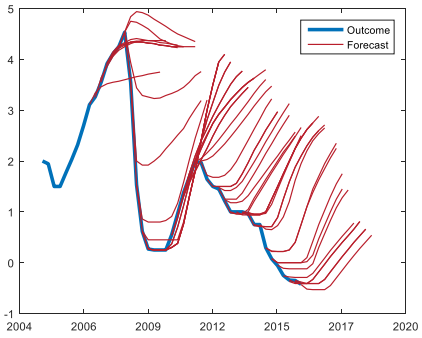
Source: The Riksbank

The question is then how the Riksbank forecast compares to other forecasts of the repo rate, forecasts made by other central banks and the forecasts that can be calculated or directly extracted from financial market data (based on forward rates).

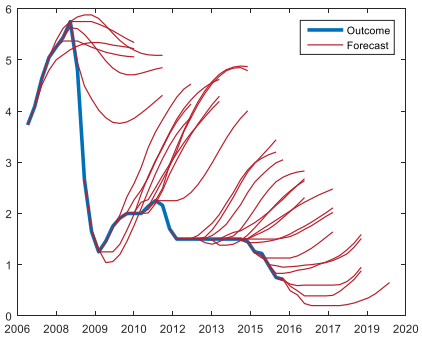
The forecasts by the Riksbank and other central banks
As already mentioned, the Riksbank, Norges Bank and RBNZ together with the CNB are among the few central banks that regularly publish a forecast of the policy rate (or of a short maturity interest rate closely related to the policy rate). When we analyse the forecasts, we can conclude that, in general, central banks have had a tendency to overestimate the outcome. This tendency is evident in Figure 2.

Figure 2: Interest rate forecasts by central banks
Percent

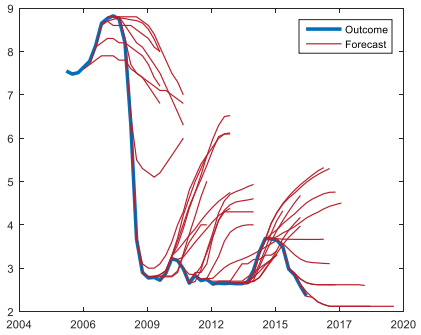
The Riksbank



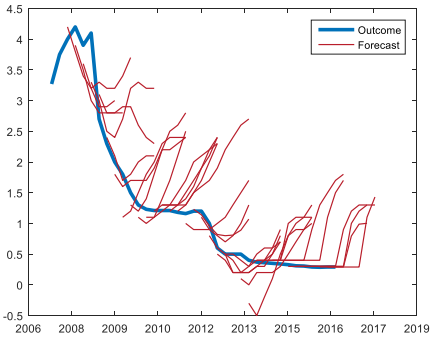
Norges Bank



Reserve Bank of New Zealand (RBNZ)



Czech National Bank (CNB)

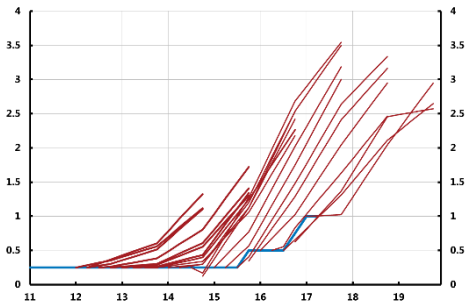


Source: The Riksbank, Norges Bank, Reserve Bank of New Zealand and Czech National Bank

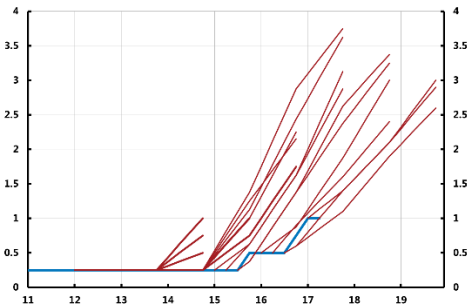
Even if the FOMC's so-called dot plots are not forecasts in the strict sense, and the period for publications of the Dot Plot has been relatively short, we show the FOMC's mean and median view of the future federal funds rate in figure 3.[3] Also these forecasts has tended to overestimate the actual interest rate.

Figure 3: FOMC's mean and median forecast of the future federal funds rate
Percent

Mean



Median



Source: Board of Governors

There are differences in the frequency at which central banks publish forecasts. The Riksbank publishes forecasts six times a year, and Norges Bank previously published three times a year

[3] The FOMC 'forecasts', the so-called dot plot depicts how individual voting members of the FOMC view the future path of the federal funds rate over the coming years. The dot plot has been published since 2012 and considering the short period and the special nature of the dot plot, we have no formal forecast evaluation for the FOMC.

but has more recently published four forecasts a year. This is also with the same frequency at which the RBNZ, CNB and the FOMC publish forecasts.

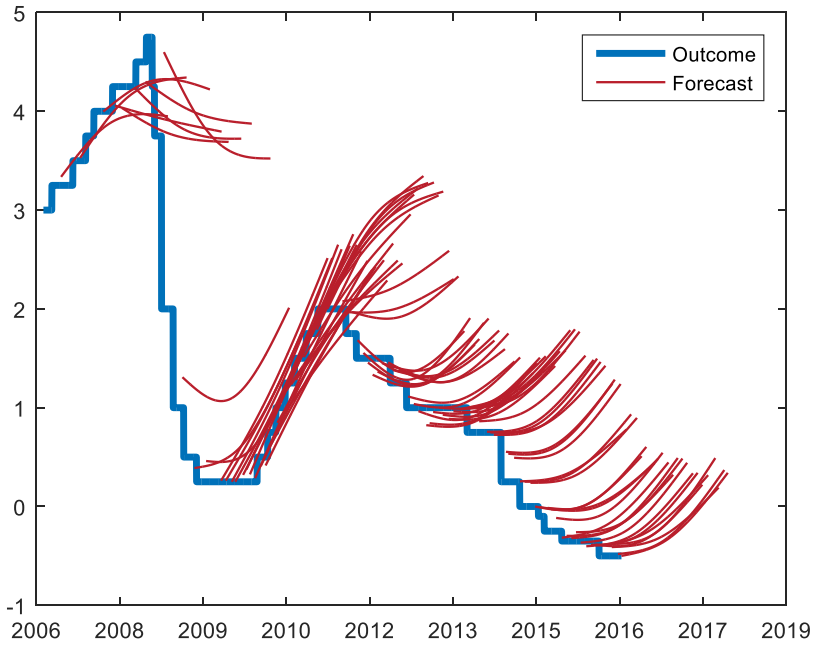
There is a tendency for the overestimation made by the Riksbank to be somewhat larger compared to other central banks. On a two year horizon the average overestimation for the Riksbank and the RBNZ is 2.0 percentage points whereas the average error for Norges bank is 1.8 percentage points. On a three year horizon, the overestimation by the Riksbank is 2.8 percentage points and 2.5 and 2.6 percentage points by the RBNZ and Norges bank, respectively. **On the whole, the most prominent feature is that the forecast errors by central banks look very similar and are of the same order of magnitude**.

Other forecasts of the repo rate

In Sweden, the repo rate expectations of money market players and others have been measured since the end of the 1990s by Prospera on behalf of the Riksbank.[4] Prospera asks money market players about their expectations regarding the future repo rate at one-, four- and eight-quarter horizons (three months, one and two years). The survey was initially carried out four times a year, but since 2010 it has also been carried out once a month for a subset of the respondents (money market players).

Figure 4: Money market players' expectations of the repo rate according to the survey by Prospera[5]

Percent



Source: Prospera

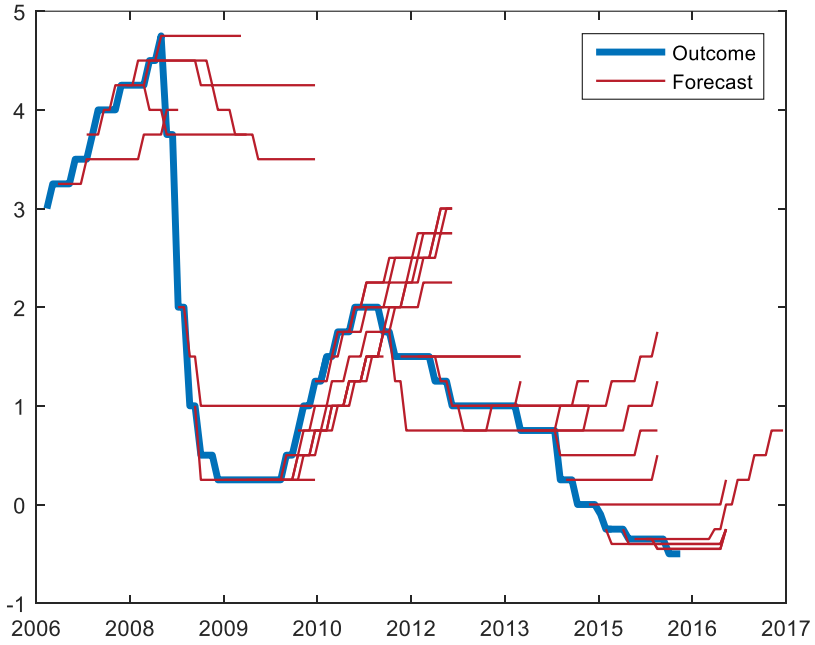
Figure 4 shows money market players' expectations of the future repo rate according to the survey by Prospera. At the one-year horizon, the average overestimation (bias) is about 0.5 percentage points. At the two-year horizon, the overestimation has been, on average, about 1.6 percentage points since 2007. Since the late 1990s, the overestimation on a two-year horizon is slightly lower at 1.3 percentage points. The survey by Prospera also measures respondents' expectations of the level of the repo rate in five years' time. In the survey, respondents' expectations have been revised downwards from about 3.25 percent in 2005, when the survey first started to include the question, to roughly 2.0 percent today.

[4] http://www.prospera.se/

[5] In the figure, the forecasts have been interpolated to create a smooth curve.

The National Institute of Economic Research (NIER) has published forecasts of the repo rate since the mid-2000s, but the forecast horizon varies and the longest horizon can sometimes be even less than two years. Forecasts for the three-year horizon are not published.[6] NIER publishes forecasts three to four times a year and has published forecasts of the repo rate since 2007.

Figure 5: NIER's forecasts of the repo rate
Percent



Source: National Institute of Economic Research

Figure 5 shows that also the NIER has had a tendency to overestimate the repo rate on average. On forecast horizons one and two years ahead, the overestimation is 0.5 and 1.4 percentage points, respectively. This is in line with the errors made by money market players in the Prospera survey.

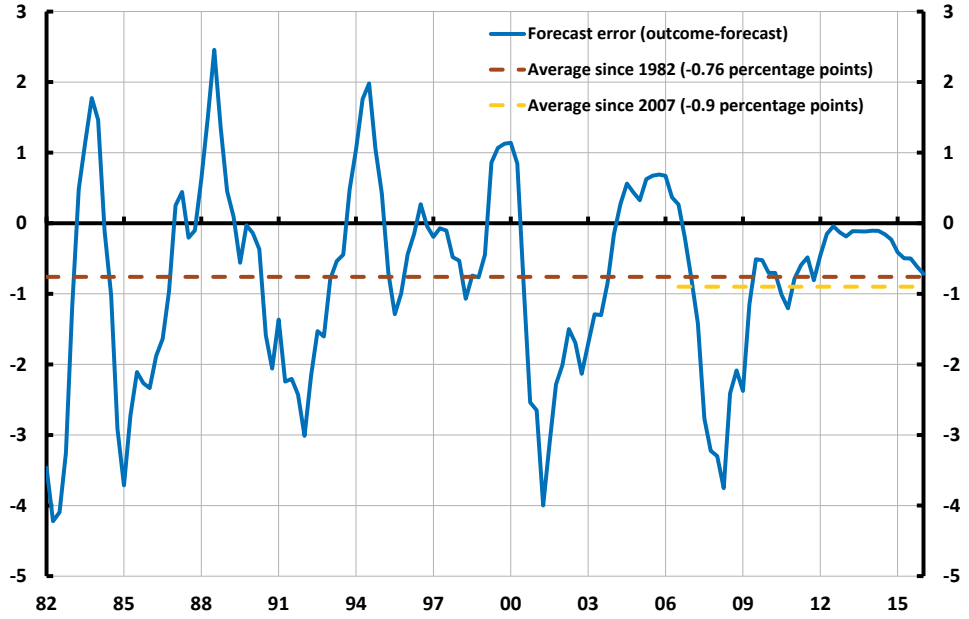
Examples of overestimation of the short interest rate in other countries

The tendency of market participants and others to overestimate the interest rate is not just a Swedish phenomenon. In the US, different professional forecasters' expectations of the short maturity interest rate at the one-year horizon have been measured in the 'Survey of Professional Forecasters' since the early 1980s by the Federal Reserve Bank of Philadelphia. This survey is the oldest running quarterly survey in the US.[7] The interest rate expectations of a three-month maturity T-bill is usually closely related to the federal funds rate, the main policy instrument of the Federal Reserve. The survey shows that respondents have overestimated the interest rate at the one year horizon between 0.7 and 0.8 percentage points on average since the beginning of the 1980s. Since 2007, the overestimation is 0.9 percentage points. The forecast errors (the difference between the actual outcome and the forecast) are illustrated in Figure 6.

[6] Here it is important to distinguish between 'forecasts' and 'scenarios'. NIER publishes scenarios for the repo rate for longer horizons. But, these are not forecasts in the usual meaning and according to NIER, it is a misrepresentation to consider them forecasts.
[7] https://www.philadelphiafed.org/research-and-data/real-time-center/survey-of-professional-forecasters/

Figure 6: Forecast errors at the one-year horizon for the three-month T-bill interest rate, according to the Survey of Professional Forecasters

Percent



Source: Federal Reserve Bank of Philadelphia

Since the beginning of the 1990s, the survey also measures the expectations of the three-month T-bill interest rate five and ten years ahead on an annual basis. In the early 1990s, the T-bill was expected to be around 5 percent in five years' time, but in the latest survey, the expectations had fallen to 2.5 percent.

Interest rate forecasts according to financial market data (forward rates)[8]

A common benchmark for forecasts of a short maturity interest rate is to compare with information from financial market data.[9] The Riksbank, Norges Bank and RBNZ are among the few central banks that publish interest rate forecasts and have also functioning money and bond market to be able to extract interest rate forecasts from financial market data (forward rates). However, these economies are small open economies with relatively small domestic financial markets. It is therefore interesting to compare with forecasts derived from US financial market data as it is one of the world's largest and most liquid markets. The US money market also has derivatives that are directly tied to the federal funds rate.[10] A summary of forecasts from financial market data in Sweden, Norway, and New Zealand and in the US is illustrated in Figure 7.[11]

[8] It is important to note that the existence of forward term premiums makes the interpretation of forward rates as measures of expectations less straight forward. This is discussed in Alsterlind (2017).

[9] Either the future policy rate (or a closely related interest rate) is directly priced, or else it can easily be computed. Whether directly or indirectly measured, the future interest rate so calculated is not the same as the expected future interest rate as financial market data usually contains compensation for risk.

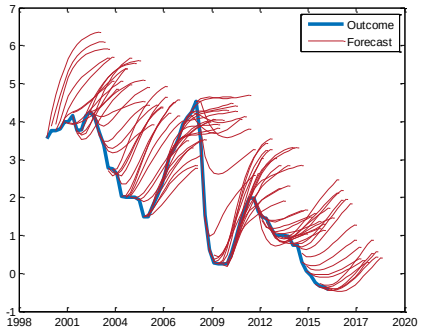
[10] In Sweden, there is a corresponding derivative contract in the RIBA. The RIBA is a forward rate agreement where the underlying security is the repo rate. However, liquidity, especially on longer horizons, is limited.

[11] For Sweden, the financial market data used is RIBA and FRA, stripped of credit risk premiums. For the US, the corresponding data is a combination of federal funds futures and OIS swap rates, stripped of credit risk premiums. The calculations have been carried out by the Riksbank. For New Zealand and Norges Bank, the data used is derivatives on the interbank market and calculated by the respective central bank. A mechanical procedure, that produces the most favourable forecast error, has been applied to strip out the credit risk premium for Norway and for New Zealand. Documentation of that procedure is available on request (in Swedish).

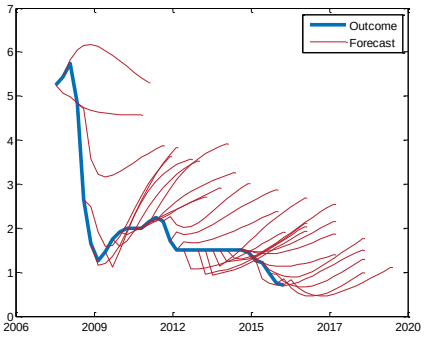
Figure 7: Forecasts of the short maturity interest rate according to financial market data (forward rates) in Sweden, Norway, New Zealand and the US

Percent

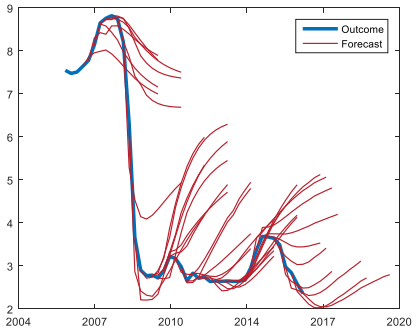
Sweden



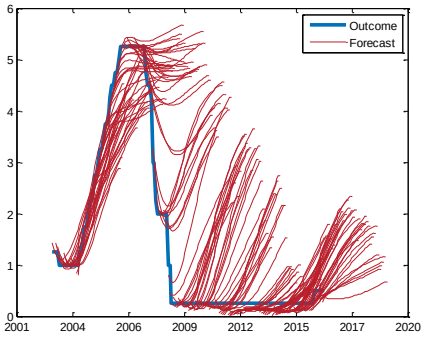
Norway



New Zealand



USA



Source: The Riksbank, Norges Bank, Reserve Bank of New Zealand and Bloomberg

Figure 7 illustrates that in all countries, for most of the time, financial market data have overestimated the interest rate on average. In both Sweden and the US (where we show a longer sample), it is evident that the financial market data underestimated the outcome during the period of interest rate increases in 2005-2007. The tendency to overestimate in times of interest rate cuts and underestimate in times of interest rate increases is something that is discussed by Goodhart and Lim (2011).

On average, especially since the financial crisis, the level of the short maturity interest rate has been overestimated. Most remarkable is that the average overestimation (bias) is of the same magnitude for all countries. At the one-year horizon, the bias is about 0.5 percentage points and on the two- and three-year horizon respectively, the bias is 1.5 percentage points and about 2.0 percentage points.[12] This is remarkable as monetary policy in the different countries has been different, and changes in interest rates have been dissimilar.[13] It is not unreasonable to expect small open economies to be subjected to larger economic disturbances. Also, despite the clear signalling from the FOMC to keep the interest rate low over a prolonged period, financial market data suggest that for a long time, money market players expected the interest rate to increase well before it actually did.

A summary of forecast errors for short maturity interest rates

The tendency to overestimate on average is something that seems to be robust across different time periods, between different forecasters and in different countries.

In Table 1, we summarize two common statistical measures for forecast errors. The mean error (ME) indicates whether the forecast tends to be an over- or underestimation on

[12] Financial market data in New Zealand has a relatively larger mean error, whereas in the US, the mean error is relatively smaller.
[13] Here, we disregard the fact that monetary policy is not solely conducted using the policy rate.

average. As the forecast error in this case is defined as the outcome minus the forecast, a negative number is indicative of an overestimation on average. Another usual statistical measure of forecast errors is the root mean square error (RMSE). This is a measure of forecast accuracy, as it especially punishes large deviations.

Table 1: ME and RMSE for forecasting errors for the short maturity interest rate, different forecasters

Percentage points

	ME (bias)[o]				RMSE			
Forecaster/country	Q1	Q4	Q8	Q12	Q1	Q4	Q8	Q12
Sweden								
The Riksbank (2007-2016)	-0.0	-0.7	-2.0	-2.8	0.1	1.2	2.4	2.9
NIER (2007-2015)	0.0	-0.5	-1.4	NaN	0.2	1.2	2.1	NaN
Prospera (2007-2016)	-0.1	-0.5	-1.6	NaN	0.3	0.9	1.8	NaN
Prospera (1998-2016)	*-0.1*	*-0.5*	*-1.3*	*NaN*	*0.3*	*0.9*	*1.7*	*NaN*
Financial market data (2007-2016)	-0.1	-0.7	-1.6	-2.0	0.3	1.3	2.0	2.2
Financial market data (2000-2016)	*-0.1*	*-0.6*	*-1.4*	*-2.1*	*0.3*	*1.1*	*1.9*	*2.5*
Norway								
Norges Bank (2007-2016)	-0.0	-0.6	-1.8	-2.6	0.0	1.2	2.1	2.7
Financial market data (2008-2016)	0.0	-0.5	-1.2	-1.8	0.0	1.3	2.0	2.2
New Zealand								
RBNZ (2007-2016)	-0.0	-0.7	-2.0	-2.5	0.1	1.3	2.7	2.9
Fin. market data, OCR (2007-2016)	0.1	-0.5	-1.8	-2.6	0.3	1.2	2.5	3.0
Czech Republic								
CNB (2008-2016)	0.1	-0.4	-1.2	NaN	0.3	0.6	1.3	NaN
US								
Financial market data (2007-2016)	0.0	-0.5	-1.5	-2.3	0.2	1.0	2.1	2.7
Financial market data (2003-2016)	*0.0*	*-0.3*	*-1.0*	*-2.0*	*0.2*	*1.0*	*2.0*	*2.9*
Survey of Prof. Forc. (2007-2016)	-0.3	-0.9	NaN	NaN	0.5	1.3	NaN	NaN
Survey of Prof. Forc. (1982-2016)	*-0.2*	*-0.8*	*NaN*	*NaN*	*0.6*	*1.6*	*NaN*	*NaN*

[o] The forecast error is defined as outcome minus forecast. A negative value is indicative of an overestimation.

In Table 1, the tendency is consistent between forecasters. Forecast errors in terms of the RMSE are similar and grow over the forecast horizon. From the mean error (ME), it is evident that every forecaster has on average overestimated the outcome of the short maturity interest rate, for almost every forecast horizon. The mean error also grows over the forecasting horizon. There is a tendency for central banks to have slightly larger forecast errors, compared to financial market data, but there are exceptions for some horizons and some measures. Also, there is a tendency for the Riksbank to have somewhat larger forecast errors compared to its peers and others, especially at the two- and three-year horizon.

All forecasts of the repo rate are uncertain: a digression

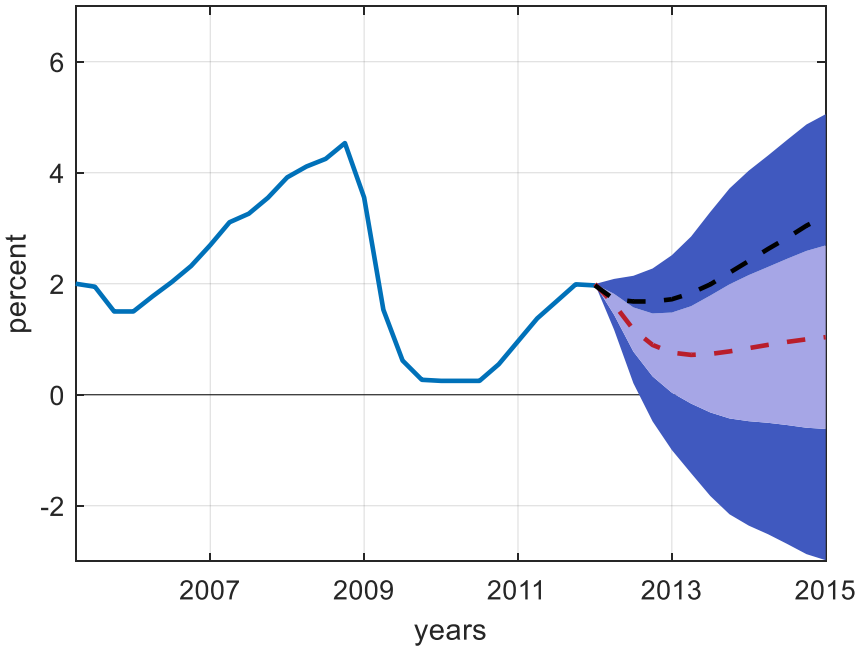
In the debate on monetary policy, there has been particular focus on the large deviations between the Riksbank forecast of the repo rate and the forecast derived from financial market data during the period 2011-2012. But considering the normal level of forecast uncertainty, it is not clear what should be considered a large deviation.

All forecasts are uncertain

The marked difference between the Riksbank forecast and the forecast derived from financial market data during the period 2011-2012 is evident in Figure 2 and 7. However, the common practice is to relate the forecast to the uncertainty that prevails. The Riksbank routinely shows the forecast of the repo rate in conjunction to its own historical forecast uncertainty. In Figure 8, we instead show the forecast of the repo rate, derived from financial market data, in December 2011 together with the historical forecast uncertainty, also derived from financial market data. We also include the Riksbank forecast, as at the time, the discrepancy between the forecast by the Riksbank and by the market where at the largest.

Figure 8: Financial market forecast (in red), forecast uncertainty and the Riksbank forecast (in black) in December 2011

Percent



Source: The Riksbank and the author's own calculation

Remark: The lightly blue shaded area represents a 50 percent forecast uncertainty interval and the darker area represents the 90 percent forecast uncertainty interval.

There is undeniable evidence that the Riksbank forecast was higher than the market view at the time.[14] In economic terms, the difference was substantial but considering the average uncertainty in the forecasts derived from financial market data, the Riksbank forecast was well within the normal 90 percent forecast uncertainty interval for financial market data.[15]

Is there a simple explanation for the forecast errors?

A simple explanation for why so many seem to have overestimated the interest rates could be that the global real rate has fallen, in a trend-like manner since the last two decades, at least. This gradual decline would have been hard to detect and forecast, at least initially. It is also reasonable to believe that the long-run level of the real interest rate in a small open economy like the Swedish economy, to a large extent, is determined by such international factors.

[14] The Riksbank forecast was also higher compared to surveys, but the difference was considerably smaller, see Alsterlind (2017).

[15] One could always argue that there are better ways of constructing a forecast interval. It might be the case that a different way of constructing an interval would result in a much smaller interval and that the Riksbank forecast would be well outside such an interval.

Global real interest rates has trended downwards

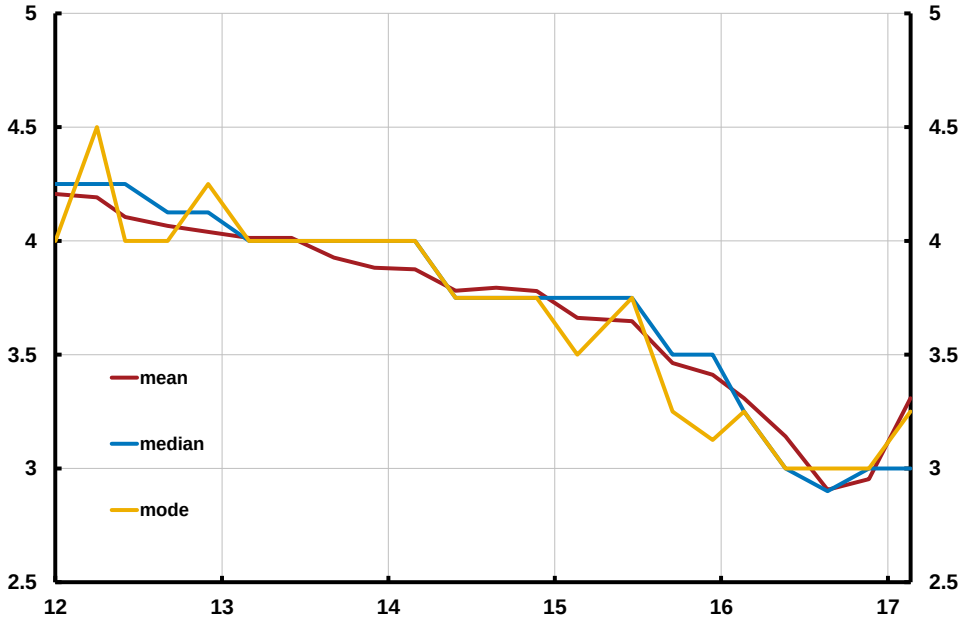
One part of the explanation for the general overestimation is that for more than 20 years, global real (risk free) interest rates have fallen and are at historically low levels. Armelius et al. (2014) discuss some of the underlying factors behind this. Factors like changes in productivity growth, demand for capital, a high demand for savings globally and an increased demand for safe assets can, to different extent, explain why global (risk-free) interest rates might have fallen. There are numerous examples of this discussion internationally: one example being Rachel and Smith (2015), who also try to quantify and rank the various factors' contribution to the fall in global real interest rates.

One could ask why so many, for so long time, have overlooked such an important phenomenon. But this change has been gradual, structural and taken place over a longer period of time. One usually requires a longer period of time to establish the full extent of such a change. Many analysts might have attached some weight to the change, but still had issues understanding the full magnitude of it.

A reasonable hypothesis is therefore that the forecasts made by different analysts underestimated the declining trend of the real interest rate. One example of this is that the FOMC has revised its estimate of the long-run (nominal) interest rate by over one percentage point over a four-year period, see Figure 9.[16] As we discussed earlier, surveys in the US that measure respondents' long-run (ten and five years ahead) expectations have also been revised downwards considerably.

Figure 9: The FOMC outlook for the federal funds rate in the long run

Percent



Source: Board of Governors

Other factors might also, at least to an extent, have contributed to the general tendency to overestimate short maturity interest rates. Since 2008, the world economy has been hit by large negative shocks, more so than the opposite. As an example, the financial crisis and the European debt crisis have had a clearly adverse effect on the world economy and might have contributed to lower interest rates.

[16] There are no signs that the FOMC adjusted their outlook for long-run inflation correspondingly. It is therefore plausible that the shifts in the nominal interest rate reflects a revision in the real interest rate one-to-one.

Simple models can make accurate forecasts

An interest rate forecast by a central bank often has a policy dimension and cannot be treated in a purely mechanical manner. However, a desired policy should also be realistic and possible to achieve. A natural question is then to ask if the Riksbank (and other forecasters) could have something to learn from a model with a good forecast performance. In this section, we present a very simple model that is also shown to have relatively good forecasts of the repo rate. We discuss why the simple model performs so well and what conclusion can be drawn from this. In the appendix, we discuss what more complex models could look like and what underlying assumptions matter from a forecasting perspective.

A simple forecast model for the repo rate

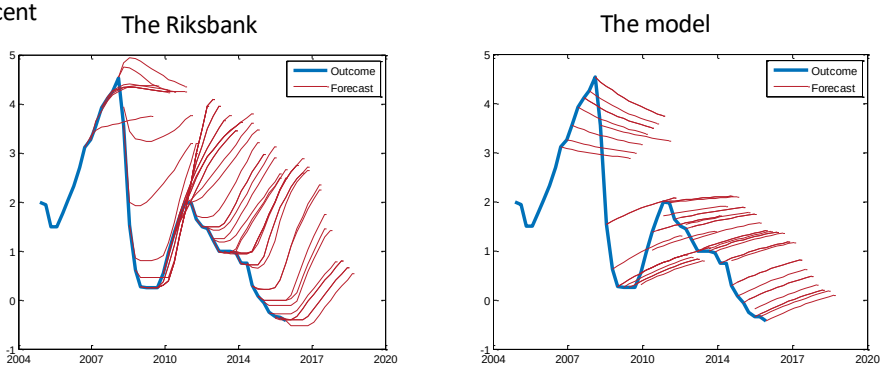
One of the simpler models we can construct describing how the repo rate moves through time is an AR (1) process:

$$i_t = c + \rho i_{t-1} + \varepsilon_t \qquad (1)$$

This model is just a time series process and says nothing about monetary policy or its relation to inflation and the economy in general. We estimate this simple model recursively to produce forecasts, only using information that is known for every forecast period, and then compare these to the Riksbank forecast.[17] In Figure 10, we see that the forecasts from the simple model (right-hand side) generally have a lower forecast of the repo rate compared to the Riksbank, especially at longer horizons.

Figure 10: The Riksbank and the AR (1)-model forecast of the repo rate

Percent



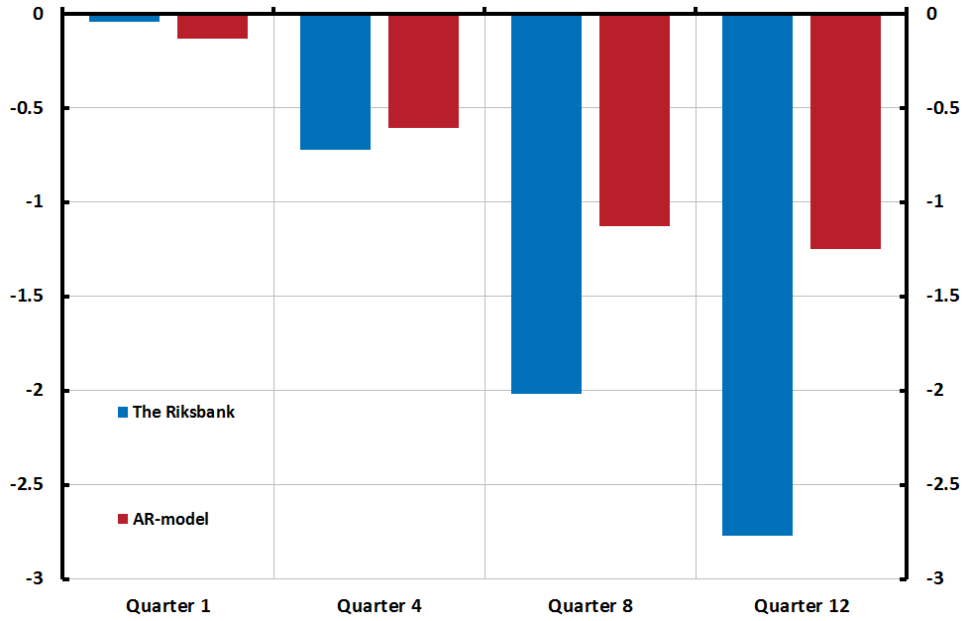
Source: The Riksbank and the authors own calculations

The mean error (ME) over a three-year horizon for the simple model is less than -1.5 percentage points, compared with the ME of the Riksbank at a three-year horizon of almost -3.0 percentage points, see Figure 11.

[17] In this case, we recursively estimate equation (1) with OLS, starting in 1993, and make a forecast that we then compare to the Riksbank forecast.

Figure 11: Mean error (ME) for the Riksbank and for the AR (1)-model forecasts of the repo rate

Percent



Source: Author's own calculation

Thus, using a simple model would have resulted in a considerably smaller forecast error compared to the Riksbank. However, it should also be noted that **the forecast errors from this simple model would also have been lower than errors from using surveys or financial market data**.

The conclusion from this simple exercise is not that forecasters should use simple models to forecast interest rates (this is especially true for central banks).[18] The relevant question here is why this simple model has such low forecast errors. The answer is simple, even trivial: Figure 10 illustrates that the simple model does not return to the mean very fast. This is a striking difference compared with the Riksbank forecast which reverts to the mean (or equilibrium level) much faster. Because the interest rate has shown a decreasing trend, the model forecasts have been relatively successful in generating smaller forecasts errors, due solely to a low degree of mean reversion. The model, with a slow mean reversion, is better adapted to capture (in a mechanical way) the slow moving gradual decline of the repo rate. Another feature is that the model constantly updates the estimate of the mean value for the repo rate. This value is successively revised downwards as the outcome of the repo rate gradually gets lower and lower.[19]

Concluding discussion

In this memo, we have discussed the overall features of forecasts of central bank policy rates (or a closely related short maturity interest rate). Over the period there has been a general tendency to overestimate the outcome, and though there are differences between forecasters, **it is the similarity among the forecast errors that is the most striking aspect**. Central banks are not alone in overestimating policy rates, even if central banks tend to have slightly larger forecast errors and the Riksbank forecast is among the worst.

The general tendency to overestimate interest rates has probably a lot to do with a global declining trend in real (risk-free) interest rates and that this trend has been difficult to forecast. Whether this really is the case, and what the underlying factors might be, is subject

[18] Quite the opposite, according to some theoretical models, a good policy also means that the interest rate is very difficult to predict.
[19] In very persistent time series processes, it is sometimes difficult to separately identify the mean and the degree of mean reversion. If the sample is short, one just has to accept that neither the mean nor the speed of mean reversion can be assessed with any precision.

to discussion internationally. In this memo, we do not discuss this further but we find it to be a reasonable hypothesis that could explain why so many different forecasters have overestimated the interest rates. Also the existence of predominantly negative shocks, with lower interest rates as a consequence, can play a part.

We also show that a simple time series model can produce forecasts with smaller forecast errors than the Riksbank, **but also lower than surveys and forecasts derived from financial market data (forward rates)**. The simple mechanism behind the model's relative success is a slow mean reversion that better captures the declining trend in the interest rate.[20] The lesson here is that, historically, a successful strategy has been not to forecast quick mean reversions of the repo rate. In a monetary policy context, this could translate to work with a time-varying neutral interest rate in the policy process or other ways to have a more flexible view on the level of the interest rate in five to ten years' time.

There is a tendency for the Riksbank to have larger forecast error on average relative to other forecasters. One possibility is that the Riksbank has assumed a faster reversion back to normal levels (that might have been too high). This would also have produced more prominent errors for longer horizon forecasts. However, even if a slow mean reversion has been a successful strategy addressing the declining trend in the interest rate, this does not mean it will be a successful strategy going forward.

[20] Even if one might prefer a time-varying mean, a slow mean reversion is sufficient to decrease the forecast errors.

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Appendix

Dynamic affine term structure models and the declining trend in interest rates

In the main text, we conclude that forecast errors for short maturity interest rates are similar, for different forecasters, over different time periods and in different countries. We argue that a likely cause of this is, for at least 20 years, a declining trend in global (risk-free) real interest rates. We show that simple time series models can produce forecasts with smaller forecast errors compared to the Riksbank and others. In this appendix, we discuss more in detail the crucial assumptions.

A dynamic affine term structure model

One of the usual ways of analysing interest rates in empirical finance is to use a dynamic (affine) term structure model. This class of models uses no-arbitrage theory, see Duffie and Kan (1996) for a general description. Here we follow De Rezende (2017a) in assuming that there is a $k \times 1$ vector of pricing factors x_t that follows a VAR (1) process under an objective probability measure $\mathbb{P}$:[21]

$$x_{t+1} = \mu + \phi x_t + \Sigma \varepsilon_{t+1}, \tag{A1}$$

where $\varepsilon_t \sim iid\ N(0, I_k)$ and Σ is a $k \times k$ lower triangular matrix. The stochastic discount factor (SDF) that prices all assets under the absence of arbitrage is assumed to be conditionally lognormal:

$$M_{t+1} = exp\left(-i_{1t} - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\varepsilon_{t+1}\right), \tag{A2}$$

where $\lambda_t = \lambda_0 + \lambda_1 x_t$ is a $k \times 1$ vector of risk prices. The one-month interest rate is then affine in the pricing factors, $i_{1t} = \delta_0 + \delta_1' x_t$. Under the risk-neutral probability measure $\mathbb{Q}$, the vector of pricing factors follows the dynamics:

$$x_{t+1} = \mu^{\mathbb{Q}} + \phi^{\mathbb{Q}} x_t + \Sigma \varepsilon_{t+1}, \tag{A3}$$

where $\mu^{\mathbb{Q}} = \mu - \Sigma\lambda_0$ and $\phi^{\mathbb{Q}} = \phi - \Sigma\lambda_1$. Under no-arbitrage, bond prices are then exponential affine functions of the state variables, $P_t^n = exp(A_n + B_n' x_t)$, where A_n is a scalar and B_n is a $k \times 1$ vector that satisfies the recursions:

$$\begin{aligned} A_{n+1} &= A_n + B_n'\mu^{\mathbb{Q}} + \frac{1}{2}B_n'\Sigma\Sigma'B_n - \delta_0 \\ B_{n+1} &= \phi^{\mathbb{Q}\prime}B_n - \delta_1 \end{aligned} \tag{A4}$$

which starts from $A_1 = -\delta_0$ and $B_1 = -\delta_1$. The n-maturity zero coupon bond yield is then computed as $i_{nt} = -n^{-1}ln(P_t^n)$.

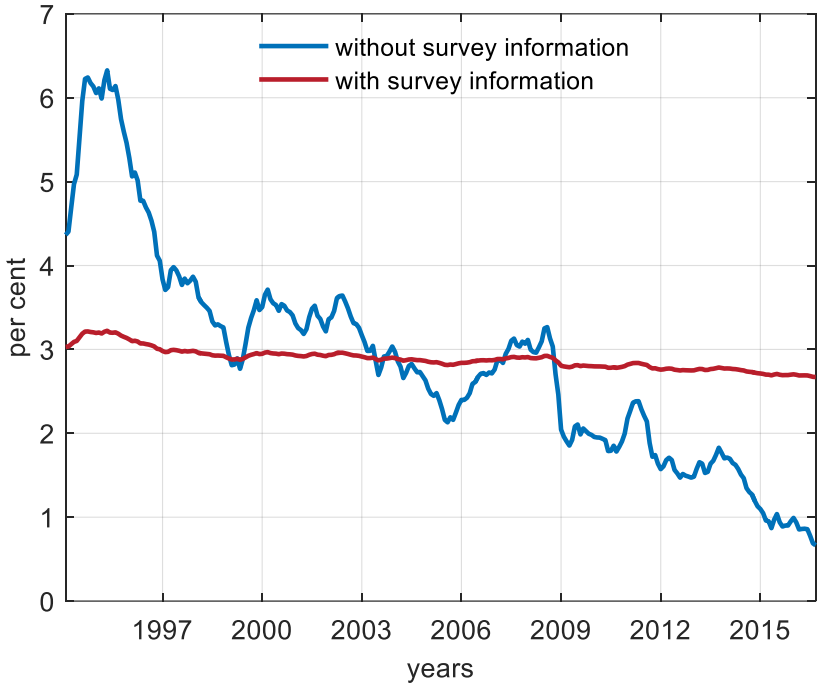
To illustrate how a slight change in the underlying assumptions can have a major effect on the models' ability to capture the declining trend in the interest rate, we also estimate a version of the model that includes survey information, following Kim and Orphanides (2012). In both models, we use monthly data for continuously compounded zero coupon yields from 1994 to 2016. In the version where we also use survey information, we include money market players' interest rate expectations at a 3-, 12-, 24- and 60-months horizon from Prospera.[22] This way, one can hope to better pinpoint the actual expectations in the yield curve, if one trusts the survey measures to reflect market participants' unobserved expectations. Both models fit the observed yields about equally well but the difference

[21] In the empirical application, we follow De Rezende (2017a) and estimate the model with two observable factors that are the two first principal components of the nominal yield curve. We choose to use demeaned factors in the estimation so μ is effectively set to zero in the estimation. We allow for a common measurement error for all yields and the model is then estimated by maximum likelihood in the Kalman filter. The code for the estimation is written in Matlab and available from the author on request.

[22] The survey was available four times a year before 2010 and every month after that. In the estimation, the survey data is treated as missing observations in the Kalman filter for those months where there is no available information. The estimation code in Matlab is available from the author on request.

between how the two models view the expected short rate in ten years' time is apparent, see Figure A1.

Figure A1: Expected one-month interest rate in ten years' time according to the models
Percent



Source: Author's own calculation

It is important to notice that the models estimate of the asymptotical one-month interest rate is almost identical to the sample average (2.9 percent) in both cases. The differences in the long-run expectations (ten years ahead) depend on the degree of mean reversion in the respective model. The model estimated without using survey information shows a relatively low degree of mean reversion, whereas the opposite is true for the model using survey information. Needless to say, the model using survey information has interest rate expectations that match the survey by Prospera rather well.

The point here is not to discuss whether one model's view on the long run expected interest rate is more plausible than the other's. Rather, the point is to highlight that the same model can deliver very different views in the long run, all depending on the assumptions.[23] In this particular case, using additional information leads to a different, and higher, estimate of the degree of mean reversion. This will also have consequences for the forecast performance of the models.

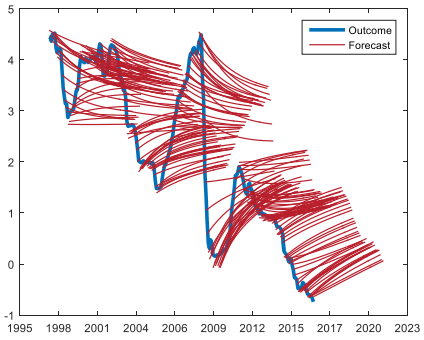
Model forecasts of the short rate

The two models have different view on the long-run interest rate, and this will also affect the respective forecasts. In Figure A2, we see that the forecast from the model without survey information is better adapted to the declining trend in the interest rate. The model where we use survey information returns faster to the sample average.

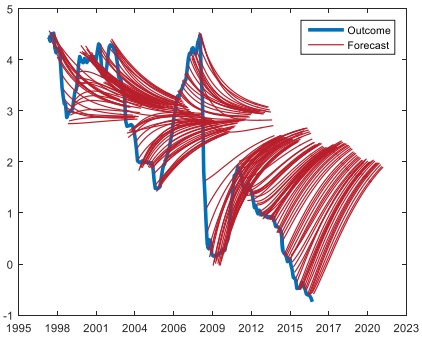
[23] This example also highlights that models with a constant mean can deliver very variable long-run expectations. A constant mean is, in this particular interpretation, not that important.

Figure A2: One-month interest rate forecasts, according to the two models
Percent

Model without surveys



Model with surveys

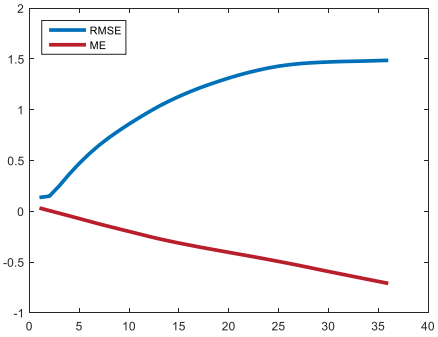


Source: authors own calculation

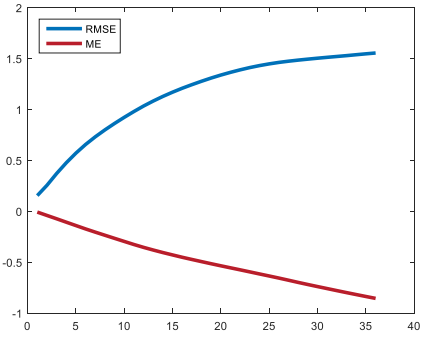
In addition to presenting the forecasts in figures, we can also calculate forecast errors and some simple forecast measures in a more formal way. In Figure A3, we show the RMSE and mean error (ME) for each model. The frequency in this case is monthly, so the scale on the x-axis is in months, e.g. the number 36 indicates RMSE and ME at a three-year forecast horizon.

Figure A3: Forecast errors, mean errors (ME) and RMSE over forecast horizons according to the two models
Percent

Model without surveys



Model with surveys



Source: Author's own calculation

Figure A3 illustrates that both models have similar RMSE. Compared to the forecast performance in Table 1 in the main text, the forecast errors of the models are rather low, at least for longer forecast horizons.

The mean error (ME), or the bias, is lower for the models without survey information. This is especially true at the two- and three-year horizons. It is clear that this model can better capture the declining trend in the interest rates, as a consequence of the lower degree of mean reversion. This improves the forecasting performance. However, this does not mean that this model is a better presentation of market participants' true expectations. If one is to believe that the survey of Prospera is at least approximately correct, the model with the slightly larger forecast error is actually the better representation of the true interest rate expectations.

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