Hedging against exchange rate risk – maturity choice and roll-over risk

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The Swedish market for hedging foreign exchange (FX) risk is about double the size of the annual Swedish gross domestic product. Key buyers of FX risk protection are Swedish insurance companies and pension funds who regularly invest in foreign currency assets, which exposes them to exchange rate risk. The dominant sellers of FX risk protection are Swedish banks. The most commonly used financial instruments are FX swaps with a duration of 3 months or less. Since the typical investment horizon of insurance companies or pension funds can span multiple years, shorter-term FX hedging arrangements need to be rolled over repeatedly. In this article, we offer a conceptual framework to discuss the risks and benefits associated with short-term hedging for six risk categories: FX risk, asset price risk, FX market distress, premature liquidation risk, counterparty risk and inflation risk. The focus is on economic considerations that have to do with uncertainty and information.

1 Introduction

A substantial part of the assets held by Swedish insurance companies, pension funds and other asset managers are issued by foreign entities and denominated in foreign currency. This is because Swedish investors seek to take advantage of investment opportunities abroad and to diversify their investment returns. Figure 1 shows a currency breakdown of the asset holdings of Swedish insurance companies and national pension funds (AP funds), who are the dominant Swedish asset managers. We can see that in 2020 almost 43% of the assets on their balance sheets were denominated in foreign currency with an important role played by US dollar and euro investments. At the same time, the vast majority of the liabilities of Swedish insurance companies and pension funds are denominated in Swedish krona. Therefore, Swedish asset managers have a “currency mismatch” on their balance sheets. In other words, the currency composition of their assets (domestic and foreign currency) and liabilities (domestic currency) differs markedly.

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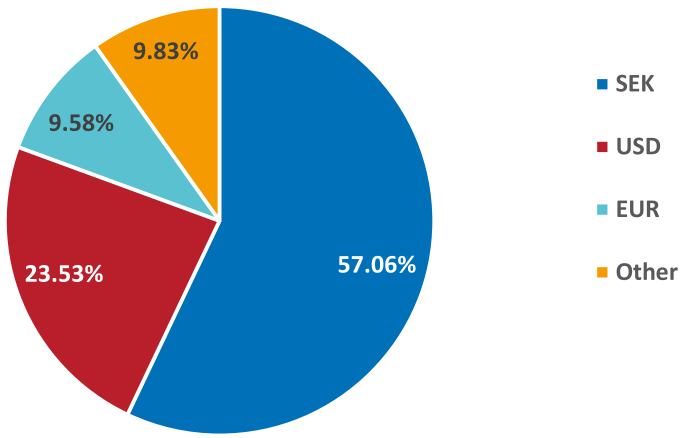
Figure 1. Currency decomposition of assets held by Swedish insurance companies and national pension funds
Per cent



Note. Currency breakdown of the consolidated asset side of Swedish insurance companies and Swedish national pension funds on December 30, 2020. The total market value of assets held amounted to 5,629 billion Swedish kronor (or 687 billion US dollar).

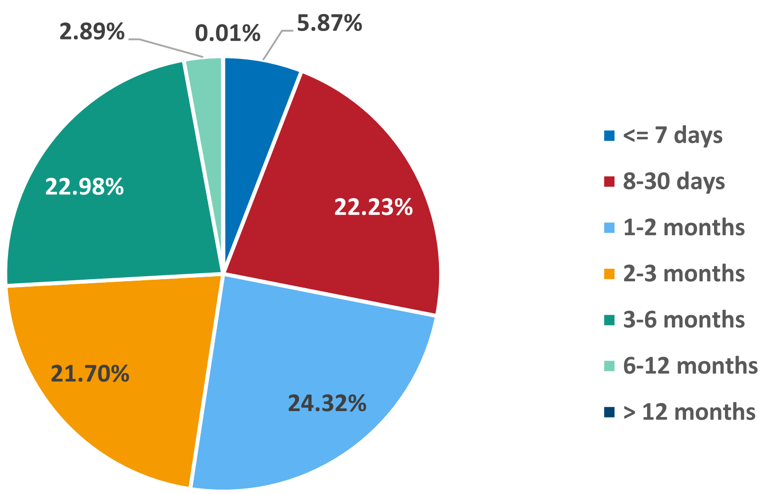
Source: Sveriges Riksbank.

The currency mismatch exposes Swedish investment portfolios to risk due to fluctuations in foreign exchange rates; an unexpected weakening of the US dollar (USD) against the Swedish krona (SEK) will reduce the krona value of assets denominated in dollars and lead to losses on the investment portfolio, and an unexpected strengthening of the dollar will lead to portfolio gains. To reduce such risk, financial market participants use various financial instruments to “hedge” their risk exposure. The most common FX hedging instruments are “FX swaps”, which consist of an FX spot transaction and a “forward contract”. In essence, FX swaps allow Swedish insurance companies and pension funds to raise foreign currency funding and to protect against exchange rate risk by agreeing to swap cash flows in different currencies at an agreed conversion rate on a future date. For example, a Swedish pension fund (protection buyer) who wants to invest in a US corporate bond can engage in an FX swap contract with a Swedish bank (protection seller) that comprises two parts: (1) the exchange of SEK against USD today through a spot transaction, which the pension fund then uses to purchase a US corporate bond; and (2) a forward contract that specifies a future date when the pension fund has to return USD and receives SEK at a pre-specified conversion rate. In this way, the pension fund can protect the income from its US corporate bond investment from an unexpected weakening or strengthening of the US dollar.

The Swedish market for hedging FX risk is large. In 2020, the total amount of outstanding contracts averaged around 10,000 billion Swedish kronor, roughly twice as large as the annual gross domestic product. The average FX hedging contract had a volume of 80–90 million Swedish kronor (Levander et al. 2021). Figure 2 shows a decomposition of the maturity profile of these contracts.[1] Importantly, more than 70% of the nominal amount of outstanding FX hedging contracts had a remaining duration of 3 months or shorter. We can see that 5.87% had a duration of less than 7 days, 22.23% had a duration of 8-30 days, 24.32% had a duration of 1-2 months, 21.70% had a duration of 2-3 months and 22.98% had a duration of 3-6 months.

Figure 2. Maturity composition of FX hedging contracts

Per cent



Note. Decomposition of the maturity profile of the nominal outstanding FX hedging contracts in the Swedish market. 2020 averages for the remaining maturity calculated over the end-of-month values. The slice corresponding to maturities exceeding 12 months (0.01% of the outstanding contracts) is not visible.

Source: Sveriges Riksbank.

The short duration of the FX hedging contracts contrasts with the typically longer duration of the foreign currency denominated assets in Swedish investment portfolios. In fact, the expected investment horizon of Swedish asset managers often spans multiple years, meaning that many of the assets in their investment portfolios have long maturities. An example for a popular foreign currency denominated asset is US government bonds with a duration of 5–10 years. Nevertheless, Swedish insurance companies, pension funds and other asset managers often use short-term FX hedging contracts to insure such investments in long-dated US government bonds.

Whenever the expected duration of the foreign currency denominated investments exceeds the duration of the FX hedging arrangements, asset managers have to rely on rolling over short-term hedging contracts. In a given month, around 250 billion Swedish kronor of US dollar FX hedges mature (Sveriges Riksbank 2021). This can pose challenges for asset managers and it can have implications for financial markets and for financial stability. The challenges are especially acute in periods of financial

[1] We thank Mats Levander for sharing the data. More detailed statistics can be found in Levander et al. (2021). While the financial market turmoil in the spring of 2020 following the outbreak of the COVID-19 pandemic had an effect (Sveriges Riksbank 2020a,b), the maturity decomposition for 2019 is similar.

distress such as the financial market turbulence during the spring of 2020, which severely affected the markets for hedging foreign exchange risk around the world.[2]

This article attempts to provide a conceptual framework to discuss the maturity choice and roll-over risk in the market for hedging FX risk. Using a simple theoretical model, we try to provide answers to questions like: What exactly are the risks associated with a duration mismatch between the underlying asset and the FX hedge? What could be potential reasons for why asset managers do not prefer longer-term FX hedging arrangements, for example hedging contracts that match the expected duration of the foreign currency denominated assets? What are the implications for episodes of financial distress?

Our focus is exclusively on aspects related to economic considerations that have to do with uncertainty and information. Therefore, we abstract from certain institutional details and other factors that can be important determinants of the supply and demand for different FX hedging products and their pricing, such as financial regulation and market power. In the article, we describe potential risks and benefits of short- and long-term FX hedging strategies stemming from uncertainty about different payoff-relevant variables (such as the exchange rate, the asset return and interest rates) and from assumptions about the flow of information over time. To this end, we distinguish between six risk categories: FX risk, asset price risk, FX market distress, premature liquidation risk, counterparty risk and inflation risk.

Exactly what risks and considerations are most relevant in relation to FX hedging arrangements is ultimately an empirical question, but we hope that this article can give some guidance by offering a suitable conceptual framework to think about a series of important economic considerations, while being flexible enough to capture additional features. A complete assessment of whether the observed FX hedging strategies are individually and socially optimal crucially hinges on the mandates of asset managers and the nature of the foreign currency denominated assets (bonds, equities, etc.). It is also contingent on expectations about the volatility of exchange rates, the persistence of exchange rate trends, inflation risk and other factors such as the microstructure of financial markets.

Taking a high-level perspective, episodes of financial market distress play a crucial role for FX markets. Severe financial distress can affect both the asset managers who seek FX risk protection and the providers of FX risk protection, who are primarily domestic banks. In normal times, asset managers appreciate the flexibility of short-term FX hedging arrangements, and we describe a number of potential advantages and disadvantages of short-term hedging strategies. However, from a policy viewpoint, an overreliance on short-term FX hedging strategies can have repercussions that are primarily felt in periods of financial distress. This is seen as a potential concern (Sveriges Riksbank 2020a), which resonates with the broader regulatory debate about the build-up of systemic risks. While this article does not intend to make normative

[2] See Sveriges Riksbank (2020a,b) for the Swedish dimension and Avdjiev et al. (2020) for the international dimension.

statements, we hope that it can offer inputs to the discussion of drivers behind the current shape of the market and point at potential policy trade-offs.

The article is organized as follows. In Section 2, we present a simple model framework that we will use for our analysis. In Section 3, we illustrate and evaluate different hedging strategies in situations characterized by different types of risk. In Section 4, we summarize our main findings and discuss how one may think about the trade-offs, the overall risk assessment and potential policy concerns.

For the fast reader, Table 2 in Section 4 offers for each of the six risk categories a summary of the potential advantages and disadvantages associated with short- and long-term FX hedging strategies.

2 Basic model

The formal analysis is based on a stylized theoretical model with two periods. This section describes the basic model used for the analysis in Section 3, where we also consider various extensions to capture different risk categories.

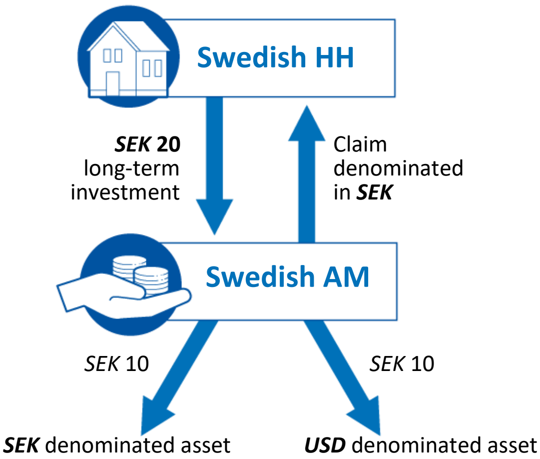
In the model, there are three dates $(t = 0, 1, 2)$ and three risk-neutral actors: a Swedish household (henceforth HH), a Swedish asset manager (AM), who is seeking FX risk protection, and a Swedish or international bank (BANK), who sells FX risk protection. The HH has an endowment of $SEK\ 20$ at $t = 0$, which she places with the AM. We consider an investment horizon that spans two periods, that is, from $t = 0$ to $t = 2$. The investment decision is delegated to the AM and we assume that the AM's objective is to seek a balanced (50%–50%) exposure to a domestic SEK-denominated asset and to a foreign USD-denominated asset, as depicted in Figure 3. It shows that the HH invests $SEK\ 20$ with the AM and receives a claim denominated in SEK. The AM, in turn, invests $SEK\ 10$, respectively, in the domestic and foreign assets. The underlying rationale is that Swedish households do not want to rely exclusively on the Swedish financial market. Instead, they want to take advantage of investment opportunities in Sweden and abroad by constructing a portfolio that comprises domestic and foreign assets.[3]

Given the HH's two-period investment horizon, the AM pursues a long-term investment strategy and expects to divest the assets at $t = 2$, thereafter converting the proceeds from the USD denominated asset into SEK. Then the funds are used to serve the SEK denominated claim of the Swedish HH. Crucially, the AM wants to insure against exchange rate risk.[4]

[3] In practice, Swedish asset managers diversify their asset holdings globally. Characteristically, small open economies have a large share of investments in foreign currency denominated assets. Important benefits of such an investment strategy include the possibility to insure against domestic shocks and to take advantage of a wider range of investment opportunities. Our stylized model does not offer micro foundations for the diversification motive and we just assume that HHs want a 50%–50% exposure.

[4] The assumption that the AM wants to insure against FX risk can be justified by the regulation of pension funds and insurance companies who are obliged to hedge certain exchange rate risks. In practice, Swedish asset managers like mutual funds and Swedish households investing directly in foreign currency

Figure 3. Investment in the basic model



We assume that the SEK and USD denominated assets deliver the cash-flows depicted in Table 1. Both investments are "long-term" with a duration of two periods. One USD invested at $t = 0$ delivers a gross return of $USD\ \tilde{r}_2^*$ at $t = 2$ if held to maturity. The investment is potentially risky, meaning that it has a stochastic return (indicated by the tilde ~). In practice, the AM may not always wait until the asset matures but sell it on the financial market at a gain or loss. If the USD denominated asset is divested prematurely at $t = 1$, the return is $USD\ \tilde{p}_1^*$. Similarly, one SEK invested at $t = 0$ delivers a gross return of $SEK\ \tilde{r}_2$ at $t = 2$ if held to maturity and a return of $SEK\ \tilde{p}_1$ if divested at $t = 1$.

Table 1. Asset returns

Returns per unit of USD or SEK invested

Time		t=0	t=1	t=2
Payoff from USD asset	If held to maturity	$-USD\ 1$		$+USD\ \tilde{r}_2^*$
	If sold prematurely	$-USD\ 1$	$+USD\ \tilde{p}_1^*$	
Payoff from SEK asset	If held to maturity	$-SEK\ 1$		$+SEK\ \tilde{r}_2$
	If sold prematurely	$-SEK\ 1$	$+SEK\ \tilde{p}_1$	

There is a foreign exchange market, which is open at dates $t = 0, 1, 2$ for spot transactions and at dates $t = 0, 1$ for forward contracts that can be used to hedge against exchange rate risk.[5] These contracts describe an agreement that stipulates the exchange of currency at a specified future date using a contractually agreed conversion rate, the so-called forward rate. In our model, the BANK offers protection

denominated assets may deliberately refrain from hedging exchange rate risk for speculative reasons, because the cost of hedging FX risk is too high for them or because of risk management considerations.

[5] In practice, FX swaps are more common than forward contracts. As explained in the introduction, FX swap contracts consist of a spot transaction where the AM "borrows" USD at $t = 0$ and "lends" SEK, and a forward transaction with reversed payments of a pre-specified amount.

at $t = 0$ and $t = 1$; the cost of the protection depends on the duration of the forward contract and may be time-varying.

Formally, we denote with S_0 the SEK/USD spot exchange rate at $t = 0$. The future spot exchange rates at $t = 1$ and $t = 2$ are stochastic (indicated by the tilde ~) and denoted by $\tilde{S}_1$ and $\tilde{S}_2$, respectively. Next, let F_{01} be the $t = 1$ forward exchange rate which is agreed upon at $t = 0$. Similarly, F_{02} is the $t = 2$ forward exchange rate agreed at $t = 0$ and F_{12} is the $t = 2$ forward exchange rate agreed at $t = 1$. Finally, let τ_{01} be the insurance premium (or contractual cost) in SEK for a forward contract spanning from $t = 0$ to $t = 1$. The parameters τ_{12} and τ_{02} are defined analogously.[6]

Besides the foreign exchange market, there is a domestic and a foreign credit market at dates $t = 0, 1$. Let i_{01} be the short-term interest rate in the domestic credit market at $t = 0$ and let i_{02} be the long-term interest rate; the future short-term rate at $t = 1$ is potentially stochastic and denoted by $\tilde{\imath}_{12}$. Analogously, we denote with i^*_{01} the short-term interest rate in the foreign credit market at $t = 0$ and with i^*_{02} the long-term interest rate; the potentially stochastic future short-term rate at $t = 1$ is $\tilde{\imath}^*_{12}$.

2.1 No arbitrage

Suppose for now that the insurance premia for forward contracts are zero, so $\tau_{01} = \tau_{12} = \tau_{02} = 0$. In a perfectly competitive risk-neutral environment with no arbitrage opportunities, interest rates and prices are in the following relationship. First, the price of the domestic asset at $t = 1$ has to equal its discounted expected cash flow: $p_1 = E[\tilde{r}_2/(1 + \tilde{\imath}_{12})]$. The same holds for the foreign asset: $p^*_1 = E[\tilde{r}^*_2/(1 + \tilde{\imath}^*_{12})]$. Second, the domestic SEK long-term interest rate has to equal the expected return of the domestic asset, so $i_{02} = (1 + i_{01}) \times E[1 + \tilde{\imath}_{12}] - 1 = E[\tilde{r}_2] - 1$, and the foreign USD long-term interest rate has to equal the expected return of the foreign asset: $i^*_{02} = (1 + i^*_{01}) \times E[1 + \tilde{\imath}^*_{12}] - 1 = E[\tilde{r}^*_2] - 1$.

If one of the above relationships does not hold, there are arbitrage opportunities in the domestic and foreign financial markets, which could be exploited by financial market participants. As an example, consider a situation where $i_{02} < E[\tilde{r}_2] - 1$. Here a risk-neutral arbitrageur could borrow funds cheaply in the domestic credit market and generate vast profits by investing them in the domestic asset, which offers a higher expected return. In practice, competitive financial markets do not offer such opportunities to generate potentially unlimited profits, provided there are no relevant limits to arbitrage (such as tight credit limits) that prevent arbitrageurs from exploiting arbitrage opportunities.

Since we are operating in an environment where there is a domestic and a foreign financial market, the assumption of no arbitrage implies additional conditions that link interest rates, exchange rates and forward rates.

To rule out the possibility that there is an opportunity to earn riskless profits from covered interest arbitrage, the covered interest parity (CIP) demands that $F_{01}/S_0 =$

[6] We can interpret τ as a catch-all that also includes factors like liquidity risk and counterparty risk, which are important determinants of how expensive it is to protect against exchange rate risk.

$(1+i_{01})/(1+i_{01}^*)$ and $F_{02}/S_0=(1+i_{02})/(1+i_{02}^*)$ at $t=0$, and $F_{12}/S_1=(1+i_{12})/(1+i_{12}^*)$ at $t=1$.[7] These relationships assure that it is not possible to generate a profit by borrowing in the credit market in one currency and investing in the credit market in another currency.

Finally, in our risk-neutral world, the equalization of expected returns (domestic and foreign) demands that $S_0\times(1+i_{01})=E[\tilde{S}_1]\times(1+i_{01}^*)$ and $S_0\times(1+i_{02})=E\left[\tilde{S}_2\right]\times(1+i_{02}^*)$ at $t=0$, and $S_1\times(1+i_{01})=E\left[\tilde{S}_2|S_1\right]\times(1+i_{12}^*)$ at $t=1$. The uncovered interest parity (UIP) conditions imply $E\left[\tilde{S}_1\right]=F_{01}$ and $E\left[\tilde{S}_2|S_1\right]=F_{12}$.[8]

2.2 Baseline example

To guide our analysis of the basic model, we use an example with some numbers. Figure 4 provides an illustration. As seen in Figure 4, we assume that the spot exchange rate at $t=0$ is $S_0=10\ SEK/USD$ and that the future exchange rates at $t=1$ and $t=2$ are unknown. We also assume that the domestic one-period interest rate is $i_{01}=i_{12}=10\%$, while the foreign one-period interest rate is $i_{01}^*=i_{12}^*=20\%$. The asset returns follow from the assumption of no arbitrage as specified above. That is, the domestic long-term interest rate is given by $i_{02}=(1+i_{01})(1+i_{12})-1=21\%$, and if the return of the foreign asset is riskless, then it must be given by $r_2^*=1.44$ to match the foreign long-term rate of $i_{02}^*=(1+i_{01}^*)(1+i_{12}^*)-1=44\%$.

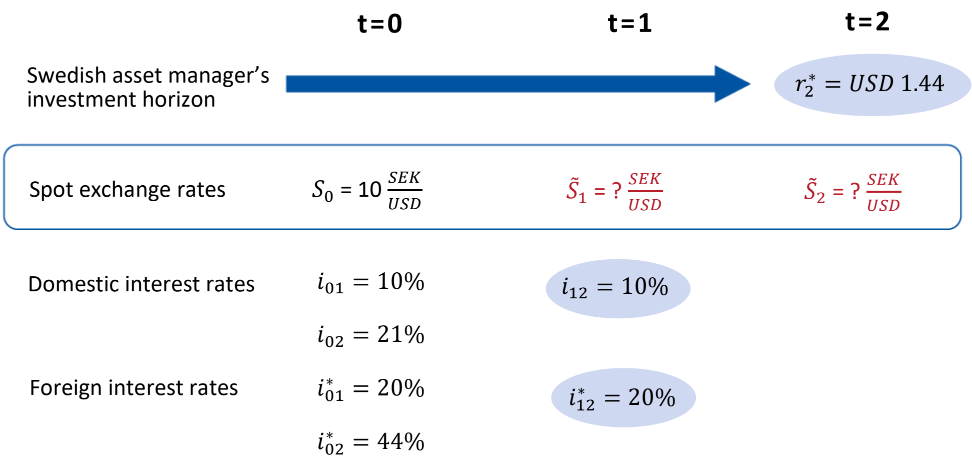
In the baseline environment, we assume that the foreign asset return is known at $t=0$, as well as the future domestic and foreign interest rates. We will relax these assumptions for the variables marked with blue circles in Figure 4 when considering various extensions.

In our example, the Swedish asset manager then invests SEK 10 in the foreign asset, with a known long-term return of $\tilde{r}_2^*=USD\ 1.44$. To do so, she must first purchase dollar in the spot market at the exchange rate S_0. While the asset return in dollar is known, the future exchange rate is not, and therefore the asset manager does not know how much SEK the investment will generate. Thus, she faces FX risk.

[7] In practice, deviations from the CIP can occur (see, for example, Borio et al. 2016). Through the lens of our model, we can capture deviations from the CIP by manipulating the insurance premia $\tau_{01},\tau_{12},\tau_{02}$.

[8] Risk aversion is one reason why the spot price of a foreign currency can deviate from the prevailing forward rate.

Figure 4. Foreign asset return, spot exchange rates and interest rates



Note. The spot exchange rates $\tilde{S}_1$ and $\tilde{S}_2$ in red are stochastic and their realization is not known at $t = 0$. In the baseline example variables marked with blue circles are assumed to be risk-free and known at $t = 0$.

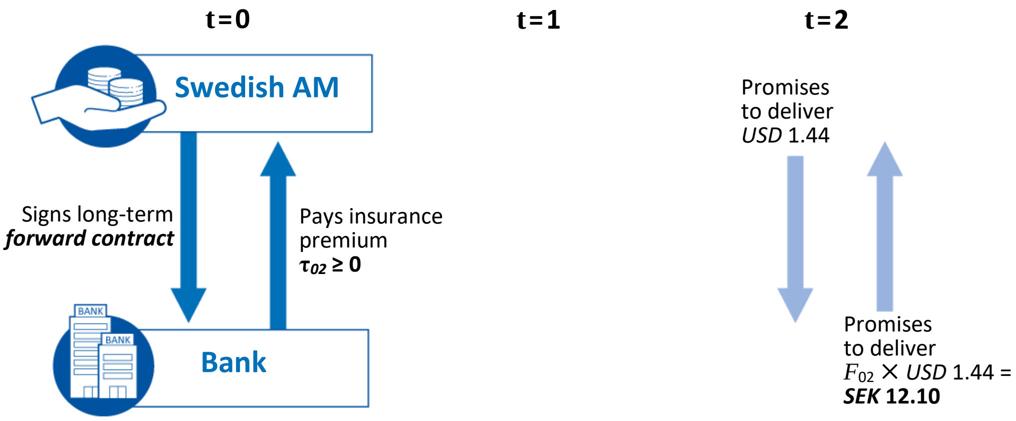
2.2.1 The long-term FX hedging strategy

One way to protect the asset manager from FX risk is to enter a long-term hedging arrangement. Figure 5 illustrates schematically how this works. After the Swedish asset manager places her investment of $SEK\ 10$ in the foreign asset, which requires her to purchase US dollar in the spot market at the rate S_0, she engages in a long-term forward contract with the BANK at $t = 0$. The forward contract stipulates a promise by the AM to deliver $USD\ 1.44$ at $t = 2$, which is the anticipated payoff from the foreign asset, and to receive $F_{02} \times USD\ 1.44$ from the BANK. Provided that covered interest parity holds, the payment received by the AM is $SEK\ 12.1$, which is the same return as investing $SEK\ 10$ in the domestic asset at the interest rate $i_{02} = 21\%$.[9]

In effect, the exchange rate risk associated with the foreign asset return is fully eliminated. Figure 5 also shows that the AM pays an insurance premium at $t = 0$ to the BANK. Since we assume in the baseline that the insurance premium (τ_{02}) charged by the BANK is zero, the long-term FX hedging assures that the rate of return on the foreign asset is identical to that on an investment in the domestic credit market.

[9] The return in SEK from the foreign asset return must satisfy $F_{02} \times USD\ 1.44 = (1 + i_{02})/(1 + i_{02}^*) \times S_0 \times USD\ 1.44 = SEK\ 12.1$.

Figure 5. The long-term FX hedging arrangement



Note. The forward contract spans from $t = 0$ to $t = 2$. The Swedish asset manager (AM) promises to deliver USD 1.44 at $t = 2$ in return for SEK 12.1 from the protection seller (BANK).

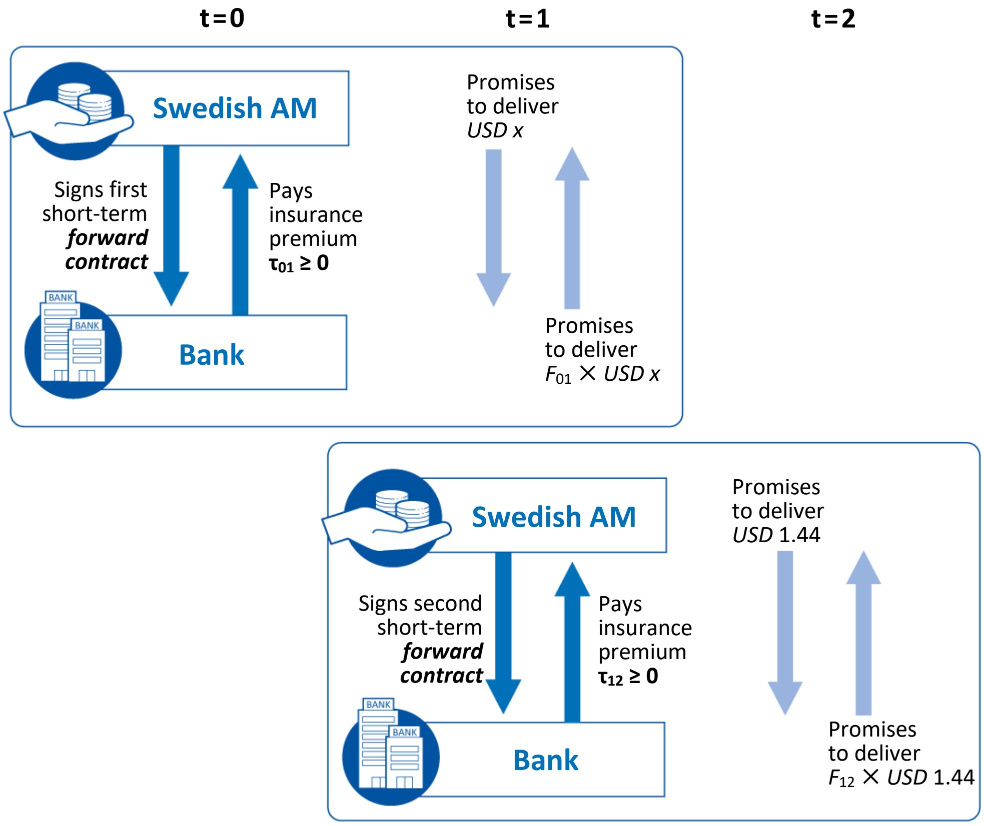
2.2.2 The short-term FX hedging strategy

An alternative to the long-term hedging arrangement is to enter two short-term hedging arrangements. This is illustrated in Figure 6. The first forward contract signed at $t = 0$ foresees a promise by the Swedish asset manager to deliver $USD\ x$ to the protection seller in exchange for $F_{01} \times USD\ x$ at $t = 1$. The second forward contract signed at $t = 1$ foresees a promise by the AM to deliver USD 1.44 to the BANK in exchange for $F_{12} \times USD$ 1.44 at $t = 2$. For each forward contract, an insurance premium may have to be paid (τ_{01}, τ_{12}), but we assume for the baseline that insurance premia are zero.

We will discuss the optimally chosen amount x of the first forward contract in more detail below. In essence, the optimal x minimizes payoff risk by tailoring hedging gains and losses to the desired levels, taking into account expected future interest rates.

This concludes the discussion of the model framework. We now use this framework to analyze how the long- and short-term FX hedging strategies perform in different risk scenarios. We discuss, in turn: risk originating from movements in the foreign exchange rate (FX risk); risk from movements in the price of the foreign asset (price risk); risk related to FX market distress; risk related to the need to sell the asset and to unwind the FX hedge in advance; risk related to the counterparty in the FX hedging arrangement; and, finally, risk of increased inflation in the domestic economy. Throughout, our aim is to address two questions: *How does the performance of short- and long-term FX hedging strategies differ? Under what conditions will the two strategies lead to the same outcome?*

Figure 6. The short-term FX hedging arrangements



Note. The first forward contract spans from $t = 0$ to $t = 1$. The Swedish asset manager (AM) promises to deliver $USD\ x$ at $t = 1$ in return for $F_{01} \times USD\ x$ from the protection seller (BANK). The second forward contract spans from $t = 1$ to $t = 2$ and AM promises to deliver $USD\ 1.44$ at $t = 2$ in return for $F_{12} \times USD\ 1.44$.

3 FX hedging strategies under different types of risk

This section considers how different hedging strategies perform under six different risk categories: FX risk, asset price risk, FX market distress, premature liquidation risk, counterparty risk and inflation risk.

3.1 FX risk

We begin by discussing risk originating from movements in the exchange rate, building on the baseline environment described above. The attractiveness of FX hedging depends in general on what types of risk the investor faces. If most of the risk is due to movements in the exchange rate, for instance if the investment return in foreign currency is easy to predict, then the case for FX hedging is most pervasive. This is typically the case for highly rated fixed-income investments. Conversely, an asset manager investing in foreign currency denominated equity may have little incentive to seek a FX hedge since the risk associated with the equity exposure is likely to dwarf the currency risk.[10]

We first abstract from asset return risk, that is, we consider the simpler baseline model with riskless asset cash flows, $\tilde{r}_2^* = r_2^* = 1.44$ and $\tilde{r}_2 = r_2 = 1.21$. Section 3.1.1 focuses on shocks to spot exchange rates and switches off interest rate risk. In this scenario, a long-term FX hedging strategy and a carefully calibrated short-term FX hedging strategy can lead to identical outcomes. We discuss limitations to the result and present in Section 3.1.2 what happens when we introduce interest rate risk as one of the drivers of exchange rate risk. Thereafter, Section 3.1.3 introduces asset return risk.

3.1.1 Shocks to the spot exchange rates without interest rate risk

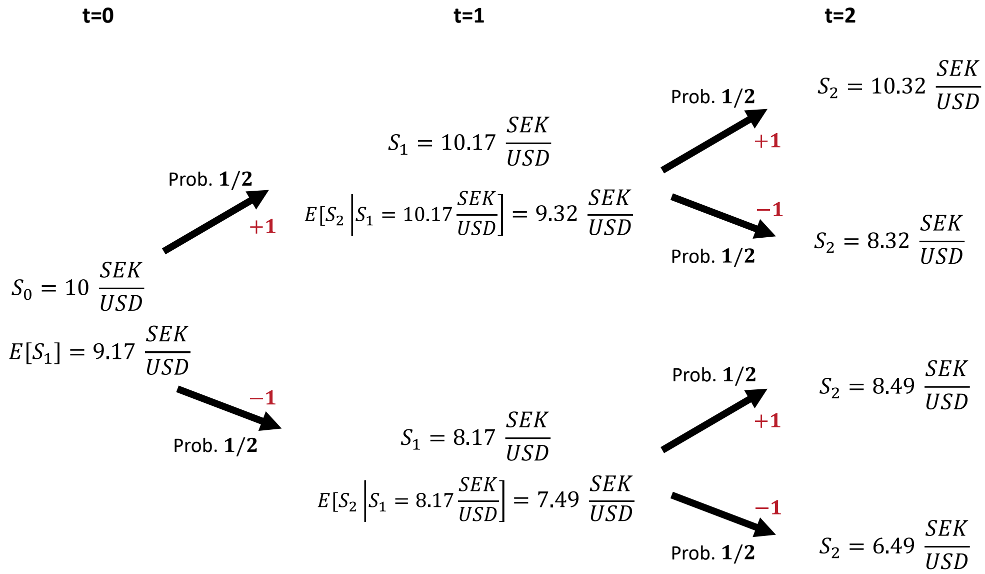
In our first comparison of FX hedging strategies, we assume that there is neither domestic, nor foreign interest rate risk, that is, the domestic and foreign one-period interest rates are known and equal to 10% and 20%, respectively ($\tilde{\iota}_{12} = i_{12} = 10\%$ and $\tilde{\iota}_{12}^* = i_{12}^* = 20\%$). As in the baseline example, this means that the two-period interest rates are $i_{02} = 21\%$ and $i_{02}^* = 44\%$, respectively.

Since the domestic interest rate is below the foreign interest rate, the SEK is on average expected to appreciate against the USD. But the exact realization of the future exchange rate can be both stronger or weaker than in period 0. We will consider an environment where it is equally likely that the exchange rate will appreciate or depreciate, with shocks equal to $+1$ or -1. Assuming that covered interest parity (CIP) holds, the forward exchange rate at $t = 0$ is 9.17 SEK/USD (derived as $F_{01} = (1 + i_{01})/(1 + i_{01}^*) \times S_0$), which equals the expected $t = 1$ spot rate, that is, $E[\tilde{S}_1] = F_{01}$. Given the symmetric shocks of $+1$ or -1, the actual spot exchange rate is either $S_1 = 10.17$ SEK/USD or $S_1 = 8.17$ SEK/USD with probability one-half each, as illustrated in Figure 7.

The CIP then also demands that the implied forward exchange rate at $t = 1$ depends on the realized spot exchange rate: $\tilde{F}_{12} = (1 + i_{12})/(1 + i_{12}^*) \times \tilde{S}_1$ SEK/USD. Given that the future exchange rate is either 10.17 or 8.17 with probability one-half each, the implied forward rate is either 9.32 or 7.49 SEK/USD. This equals the expectations about $t = 2$ spot rates shown in Figure 7. Again, because of the symmetric shocks of $+1$ or -1, the actual realizations of the $t = 2$ spot exchange rates differ. Specifically, Figure 7 shows that there are four different possible realizations with probability one-quarter each, which are 10.32, 8.32, 8.49 and 6.49 SEK/USD.

[10] See, for example, Dimson et al. (2012) for a discussion of empirical evidence.

Figure 7. Spot exchange rates and expectations with FX risk only



Note. Prob. stands for probability. Exchange rate shocks in red.

FX hedging strategies

We consider the following investment strategies for the asset manager:

1. **Long-term**: Invest in the foreign asset and seek a long-term FX hedging arrangement.

2. **Short-term**: Invest in the foreign asset and seek two short-term FX hedging arrangements (for example from $t = 0$ to $t = 1$ and from $t = 1$ to $t = 2$).[11]

3. **No FX hedge**: Invest in the foreign asset without a hedging arrangement.

Strategy: Long-term

Suppose that the Swedish asset manager pursues a long-term FX hedging strategy. We will show that the asset manager can then fully eliminate FX risk in the described environment and achieve a return of 21% for the foreign dollar denominated asset, which equals the return of the domestic asset.

The long-term FX hedging strategy works as follows: At $t = 0$, the AM sells $SEK\ 10$ on the FX spot market at the rate $S_0 = 10\ SEK/USD$ and obtains $USD\ 1$, which she invests in the dollar denominated foreign asset. To insure the FX risk, the AM enters a contract with the BANK, where the BANK agrees to deliver $F_{02} \times USD\ r_2^* = (1 + i_{02})/(1 + i_{02}^*) \times S_0 \times USD\ r_2^* = SEK\ 8.40 \times r_2^*$ in exchange for $USD\ r_2^*$ at $t = 2$. At $t = 0$, the foreign asset payoff is realized and the forward contract is settled.

As a result, the AM's rate of return at $t = 2$ for the domestic investment is

[11] The two FX hedges may be provided either by two different protection sellers or by the same protection seller who agrees to roll over the hedge.

$$r_2 - 1 \equiv \frac{SEK\ r_2 - SEK\ 10}{SEK\ 10} = 21\%$$

and the rate of return for the foreign investment is

$$R^{LT} \equiv \frac{F_{02} \times USD\ r_2^* - S_0\ USD}{S_0\ USD} = 21\%,$$

where the gross rate of return is computed as the forward rate times the foreign asset return in USD divided by the initial investment of $SEK\ 10$. Subtracting one gives the net rate of return of 21%.

Due to the assumption of no-arbitrage, the returns on the domestic and the foreign investment are identical. Moreover, the AM is *fully insured* against both an appreciation or depreciation of the krona relative to the $t = 0$ expectation given by $E[\tilde{S}_2] = 8.40\ SEK/USD$; the realization of the spot exchange rates at $t = 1{,}2$ does not matter.

Strategy: Short-term

Next, suppose that the Swedish asset manager pursues a short-term FX hedging strategy, and enters a one-period hedging arrangement at $t = 0$ and then a second arrangement at $t = 1$.[12] We will show that the asset manager can fully eliminate FX risk in the described environment if the first FX hedge is calibrated to the right amount. In this case, the expected return of the foreign dollar denominated asset is always identical at 21%.

The short-term FX hedging strategy works as follows: At $t = 0$, the AM sells $SEK\ 10$ on the FX spot market at the rate $S_0 = 10\ SEK/USD$ and obtains $USD\ 1$, which is invested in the dollar denominated foreign asset. To insure the AM's FX risk; the BANK agrees to deliver $F_{01} \times USD\ x = \frac{1+i_{01}}{1+i_{01}^*} \times S_0 \times USD\ x = SEK\ 9.17 \times\ x$ in exchange for $USD\ x$ at $t = 1$. At the beginning of $t = 1$, the realization of the spot exchange rate becomes known. It is either $S_1 = 10.17\ SEK/USD$ or $S_1 = 8.17\ SEK/USD$. We start with the former case.

First, the AM has to deliver $USD\ x$ to the counterparty of the first forward contract and receives $SEK\ 9.17 \times x$, meaning that she faces a hedging loss of $SEK\ x$, as the true exchange rate is $10.17\ SEK/USD$. Second, the AM seeks a new FX hedge with another BANK, which involves a promise to deliver $USD\ r_2^*$ at $t = 2$ to the BANK in exchange for $F_{12} \times USD\ r_2^* = (1 + i_{12})/(1 + i_{12}^*) \times 10.17\ SEK/USD \times USD\ r_2^* = SEK\ 13.42$. The hedging loss necessitates that the AM borrows $SEK\ x$ domestically at the interest rate $i_{12} = 10\%$ in order to meet the $t = 1$ payment obligation from the first forward contract. At $t = 2$, the foreign asset payoff is realized, the second forward contract is settled and the debt is repaid with interest.

[12] The protection seller may be the same or a different BANK. We discuss the case when the new protection seller is another BANK. The outcome is identical if the same BANK is used for a roll-over of the forward contract, which requires to account for hedging gains and losses, as to make the BANK indifferent about whether or not to roll over the forward contract.

This time the result differs. If $S_1 = 10.17\ SEK/USD$, the AM's rate of return on the foreign investment at $t = 2$ is

$$R_A^{ST}(x) \equiv \frac{F_{12}\times USD\ r_2^* - S_0\ USD - (1+i_{12})\times SEK\ x}{S_0\ USD} = 34.2\% - \frac{(1+i_{12})\times SEK\ x}{SEK\ 10}.$$

Similar to before we compute the gross rate of return as the forward rate times the foreign asset return in USD divided by the initial investment of $SEK\ 10$. However, we now also need to correct for the $t = 2$ value of the hedging loss. Note that the outcome of the short-term FX hedging strategy is *only identical* to the outcome of the long-term FX hedging strategy if the first FX hedge is over the amount $x = 1.2$, that is, *the discounted* $t = 2$ *return of the foreign asset*. Formally, $R_A^{ST}(x = 1.2) = 21\%$. The intuition for this result is that the lower cost for the second FX hedging contract has to be exactly off-set by the hedging loss.

We next look at the case where the realization of the spot exchange rate is $S_1 = 8.17\ SEK/USD$. At $t = 1$ the AM seeks to roll over the expiring forward contract. First, the AM has to deliver $USD\ x$ to the counterparty of the first forward contract and receives $SEK\ 9.17 \times x$, meaning that she faces a hedging gain of $SEK\ x$. The AM seeks a new FX hedge with another BANK, which involves a promise to deliver $USD\ r_2^*$ at $t = 2$ to the BANK in exchange for $F_{12} \times USD\ r_2^* = SEK\ 10.78$.

The AM's rate of return on the foreign investment at $t = 2$ then is

$$R_B^{ST}(x) \equiv \frac{F_{12}\times USD\ r_2^* - S_0 USD + (1+i_{12})\times SEK\ x}{S_0\ USD} = 7.8\% + \frac{(1+i_{12})\times SEK\ x}{SEK\ 10}.$$

Relative to the previous case, the cost for the second FX hedging contract is higher, which shows up as a reduction in the first term. However, we now have to account for a hedging gain in the second term. Again the outcome of the short-term FX hedging strategy is *only identical* to the outcome of the long-term hedging strategy if $x = 1.2$.

In sum, we find that the asset manager's payoff is constant (that is, the payoff variance is zero) if $x = 1.2$. Otherwise, the payoff variance is positive.

Strategy: No hedge

Now suppose that the Swedish asset manager pursues no FX hedging. It can be shown that in the described environment the expected return is the same as for the previous strategies, that is, $E[\tilde{R}^N] = 21\%$. However, the payoffs under the different exchange rate realizations are associated with substantial risk.

Specifically, without FX hedging, the return on the foreign investment at $t = 2$ depends on which of the four equally likely realizations of the spot exchange rate S_2 prevails (recall Figure 7).

We have now set the stage to address some of our questions and to identify variations in the economic environment that help us to gain additional insights.

Can the FX risk be fully eliminated under the short-term FX hedging strategy?

Yes, as shown above, the short-term FX hedging strategy can replicate the outcome of the long-term FX hedging strategy in the described environment. This is, however, only the case if the first forward contract targets the discounted cash-flow of the foreign currency denominated asset, that is, if $x = 1.2$. A wrongly calibrated first forward contract yields *the same expected return* of 21% as the long-run strategy, *but exposes* the Swedish asset manager to some risk. To see this, suppose that the first forward contract targets the non-discounted cash-flow of the foreign asset, that is, if $x = 1.44$, then the short-term FX hedging strategy has a risky return of $R_A^{ST} = 18.36\%$ with probability one-half and $R_B^{ST} = 23.64\%$ with probability one-half.

Importantly, the short-term hedging strategy is associated with gains or losses. These gains or losses occur because the FX hedge becomes either cheaper if the Swedish krona appreciates or more expensive if it depreciates. Only if $x = 1.2$ will the gains and losses exactly offset the variability in the cost of the second FX hedging contract. As a result, the first forward contract needs to be carefully calibrated.

What are the challenges in calibrating the first forward contract?

In our simple example, not only the cash-flow is known, but also the future foreign interest rate. In practice, both variables are likely to be uncertain. In fact, we show in Section 3.1.2 that the equivalence result breaks down if we consider a scenario with interest rate risk. Specifically, we show that discounting with the expected future foreign interest rate inevitably generates payoff variability, making the short-term FX hedging strategy risky.

Why is it that short-term FX hedging arrangements are used much more frequently in practice?

One aspect that can draw a wedge between the outcomes of the long- and short-term FX hedging strategies is related to the insurance premia. Specifically, the comparison of the two strategies is influenced by insurance premia if $\tau_{01} + \tau_{12} \neq \tau_{02}$. Consistent with the prevalent use of short-term FX hedging strategies, it is possible that longer-term hedges are, at least for certain maturities, more expensive due to a less liquid market. In fact, market surveys reveal that the most liquid segment of the FX swap market is typically concentrated over maturities of a few months. For the Swedish krona, the outstanding nominal amounts of FX swaps with maturities over 6 months are very small compared to shorter maturities, as illustrated in Figure 2. It is an interesting empirical question to understand how much of this outcome can be explained by demand and supply factors.

Investors may also choose to use short-term hedging strategies if demand and supply are more misaligned for longer maturities. Specifically, supply shortages for longer maturities can imply that the short-term FX strategy has a more favorable expected return. Formally, this can be captured in our model as $\tau_{01} + \tau_{12} < \tau_{02}$. Conceptually, there are a number of possible reasons for such a scenario. Important aspects have to do with institutional and regulatory factors. From the perspective of domestic banks, who are the dominant counterparties for Swedish asset managers, there is a desire to align the duration of the offered FX swaps with their desired FX funding profile. If their own funding costs in foreign currency are more favorable at shorter maturities, banks will have an incentive to offer better terms to Swedish asset managers for short-term FX hedging arrangements with similar durations.

Another aspect that can draw a wedge between the outcomes of the long- and short-term FX hedging strategies is related to risk aversion. While risk aversion tends to favor long-term hedging strategies, it can interact with other factors that may induce asset managers to favor short-term FX hedging arrangements. These include the risk of over- and under-hedging, which we discuss in detail in Section 3.1.3, considerations that have to do with flexibility, which we discuss in Sections 3.4 and 3.5, and considerations related to domestic inflation risk, which we discuss in Section 3.6.

3.1.2 Shocks to the spot exchange rates and to the foreign interest rate

Next, we examine the role of uncertainty about foreign interest rates, which is one of the determinants of exchange rate risk. In the previous analysis, we modeled interest rates as deterministic variables and exchange rates as stochastic variables that are driven by a symmetric exogenous shock. Now we add a stochastic foreign interest rate, which is also driven by a symmetric exogenous shock. Specifically, we consider the following modified environment.

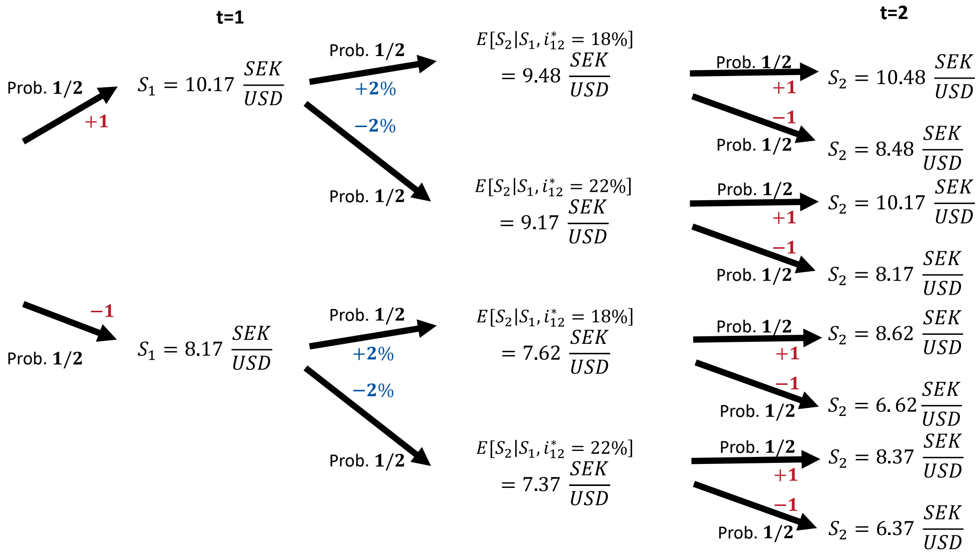
As before, the spot exchange rates at $t = 1, 2$ are assumed to be equally likely to strengthen or weaken by one unit. In contrast to Section 3.1.1, the foreign interest rate $\tilde{\imath}^*_{12}$ is now stochastic and becomes known at the beginning of $t = 1$. Consequently, the interest rate differential is now an additional driver of the spot exchange rate at $t = 2$ and of forward rates at $t = 1$.

We continue to assume that the short-term foreign interest rate in period $t = 0$ is given by $i^*_{01} = 20\%$. But the realization of the future short-term interest rate is equally likely to be $i^*_{12} = 18\%$ or $i^*_{12} = 22\%$, with an expected value of $E[\tilde{\imath}^*_{12}] = 20\%$. The long-term interest rate is unchanged at $i^*_{02} = 44\%$, and we assume that the domestic interest rates are unaltered with $i_{01} = i_{12} = 10\%$ and $i_{02} = 21\%$.

Assuming that CIP holds, the spot rate and implied forward exchange rate at $t = 0$ remain $S_0 = 10\ SEK/USD$ and $F_{01} = 9.17\ SEK/USD$. The implied forward exchange rate at $t = 1$ depends on both the realized spot exchange rate at $t = 1$ (which is either 10.17 or 8.17) and the realized short-term foreign interest rate (either 18% or 22%). As the forward rate at $t = 1$ is stochastic and given by $\tilde{F}_{12} = (1 + i_{12})/(1 + \tilde{\imath}^*_{12}) \times \tilde{S}_1$, there are four possible realizations: 9.48, 9.17, 7.62, or 7.37 SEK/USD, each of which occurs with equal probability of one-quarter. Figure 8 shows how the environment is modified at dates $t = 1,2$. We can see that there are eight possible realizations with probability one-eights each, which are 10.48, 8.48, 10.17, 8.17, 8.62, 6.62, 8.37 and 6.37 SEK/USD.

As in Section 3.1.1, the Swedish krona is expected to appreciate due to the interest rate differential, but the actual realization of the spot exchange rate can be either stronger or weaker than the expected value. We discuss the outcomes of the two FX hedging strategies in turn.

Figure 8. Spot exchange rates, foreign interest rate risk and expectations with FX and foreign interest rate risk



Note. Prob. stands for probability. Exchange rate shocks in red; foreign interest rate shocks in blue.

Strategy: Long-term

If the Swedish asset manager pursues a long-term FX hedging strategy, the analysis of Section 3.1.1 is unchanged. The long-term FX hedging strategy allows the asset manager to fully insure against either a depreciation or an appreciation of the Swedish krona. The spot exchange rate and the foreign interest rate realized at $t = 1$ do not matter since the contracting at $t = 0$ is based on the known (non-stochastic) long-term foreign interest rate $i^*_{02} = 0.44$ as in Section 3.1.1.

Strategy: Short-term

Next, suppose that the Swedish asset manager pursues a short-term FX hedging strategy. We then find that the strategy now delivers a risky payoff. The expected return is with 21% identical, but in contrast to Section 3.1.1, the hedging gains and losses are not fully offset.

As before, the AM sells $SEK\ 10$ at $t = 0$ on the FX spot market at the rate $S_0 = 10\ SEK/USD$ and obtains $USD\ 1$, which is invested in the dollar denominated foreign asset. To insure the AM's FX risk; the BANK agrees to deliver $F_{01} \times USD\ x = (1 + i_{01})/(1 + i^*_{01}) \times S_0 \times USD\ x = 9.17\ SEK/USD \times USD\ x$ in exchange for $USD\ x$ at $t = 1$. At the beginning of $t = 1$, the realization of the spot exchange rate and the foreign interest rate become known. As shown in Figure 8, we have to consider four different combinations of spot exchange rate and foreign interest rate realizations. Otherwise, the analysis is identical to Section 3.1.1 and we provide the derivations in Appendix A.

For the case where $S_1 = 10.17\ SEK/USD$ and $i^*_{12} = 18\%$, the AM's rate of return on the foreign investment at t =2 is $R^{STi}_A = 36.51\% - 1.1 \times SEK\ x/(S_0\ USD)$, for the case $S_1 = 10.17\ SEK/USD$ and $i_{12}^* = 22\%$, it is $R_B^{STi} = 32.05\% - 1.1 \times SEK\ x/(S_0\ USD)$, for the case $S_1 = 8.17\ SEK/USD$ and $i_{12}^* = 18\%$, it is $R_C^{STi} = 9.67\% + 1.1 \times SEK\ x/(S_0\ USD)$, and for the case $S_1 = 8.17\ SEK/USD$ and $i_{12}^* = 22\%$, it is $R_D^{STi} = 6.08\% + 1.1 \times SEK\ x/(S_0\ USD)$.

Notably the expected return from the foreign currency denominated asset is still at 21%. However, different to Section 3.1.1, the short-term FX hedging strategy now delivers a volatile payoff. This is true even if the first FX hedging arrangement is tailored to the discounted cash-flow using the expected interest rate, that is, if $x = 1.2$, which leads to a small, but positive, payoff variance. In fact, exposure to some risk cannot be avoided. The reason is that the realized interest rate inevitably differs from the expected interest rate used to discount the future cash flow. Hence, the investment return is not fully hedged.[13]

What happens if short-term FX hedging is taken to the extreme, that is, if the asset manager engages in short-term FX hedging arrangements with an ultra-short duration?

From the discussion of our baseline model in Section 3.1.1, we know that the short- and long-term FX hedging strategies can deliver identical outcomes as long as we do not introduce additional elements such as interest rate risk. As a result, even FX hedging arrangements with an ultra-short duration, in some special cases, can yield the same outcome as a long-term FX hedging arrangement. However, this result is a special case and does not generalize.

The environment with foreign interest rate risk in Section 3.1.2 is a case in point. When adding additional periods, we can get an idea of the effect of shortening the duration of the FX hedges. Intuitively, a shorter duration exposes the asset manager to more risk of the type described above. Consequently, the short-term hedging strategy becomes increasingly less favorable when compared to the long-term strategy. In the extreme, when the duration of the short-term hedge becomes ultra-short, then the outcome of the short-term strategy starts to resemble more and more the outcome without any hedging at all. See Appendix B for a formal discussion.

3.1.3 Over- and under-hedging FX risk

So far, we have shown that a long-term currency hedge can perform very well in eliminating FX risk. This result changes when we introduce asset return risk. Specifically, we have so far assumed that the foreign asset return $\tilde{r}_2^*$ is constant. We next consider a modified environment with a stochastic foreign asset return.

[13] It is worth mentioning that from a theoretical viewpoint an ideal environment with complete markets would allow the asset manager to eliminate any risk even if using a short-term FX hedging strategy. This is because the asset manager could in such an ideal world construct a self-financing trading strategy that has a cash-flow identical to the long-term FX hedging arrangement. In practice, this outcome is, however, difficult to achieve. Even when instruments for the insurance of interest rate risk are available, it is arguably challenging to accomplish a full elimination of risk with short-term hedging strategies if the exchange rate and interest rate risks are intertwined, as it is the case in the environment we considered.

If the payoff in foreign currency is variable and uncertain, then the asset manager is unable to construct a hedge that fully eliminates the FX risk. Faced with such a scenario, the manager forms expectations about the payoffs and seeks FX risk protection accordingly. We find that in such an environment the asset manager will inevitably do some degree of over- or under-hedging from an ex-post perspective. If the asset manager receives new information over time which allows her to form better expectations about the payoffs, the strategy to roll over short-term FX hedges has the potential benefit that it can be more easily adjusted at each roll-over date to reduce the extend of over- or under-hedging.

To see this formally, consider a modification to the baseline model from Section 3.1.1. Specifically, we now assume that the asset manager receives information about the stochastic foreign asset return $\tilde{r}_2^*$ at $t = 1$, which reveals that the payoff will be higher or lower than the expected value. Specifically, the foreign asset return realizations are $r_2^* = 1.44 + \Gamma$ or $r_2^* = 1.44 - \Gamma$ with equal probability, where $0 < \Gamma < 1.44$.

To be precise, we define another time the actions for the long- and short-term FX hedging strategies for the context of risky foreign asset returns and label these strategies with a star * superscript to account for the modification.

1. **Long-term***: Invest in the foreign asset and seek a long-term FX hedging arrangement that does not leave room for adjustments to reduce over- and under-hedging.

2. **Short-term***: Invest in the foreign asset and seek two short-term FX hedging arrangements that allow for adjustments after one period to reduce over- and under-hedging. Moreover, use the domestic credit market for gains and losses from the currency hedges.

Note that the AM could, in principle, under the strategy Long-term*, seek additional short-term FX hedges after new information comes in, so as to make adjustments for the period from $t = 1$ to $t = 2$. For brevity, we abstract from this possibility and discuss after the analysis why this assumption is plausible.

Strategy: Long-term*

Suppose that the Swedish asset manager pursues a long-term FX hedging strategy. We show that the long-term FX hedging strategy does not allow the AM to fully insure against a depreciation or an appreciation of the Swedish krona anymore. Specifically, the rate of return of the AM on the foreign asset is

$$\tilde{\mathrm{R}}^{\mathrm{LT}*} \equiv \begin{cases} 21\% + \dfrac{\tilde{\mathrm{S}}_2}{\mathrm{S}_0} \times \Gamma & \text{with probability } 1/2 \\ 21\% - \dfrac{\tilde{\mathrm{S}}_2}{\mathrm{S}_0} \times \Gamma & \text{with probability } 1/2 \end{cases}$$

where $\tilde{\mathrm{S}}_2$ has four equally likely outcomes shown in Figure 7. Evidently, the rate of return is not constant anymore and depends on the realization of the asset return and its interaction with the realization of the spot exchange rate at $t = 2$. As a result, the AM is either over- or under-hedged.

Strategy: Short-term*

Next, suppose that the Swedish asset manager pursues a short-term FX hedging strategy. We find that the short-term FX hedging strategy is typically superior if the realizations of the foreign asset return deviate a lot from its expected value (that is, if Γ is large), because it allows the Swedish asset manager to reduce risk relative to the long-term FX hedging strategy. The analysis of the asset manager's rate of return follows the same steps as before and derivations can be found in Appendix C. When comparing the short- and long-term FX hedging strategies, the average rate of return is identical and stands at 21%, but the return variability differs.

If $S_2 \approx S_1$, then the risks associated with both strategies are quite similar. Instead, if there is additional exchange rate risk between dates $t = 1$ and $t = 2$, as it is the case in our model, then the short-term strategy is typically superior if Γ is large. Analyzing the payoff variances associated with the two strategies reveals that the return variability associated with over- or under-hedging is higher for both, the larger is Γ. However, the effect is stronger for the long-term FX hedging strategy.

The critical insight is that the rolling FX hedge can be more effective in absorbing risk if better information about the returns of the dollar investment arrives over time. Whenever this aspect is important, for example if the asset return risk is high and if the asset manager expects to learn about it over time, then the short-term FX strategy is preferable.

One qualification is important to keep in mind. As mentioned earlier, the asset manager pursuing a long-term FX hedging strategy may seek additional short-term hedges for the period from $t = 1$ to $t = 2$ after receiving new information at the beginning of $t = 1$. There are, however, various practical reasons suggesting that the long-term FX hedging strategy is more difficult to adjust than the short-term strategy. First, the additional hedging arrangements are likely to involve additional costs that make the strategy Long-term* less favorable. Second, the unhedged returns are fairly small relative to the amounts rolled over under the strategy Short-term*, which can make it harder to obtain insurance at reasonable terms for additional FX hedges under the strategy Long-term*.

In practice, the model in Section 3.1.3 best captures a scenario where the underlying foreign currency denominated asset carries substantial risk, as it is the case for lower rated corporate bonds or equity. Instead, the benchmark in Section 3.1.1 best captures a scenario where the foreign asset is a riskless zero coupon bond, meaning that the future payoff in foreign currency is certain.

3.2 Price risk

We next discuss another type of risk, namely risk about the price of the foreign currency denominated asset at $t = 1$ and its effect if losses associated with the short-term FX hedging strategy need to be funded by asset sales. For this purpose, we consider a slightly modified environment where we introduce asset-side adjustments to highlight the key insights.

Suppose that the Swedish asset manager pursues a short-term FX hedging strategy as in Section 3.1.1 with the difference that losses from the first period FX hedge are not funded by borrowing, but by selling a fraction of the foreign currency denominated asset at the price p_1^*. This modified strategy may, for instance, be justified by the asset manager's inability to borrow or to sell other assets at $t = 1$.

How do asset-side adjustments affect the outcome of the short-term FX hedging strategy?

We find that the liquidation price of the foreign currency denominated asset plays an important role for the outcomes. While a buy-and-hold investor (who owns the asset until maturity and uses a long-term FX hedging strategy) is unaffected by changes in the asset price over the duration of the hedging contract, this does not hold for an investor who uses a short-term hedging strategy.[14] Specifically, a depressed liquidation price results in a higher hedging loss due to costly asset liquidation. For the short-term FX hedging strategy, the associated risks remain unhedged. Consequently, short-term FX hedging exposes the AM to a combination of FX risk and price risk.[15]

How may such a situation arise? In practice, the foreign asset price may fluctuate over time due to changing liquidity conditions in the market at the point in time when the short-term FX hedge has to be rolled over or due to other factors such as adverse selection problems. Moreover, the asset price no-arbitrage condition may not always hold, which can give rise to deviations from the outcome described in Section 3.1.1.

3.3 Foreign exchange market distress

A third type of risk is foreign exchange market distress, which crystallizes as a challenge to roll over short-term FX hedges. This section is concerned with situations of financial market stress that can occur during a financial crisis episode or because of a large shock such as the COVID-19 pandemic. We are particularly interested in "insurance premium variability", which in our context refers to the FX hedging costs of Swedish asset managers, and in "market access risk", which is a more extreme manifestation of a spike in FX hedging costs that essentially renders the FX hedging market dysfunctional. Arguably, the type of events we have in mind are rare. Nevertheless, they are important and can have significant repercussions in the financial system.[16]

Both insurance premium variability and market access risk have important negative implications for the roll-over of short-term FX hedges. We use a slightly modified setting to highlight the key insights. Specifically, consider a version of the environment used in Section 3.1.1 where the asset manager only gets access to FX

[14] A buy-and-hold investor who does not hedge the FX risk is not exposed to price risk, but fully exposed to FX risk.

[15] See Appendix D for a numerical example.

[16] See Avdjiev et al. (2020) and Sveriges Riksbank (2020a,b) for a discussion of the financial market turmoil and FX markets in spring 2020 following the outbreak of the COVID-19 pandemic.

hedging instruments at the intermediate date $t = 1$ with probability q, where $0 < q < 1$. What we have in mind with this modelling tool is to capture major market dislocations during a financial crisis that temporarily impair the functioning of the FX market, thereby disrupting the roll-over of short-term FX hedges.

The outcome of the long-term FX hedging strategy is by definition unaffected by this disruption, but the outcome of the short-term FX hedging strategy is affected as follows. At the beginning of $t = 1$, the realization of the spot exchange rate becomes known. With probability $1 - q$, everything is identical to the short-term FX hedging strategy described in Section 3.1.1. With probability q, there is the FX market distress scenario. If the spot exchange rate is $S_1 = 10.17\ SEK/USD$, the Swedish asset manager faces a situation where she cannot roll over the FX hedge. As a result, her rate of return is now risky and given by $(SEK\ 10.32 \times 1.44 - SEK\ 10 - SEK\ 1.2 \times 1.1)/SEK\ 10 = 35.4\%$ or $(SEK\ 8.32 \times 1.44 - SEK\ 10 - SEK\ 1.2 \times 1.1)/SEK\ 10 = 6.6\%$ with probability one-half each. A similar result arises if the spot exchange rate is $S_1 = 8.17\ SEK/USD$. For this case, we can derive her rate of return as 35.4% or 6.6% with probability one-half each.

Interestingly, the expected rate of return is 21% as before. However, the asset manager's payoff now has a positive variance. Notably, the payoff variance increases in the magnitude of unhedged FX risks, which are positively associated with q, the probability of the FX market being dysfunctional.

Next, we consider a variant of the baseline model where the insurance premium is positive and potentially time-varying.[17] It shows that time-varying premia drive a wedge between the outcomes of the short- and long-term FX hedging strategies. An interesting case in point is a setting where the future premium is stochastic while the expected insurance premium payments are unaltered, that is, $\tilde{\tau}_{12}$ is stochastic with $\tau_{02} = \tau_{01} + E[\tilde{\tau}_{12}]$. In this scenario, the long-term hedging strategy is more effective not only in insuring against the previously discussed risks, but it also shields from fluctuations in the insurance premium. In practice, a source of such fluctuations could be a moderate degree of FX market distress at $t = 1$ that does not make the market dysfunctional, but merely causes a spike in hedging costs. Another relevant factor could be changes in the market power held by the small number of banks who act as the key sellers of FX risk protection.

Taken together, the roll-over of short-term FX hedges exposes the Swedish asset manager to both market access risk and insurance premium variability. While the average return of the Swedish asset manager may be unaltered, the possibility of FX market distress creates additional return volatility because of unhedged FX risks between dates $t = 1$ and $t = 2$.

[17] In Section 3.1.1, we assumed that $\tau_{01} = \tau_{12} = \tau_{02} = 0$. With a positive insurance premium, it evidently matters how the long-term premium τ_{02} relates to the short-term premia τ_{01} and τ_{12}. Only if $\tau_{02} = \tau_{01} + \tau_{12}$ do the results in Section 3.1 continue to hold.

3.4 Premature liquidation risk

In this section, we discuss the fourth risk category. While the previous three risk categories pointed to advantages of long-term FX hedging strategies, premature liquidation risk shows that short-term hedging arrangements offer some “flexibility” that can in certain scenarios be beneficial.

While some asset managers such as insurance companies are often buy-and-hold investors, there is a possibility that they may have to meet unexpected outflows and, therefore, need to sell a foreign currency denominated asset earlier than originally planned, that is, before it matures. In the context of our model, the FX risk may be fully eliminated with the help of a long-term hedging arrangement if the expected duration of the asset is perfectly matched with the duration of the currency hedge (see Section 3.1.1). This is, however, only true if the asset is held for the expected duration. Instead, if it is liquidated prematurely, then the investor has to unwind the long-term FX hedge, which can prove to be costly. In this scenario, the short-term FX hedging strategy can be advantageous, as it suffices not to roll over the FX hedge.

We can identify two relevant scenarios where the flexibility of the short-term FX hedging strategy has advantages. First, there may be additional insurance costs that arise from unwinding the long-term FX hedge, that is, if $\tau_{12} > 0$, which do not arise for the short-term FX hedging strategy. Second, the need to prematurely liquidate the foreign asset may be positively correlated with a foreign currency appreciation. In Appendix E, we formally compare the outcomes of the two FX hedging strategies for this scenario. We find that the expected return of the long-term FX hedging strategy can fall short of the expected return of the short-term strategy.

3.5 Counterparty risk

Next, we discuss the fifth risk category. Similar to Section 3.4, we find that also in an environment with counterparty risk short-term hedging arrangements offer some “flexibility” that can in certain scenarios be beneficial. Counterparty risk materializes when the seller of FX risk protection is for some reason unable to deliver on her promise. As a result, the forward contract signed at $t = 0$ may become worthless, leaving the Swedish asset manager fully exposed to FX risk. Short-term FX hedging contracts may help to mitigate this type of counterparty risk. More specifically, a short-term hedging arrangement can enable the asset manager to insure a substantial part of the FX risk, while allowing for a switch of counterparties at the roll-over stage (that is, at $t = 1$) if negative information about the current counterparties comes in. Differently, long-term FX hedging does not allow for this option.

The potential advantage of the short-term FX hedging arrangements being more flexible when unfavorable information about the counterparty comes in needs to be qualified. This is because it is common for the parties of a hedging arrangement to exchange collateral.[18] Notwithstanding, collateralization only works well if margin

[18] Major parties in the FX swap market exchange collateral in accordance with the so-called 'Credit Support Annex' (CSA) agreements. The CSA agreements are voluntary add-on agreements to the standard agreements to which operators undertake through membership of the International Swaps and Derivatives

calls can be met adequately to ensure that changes in the valuation of the collateral and changes in the counterparty risk are taken into account. As a result, a higher residual counterparty risk is likely to remain for long-term FX hedging arrangements relative to short-term FX hedging arrangements; especially when it comes to extreme market stress scenarios like a financial crisis. The same logic applies to credit default swaps, which could be used to hedge the counterparty risk.

3.6 Inflation risk

Finally, we examine the role of domestic inflation risk. In the previous analysis, the domestic and foreign interest rates reflect the real return in the respective currency. However, inflation and monetary policy are, in practice, important determinants of exchange rate developments, which tend to gradually restore relative purchasing power parity in the medium- to long-term (Dornbusch 1976; Taylor and Taylor 2004). At the same time, exchange rate movements also affect domestic inflation, especially in a small open economy like Sweden (Corbo and Di Casola 2020).[19]

To incorporate inflation risk in our stylized conceptual framework, we model inflation and exchange rates as exogenously determined stochastic variables and analyze implications for FX hedging. Specifically, we construct an example where we modify the baseline model by allowing for stochastic domestic inflation, $\tilde{\pi}_{01}$, $\tilde{\pi}_{12}$ and $\tilde{\pi}_{02}$, while fixing foreign inflation to zero, that is, $\pi^*_{01} = \pi^*_{12} = \pi^*_{02} = 0$. We denote real and nominal interest rates with the superscripts 'r' and 'n', respectively. Moreover, we assume that the expected real return of investing in the domestic and foreign credit market is identical to the baseline model; formally $i^r_{02} = E[(1 + i^n_{02})/(1 + \tilde{\pi}_{02})] - 1 = 21\%$ and $i^{r*}_{02} = E[(1 + i^{n*}_{02})/(1 + \tilde{\pi}^*_{02})] - 1 = 44\%$.

We assume that domestic inflation is either high or low and that it becomes publicly know at the beginning of date $t = 1$. Specifically, suppose that inflation risk is independently distributed with the shock hitting the economy between $t = 0$ and $t = 1$ so that domestic inflation is $i^n_{01} = 10\%$ in the first period with probability one-half and $i^n_{01} = -8.33\%$ otherwise. In the second period, domestic inflation is assumed to be zero so that the real interest rate in the first period is $i^r_{01} = 0\%$ with probability one-half and $i^r_{01} = 20\%$ otherwise, with the expected domestic real interest rate given by $E[\tilde{i}^r_{01}] = 10\%$ as in the baseline model in Section 3.1.1.

When analyzing the outcomes of the long-term and short-term FX hedging strategies in the modified environment with inflation risk, we find that the long-term FX hedging strategy is no longer able to fully eliminate FX risk. In fact, it performs considerably worse than the short-term FX hedging strategy if uninsurable domestic inflation risk is an important factor. The intuition for this result is that the long-term FX hedging

Association (ISDA). All major banks in Sweden, for example, are members of ISDA, as are all the large insurance companies.

[19] Empirically, exchange rates are impossible to forecast in the short-term, as document by Meese and Rogoff (1983), who show for major exchange rates that a random walk outperforms various time series and structural models of exchange rates (for a recent study of the Swedish krona see Askestad et al. 2019).

strategy goes *long in domestic inflation risk*. This effect matters more, the higher the exposure to uninsurable domestic inflation risk. Appendix F offers a formal analysis.

4 Discussion

In Sections 3.1–3.6, we have analyzed the outcomes of short- and long-term FX hedging strategies in different environments that allowed us to focus attention on six risk categories: FX risk, asset price risk, FX market distress, premature liquidation risk, counterparty risk and inflation risk. Based on our findings, we summarize in Table 2 potential advantages and disadvantages associated with the different strategies.

Generally, the outcomes associated with short- and long-term FX hedging strategies are not identical and depend on the nature of uncertainty and the informational environment. The overall picture is nuanced and a key take-away from this article is to carefully consider what types of risk are relevant for different types of asset managers and for the different asset classes they invest in.

As illustrated in Figure 1, Swedish insurance companies and pension funds have a substantial share of their investments in foreign currency denominated assets. They may seek protection against exchange rate risk because of risk management considerations or for other reasons, such as regulatory requirements. Figure 2 shows that Swedish insurance companies and pension funds primarily use FX hedging contracts with short durations (most frequently 3-4 months). Given the longer duration of the foreign currency denominated investments, a duration mismatch emerges which requires the short-term FX hedges to be repeatedly rolled over.

Based on our theoretical framework, a roll-over of short-term FX hedges can have benefits by reducing the risk of over- and under-hedging and by providing more flexibility in case of premature asset liquidation and new information about counterparty risk. Moreover, a short-term FX hedging strategy can help to reduce exposure to uninsurable domestic inflation risk. Notwithstanding, long-term FX hedging arrangements have clear advantages, especially in periods of financial distress when market functioning is impaired. While short-term FX hedging strategies create refinancing risks, long-term FX hedging strategies effectively shield against such risks.

It is an interesting empirical question to understand how much of the FX swap market concentration on maturities below half a year can be explained by demand and supply factors. In practice, Swedish insurance companies and asset managers are likely to carefully trade off the different risks, as well as the price and non-price attributes associated with different contracts. Still, it is possible that market failures stemming from financial frictions or market concentration create a wedge between the privately optimal and the social optimal choice of the appropriate FX hedging products. While it is beyond the scope of this article to draw normative implications, there are important aspects related to our analysis that deserve consideration.

Table 2. A comparison of long- and short-term FX hedging strategies

Risk category	Long-term FX hedge	Roll-over of short-term FX hedges	No FX hedge
(1) FX risk.	FX risk is fully eliminated to the extent that the future returns on the USD asset are known or can be insured; otherwise there can be some over- or under-hedging.	In some cases, all/most FX risk can be eliminated. FX hedges need to be carefully calibrated. It is difficult to hedge FX risk associated with foreign interest rate risk. Flexibility benefit: If the USD asset returns are uncertain, the short-term FX hedge can reduce risk, as it can be more easily tailored in response to incoming information.	Full exposure to FX risk.
(2) Price risk (i.e., changes in the USD asset price over investment duration).	No exposure to FX risk and to price risk for buy-and-hold investors (i.e., if USD asset is held until maturity).	If losses from short-term FX hedges are funded with asset-side adjustments, then an exposure to unhedged asset price risk arises (e.g. due to time-varying market liquidity or other factors).	No exposure to price risk for buy-and-hold investors; full exposure to FX risk.
(3) FX market distress (e.g. FX hedges cannot be rolled over at certain times).	No exposure to future FX market distress if there is no need to sell USD asset.	Exposure to market access / refinancing risk and to insurance premium variability (e.g. higher hedging costs when rolling over in times of financial distress).	No exposure to short-term market distress if there is no need to sell the USD asset; full exposure to FX risk.
(4) Premature asset liquidation risk / transaction date uncertainty (i.e., asset sale prior to expected investment duration).	Exposure to the risk of a costly unwinding of the long-term FX hedge if there is a need to sell USD asset.	Flexibility benefit: Limited exposure—especially if the next roll-over date of the short-term FX hedge is near the date of the unexpected premature asset liquidation.	Full exposure to FX risk also when USD asset is liquidated prematurely.
(5) Counterparty risk (i.e., default of the insurer; possibly in conjunction with an impairment of the collateral value).	Exposure to the same counterparty for FX hedging during the full investment duration.	Flexibility benefit: Possibility to limit exposure by changing counterparties at the next roll-over date in case of negative information about existing counterparty.	No exposure to counterparty risk; full exposure to FX risk.
(6) Risk of a high domestic inflation.	Long-term FX hedging has the disadvantage that it goes long in domestic inflation risk.	Short-term FX hedging offers the possibility to limit exposure to domestic inflation risk.	No exposure to domestic inflation risk; full exposure to FX risk.

First, the repercussions of individual FX hedging strategies are primarily felt in periods of financial distress. Consequently, an “over-reliance” on short-term FX hedges can pose a negative externality by creating additional market congestion in periods of financial distress. In practice, Swedish asset managers cannot rely on having market access at all times. Empirically, a typical crisis scenario features an appreciation of the dollar and at the same time a rise in the cost of FX risk protection. While long-term FX hedging arrangements are shielded against such a scenario, asset managers relying on short-term FX hedging arrangements experience a spike in the costs to roll over their FX hedges, as well as a further shortening of the duration of their hedges.

Second, a substantial part of the funding of Swedish banks is short-term and in foreign currency (see Bertsch and Molin 2016). At the same time, Swedish banks play a dominant role as sellers of FX risk protection to Swedish asset managers. Consequently, a stronger reliance on more short-term FX risk protection can have a disadvantageous financial stability effect in that it translates into a further increase in the short-term foreign currency funding reliance of Swedish banks.

References

Askestad, Emil, Ana M. Ceh, Paola Di Casola and Annukka Ristiniemi (2019), 'Forecasting the krona', *Economic Commentaries* no. 12, Sveriges Riksbank.

Avdjiev, Stefan, Egemen Eren and Patrick McGuire (2020), 'Dollar funding costs during the Covid-19 crisis through the lens of the FX swap market', *BIS Bulletin*, no. 1, Bank for International Settlements.

Bertsch, Christoph and Johan Molin (2016), 'Revisiting the role of central banks as liquidity providers – old and new challenges', *Sveriges Riksbank Economic Review*, no. 2, pp. 89–158.

Borio, Claudio, Robert McCauley, Patrick McGuire and Vladyslav Sushko (2016), 'Covered interest parity lost: understanding the cross-currency basis', *BIS Quarterly Review*, September, pp. 45–64.

Corbo, Vesna and Paola Di Casola (2020), 'Drivers of consumer prices and exchange rates in small open economies', Working Paper no. 387, Sveriges Riksbank. Forthcoming, *Journal of International Money and Finance.*

Dimson, Elroy, Paul Marsh and Mike Staunton (2012), 'Currency matters', *Credit Suisse Global Investment Returns Yearbook 2012*.

Dornbusch, Rudiger (1976), 'Expectations and exchange rate dynamics', *Journal of Political Economy*, vol. 84, no. 6, pp. 1161–1176.

Levander, Mats, Carl-Johan Rosenvinge and Vanessa Sternbeck Fryxell (2021), 'The Swedish derivative market', Staff memo, Sveriges Riksbank.

Meese, Richard A. and Kenneth Rogoff (1983), 'Empirical exchange rate models of the seventies: do they fit out of sample?', *Journal of International Economics*, vol. 14, no. 1–2, pp. 933–948.

Sveriges Riksbank (2020a), *Financial Market Survey*, Spring 2020.

Sveriges Riksbank (2020b), *Financial Stability Report*, 2020:1.

Sveriges Riksbank (2021), *Financial Stability Report*, 2021:1.

Taylor, Alan M. and Mark P. Taylor (2004), 'The purchasing power parity debate', *Journal of Economic Perspectives*, vol. 18, no. 8, pp. 135–158.

APPENDIX A – Derivations for the model with FX and foreign interest rate risk

This Appendix complements the discussion in Section 3.1.2. We first consider the case where $S_1 = 10.17\ SEK/USD$ and $i_{12}^* = 0.18$. As before, the AM seeks to roll over the expiring FX forward contract at $t = 1$. The protection seller may be the same BANK or a different BANK.[20] The payments associated with the first contract are settled and the AM faces a hedging loss of $SEK\ x$. She seeks a new FX hedge, which involves a promise to deliver $USD\ r_2^*$ at $t = 2$ in exchange for $SEK\ 9.48 \times 1.44$. The hedging loss necessitates that the AM borrows $SEK\ x$ domestically at the interest rate $i_{12} = 0.1$. The AM's rate of return on the foreign investment at t =2 is

$$R_A{}^{STi} \equiv \frac{9.48 \frac{SEK}{USD} \times USD\ 1.44 - S_0\ USD - (1 + i_{12}\ ) \times SEK\ x}{S_0\ USD}$$
$$= 36.51\% - \frac{1.1 \times SEK\ x}{S_0\ USD}.$$

Similar to before, we compute the gross rate of return as the forward rate times the foreign asset return in USD divided by the initial investment of $SEK\ 10$. Differently, however, the forward rate at $t = 1$ now depends on the realization of the foreign interest rate. Again, we need to correct for the $t = 2$ value of the hedging loss.

We next look at the case where $S_1 = 10.17\ SEK/USD$ and $i_{12}^* = 0.22$. Different to before, the new FX hedge now involves a promise to deliver $USD\ r_2^*$ at $t = 2$ in exchange for $SEK\ 9.17 \times 1.44$. The AM's rate of return on the foreign investment is

$$R_B{}^{STi} \equiv 32.05\% - \frac{1.1 \times SEK\ x}{S_0\ USD}.$$

Similarly, we can derive the AM's rate of return on the foreign investment for the case where $S_1 = 8.17\ SEK/USD$ and $i_{12}^* = 0.18$ as

$$R_C{}^{STi} \equiv 9.67\% + \frac{1.1 \times SEK\ x}{S_0\ USD}$$

and for the case where $S_1 = 8.17 \frac{SEK}{USD}$ and $i_{12}^* = 0.22$ as

$$R_D{}^{STi} \equiv 6.08\% + \frac{1.1 \times SEK\ x}{S_0\ USD}.$$

The payoff variance for $x = 1.2$ is given by

$$Var^{STi}(x = 1.2) = \frac{\begin{pmatrix} (23.31\% - 21\%)^2 + (18.84\% - 21\%)^2 \\ +(22.87\% - 21\%)^2 + (19.28\% - 21\%)^2 \end{pmatrix}}{4} = 7 * 10^{-6}.$$

[20] Without loss of generality, we focus on the case where the new protection seller is a different BANK. See discussion in Appendix A for the environment of Section 3.1.1.

APPENDIX B – Shortening the duration of FX hedges

This Appendix complements Section 3.1.2 by providing a formal discussion of potential implications when shortening the duration of FX hedging arrangements. We simplify the environment in Section 3.1.2 by setting expected domestic and foreign interest rates to zero. Formally, let $i_{01} = i_{12} = i_{02} = 0$ and let the foreign interest rate process be given by $i_{01}^* = i_{02}^* = 0 = r_2^*$ and

$$\tilde{\iota}_{12}^* = \begin{cases} -\varepsilon & with\,probability\; 1/2 \\ +\varepsilon & with\,probability\; 1/2 \end{cases}$$

with $E[1 + \tilde{\iota}_{12}^*] = 1$.

Assuming the CIP holds, the spot and implied forward rates at $t = 0$ are $S_0 = F_{01} = 10\; SEK/USD$. The $t = 0$ expectation about the $t = 1$ spot exchange rate is $E[\tilde{S}_1] = S_0$. We consider a generalized version of the environment in Section 3.1.2 with $\Delta \geq 0$

$$\tilde{S}_1 = \begin{cases} (10 + \Delta)\dfrac{SEK}{USD} & with\,probability\; 1/2 \\ (10 - \Delta)\dfrac{SEK}{USD} & with\,probability\; 1/2. \end{cases}$$

Assuming the CIP holds, expectations about implied forward exchange rates at $t = 1$ are now also influenced by the realization of $\tilde{\iota}_{12}^*$ and $\tilde{F}_{12}$ can be derived as

$$\tilde{F}_{12} = \begin{cases} \dfrac{10 + \Delta}{1 - \varepsilon}\dfrac{SEK}{USD} & with\,probability\; 1/4 \\ \dfrac{10 + \Delta}{1 + \varepsilon}\dfrac{SEK}{USD} & with\,probability\; 1/4 \\ \dfrac{10 - \Delta}{1 - \varepsilon}\dfrac{SEK}{USD} & with\,probability\; 1/4 \\ \dfrac{10 - \Delta}{1 + \varepsilon}\dfrac{SEK}{USD} & with\,probability\; 1/4. \end{cases}$$

The expectations about $t = 2$ spot exchange rates are $E[\tilde{S}_2 | S_1 = (10 + \Delta)\; SEK/USD, i_{12}^* = -\varepsilon] = (10 + \Delta)/(1 - \varepsilon)\; SEK/USD$, etc. and the actual realizations are

$$\tilde{S}_2 = \begin{cases} \left(\dfrac{10 + \Delta}{1 - \varepsilon} + \Delta\right)\dfrac{SEK}{USD} & with\,probability\; 1/8 \\ \left(\dfrac{10 + \Delta}{1 - \varepsilon} - \Delta\right)\dfrac{SEK}{USD} & with\,probability\; 1/8 \\ & etc. \end{cases}$$

A comparison with the payoff variance for short-term hedging yields for all $\varepsilon > 0$ that

$$Var^N(\Delta, \varepsilon) > Var^{STi}(\Delta, \varepsilon)$$
$$= \frac{\left(\dfrac{\frac{(10 - \Delta)\varepsilon}{1 - \varepsilon}}{10} - 0\%\right)^2 + \left(\dfrac{\frac{-(10 + \Delta)\varepsilon}{1 + \varepsilon}}{10}\right)^2 + \left(\dfrac{\frac{-(10 + \Delta)\varepsilon}{1 - \varepsilon}}{10}\right)^2 + \left(\dfrac{\frac{-(10 - \Delta)\varepsilon}{1 + \varepsilon}}{10}\right)^2}{4}.$$

Consistent with Section 3.1.1, we can see that absent interest rate risk, that is, if $\varepsilon \rightarrow 0$, the short-term FX hedging strategy eliminates all risk. For $\Delta = 1$ and $\varepsilon = 0.02$ as in Section 3.1.2 we have that $Var^{N}(1{,}0.02) = 0.0204 > Var^{STi}(1{,}0.02) = 0.0004$.

Notably, the short-term FX hedging strategy does just as badly as the no FX hedging strategy if $\Delta \rightarrow 0$ and $\varepsilon > 0$. Formally, $Var^{STi}(\Delta, \varepsilon) \rightarrow Var^{N}(\Delta, \varepsilon) > 0$ if $\Delta \rightarrow 0$. As a numerical example, consider the case where $\Delta = 0.5$ and $\varepsilon = 0.1$, where $Var^{N}(0.5{,}0.1) = 0.01538 > Var^{STi}(0.5{,}0.1) = 0.0103$.

We next study a modification of our model with an additional third period which also features interest rate risk and with implied forward exchange rates at $t = 2$ given by the eight possible combinations in

$$F_{23} = \frac{1}{1 \mp \varepsilon}\left(\frac{10 \mp \Delta}{1 - \varepsilon} \mp \Delta\right)\frac{SEK}{USD}$$

which occur with equal probability of one-eighths.

For $\Delta = 1$ and $\varepsilon = 0.02$, the payoff variance for the no FX hedging strategy is now given by $Var^{N}(0.02) = 0.0313$ and the payoff variance of the short-term FX hedging strategy is $Var^{STi}(0.02) = 0.0009$. Evidently, the short-term FX hedging strategy becomes more similar to the no FX hedging strategy when adding additional periods in this fashion. This effect shows up more prominently for a higher values of ε and smaller values of Δ. To see this, consider again the numerical example where $\Delta = 0.5$ and $\varepsilon = 0.1$. Now $Var^{N}(0.5{,}0.1) = 0.03582 > Var^{STi}(0.5{,}0.1) = 0.0277$.

APPENDIX C – Derivations for over- and under-hedging

This Appendix complements the discussion in Section 3.1.3. As before, the Swedish AM sells $SEK\ 10$ at $t = 0$ on the FX spot market at the rate $S_0 = 10\ SEK/USD$ and obtains $USD\ 1$, which is invested in the dollar denominated foreign asset. To insure the AM's FX risk; the BANK agrees to deliver $SEK\ 9.17 \times x$ in exchange for $USD\ x$ at $t = 1$.

We first look at the case when $S_1 = 10.17\ SEK/USD$ and $r_2^* + \Gamma$. At $t = 1$, the asset manager seeks to roll over the expiring forward contract. The hedging loss of $SEK\ x$ is funded at the interest rate $i_{12} = 0.1$. Moreover, the AM seeks a new protection which involves a promise to deliver $USD\ r_2^* + \Gamma$ at $t = 2$ to the BANK in exchange for $SEK\ 9.32 * (r_2^* + \Gamma)$. Taken together, the rate of return of the AM is

$$R_{1A}^{ST*} \equiv \frac{SEK\ 9.32 \times (r_2^* + \Gamma) - SEK\ 10 - SEK\ 1.1 \times x}{SEK\ 10}.$$

Next, we look at the case when $S_1 = 8.17\ SEK/USD$ and $r_2^* + \Gamma$. Now there is a hedging gain and the rate of return of the AM is

$$R_{1B}^{ST*} \equiv \frac{SEK\ 7.49 \times (r_2^* + \Gamma) - SEK\ 10 + SEK\ 1.1 \times x}{SEK\ 10}.$$

The third case is characterized by $S_1 = 10.17\ SEK/USD$ and $r_2^* - \Gamma$. The rate of return of the AM is

$$R_{2A}^{ST*} \equiv \frac{SEK\ 9.32 \times (r_2^* - \Gamma) - SEK\ 10 - SEK\ 1.1 \times x}{SEK 10}.$$

Finally, the fourth case is characterized by $S_1 = 8.17\ SEK/USD$ and $r_2^* - \Gamma$, and the rate of return of the AM is

$$R_{2B}^{ST*} \equiv \frac{SEK\ 7.49 \times (r_2^* - \Gamma) - SEK\ 10 + SEK\ 1.1 \times x}{SEK\ 10}.$$

The respective payoff variances associated with the two strategies are

$$Var^{LT*}(\Gamma) \equiv \frac{\left(\frac{10.32}{10} \times \Gamma\right)^2 + \left(\frac{8.32}{10} \times \Gamma\right)^2 + \left(\frac{8.49}{10} \times \Gamma\right)^2 + \left(\frac{6.49}{10} \times \Gamma\right)^2}{4}$$

$$> Var^{ST*}(x = 1.2; \Gamma) = \frac{\left(\frac{9.32}{10} \times \Gamma\right)^2 + \left(\frac{7.49}{10} \times \Gamma\right)^2 + \left(\frac{9.32}{10} \times \Gamma\right)^2 + \left(\frac{7.49}{10} \times \Gamma\right)^2}{4}.$$

Analyzing the variance terms reveals that, for both strategies, the return variability associated with over- or under-hedging is higher, the higher Γ. However, the effect is stronger for the long-term FX hedging strategy, meaning that the differential payoff variance, $Var^{LT*}(\Gamma) - Var^{ST*}(x = 1.2, \Gamma)$, increases in Γ.

APPENDIX D – Derivations for price risk

This Appendix complements the discussion in Section 3.2. As before, the AM sells $SEK\ 10$ at $t = 0$ on the FX spot market at the rate $S_0 = 10\ SEK/USD$ and obtains $USD\ 1$, which is invested in the dollar denominated foreign asset. To insure the FX risk; the BANK agrees to deliver $SEK\ 9.17 \times x$ in exchange for $USD\ x$ at $t = 1$.

We first consider the case when $S_1 = 10.17\ SEK/USD$. The AM has to deliver $USD\ x$ and receives $SEK\ 9.17 \times x$. Since the AM lost from the appreciation of the dollar, a fraction of the foreign asset has to be liquidated, which we denote as $l > 0$. Moreover, the AM seeks a new FX risk protection, which involves a promise to deliver $USD\ (1 - l) \times x$ at $t = 2$ to the BANK in exchange for $F_{12} \times (1 - l) \times USD\ r_2^* = SEK\ 9.32 \times (1 - l) \times r_2^*$ where l can be derived as $l = x/({p_1}^* \times 9.32)$.

The rate of return of the AM at $t = 2$ is

$$R_A^{STl} \equiv \frac{SEK \frac{p_1^* \times 9.32 - x}{p_1^*} \times r_2^* - SEK\ 10}{SEK\ 10}.$$

The foreign asset is fairly priced, that is, its return at $t = 1$ corresponds to the interest rate in the credit market, if $p_1^* = 1.31$. In this case, the AM can achieve a return of 21% and fully eliminate risk as in Section 3.1.1 by calibrating the first forward contract in the same way, that is, $x = 1.2$. The reason is that the implicit funding cost $r_2^*/p_1^* - 1 = 0.1$ equals the domestic interest rate $i_{12} = 0.1$.

Instead, if the foreign asset is not fairly priced and trades at a price lower than $p_1^* = 1.31$, then the AM strictly prefers the domestic credit market (if accessible). Conversely, if the price is higher than $p_1^* = 1.31$, then the AM strictly prefers to sell the asset over borrowing in the domestic credit market.

We next consider the case when $S_1 = 8.17\ SEK/USD$. This time the AM enjoys a hedging gain and there is no need to sell a fraction of the foreign asset. If the asset price is lower than $p_1^* = 1.31$, the asset manager has no incentive to sell the asset and the outcome is the same as in Section 3.1.1.

Taken together, the main insight is that the liquidation price of the foreign currency denominated asset plays an important role for the outcomes. While a buy-and-hold investor (who owns the asset until maturity and uses a long-term FX hedging strategy) is unaffected by changes in the asset price over the duration of the hedging contract,[21] this does not hold for an investor who uses a short-term hedging strategy. Specifically, a depressed liquidation price, for example $p_1^* < 1.31$, results in a higher hedging loss due to costly asset liquidation. For the short-term FX hedging strategy, the associated risks remain unhedged.

To illustrate this point numerically, consider the outcome when the $t = 1$ asset price is $p_1^* = 1$, for example due to an adverse selection problem. In this situation, the asset manager suffers from an appreciation of the dollar. Hence, the rate of return of the AM at $t = 2$ falls short of the return achieved by the short-term FX hedging strategy in Section 3.1.1 since

$$R_A^{STl}(x = 1.2) = 16.92\% < R_A^{ST}(x = 1.2) = 21\%.$$

In sum, short-term hedging exposes the AM to a combination of FX risk and price risk.

APPENDIX E – Derivations for premature liquidation risk

This Appendix complements Section 3.4 by providing a formal discussion of potential implications of premature liquidation risk.

To make the argument, consider a modification to the baseline model where the AM has a need to liquidate the dollar investment at $t = 1$ with probability q', where $0 < q' < 1$. We discuss the outcome below using a modification of the environment in Section 3.1.1. As before, the AM can choose among three investment strategies.

We find that the strategy not to hedge FX risk performs poorly also in our modified environment and is associated with a considerably higher payoff variance than the short- and long-term FX hedging strategies. Moreover, we find that the outcomes of the short- and long-term FX hedging strategies can differ at the presence of

[21] A buy-and-hold investor who does not hedge the FX risk is not exposed to price risk, but fully exposed to FX risk.

premature asset liquidation risk. For the example with $p_1^* = 1.2$, and with a zero FX insurance premium, the two strategies deliver outcomes identical to those in Section 3.1.1. For $p_1^* \neq 1.2$, both strategies deliver risky payoffs. Notably, the expected return of the short- and long-term FX hedging strategy is equal. But the return is lower than 21% if $p_1^* < 1.2$ and larger than 21% if $p_1^* > 1.2$.

Next, we consider a variant of the previous model where the liquidation need and the foreign currency appreciation are perfectly correlated, with $\Pr\{S_1 = 10.17 | q' = 1\} = 1$ and $\Pr\{q' = 1 | S_1 = 10.17\} = q'' > 0$. Moreover, assume the foreign currency denominated asset has to be liquidated at a depressed price of $p_1^* = 1$. We discuss the outcomes of the long- and short-term FX hedging strategies in turn.

Strategy: Long-term

Everything remains the same if the spot exchange rate realization at $t = 1$ is $S_1 = 8.17$. Instead, if $S_1 = 10.17$, then the rate of return of the AM is $(11.19 \times p_1^* - 11.32)/10$. For $p_1^* = 1$ and $q'' = 1$, the ex-ante return can be derived as

$$R^{LT**}(q'' = 1) = \frac{21\% + (1-q'') \times 21\% + q'' \times \frac{11.19 \times p_1^* - 11.32}{10}}{2} = 9.85\%.$$

Strategy: Short-term

Again, everything stays the same if the spot exchange rate realization at $t = 1$ is $S_1 = 8.17$. Instead, if $S_1 = 10.17$, then the rate of return is $(4.53 + (p_1^* - r_2{}^*) \times 11.19)/10$. For $p_1^* = 1$ and $q'' = 1$, the ex-ante return can be derived as

$$R^{ST**}(q'' = 1) = \frac{23.64\% + (1-q'') \times 18.36\% + q'' \times \frac{4.53 + (p_1^* - r_2{}^*) \times 11.19}{10}}{2} = 13.79\%.$$

To conclude, the short-term FX hedging strategy may deliver a better-than-expected return if the asset manager considers it to be likely that the long-term asset needs premature liquidation with a risk that the cost of the unwinding of the long-term FX hedging arrangement cannot be covered.

APPENDIX F – Derivations for inflation risk

This Appendix complements Section 3.6 by providing a formal discussion of the environment with stochastic domestic inflation. We analyze the outcomes of the long- and short-term FX hedging strategies in turn.

Strategy: Long-term

Observe that the two-period forward rate at $t = 0$ remains the same as in Section 3.1.1, since $F_{02} = S_0 \times E[1 + i_{02}^r]/(1 + i_{02}^*) = 8.40\ SEK/USD$. The AM's expected rate of return at t=2 is also the same as in Section 3.1.1, that is, $E[\tilde{R}^{LT,r}] = 21\%$, but now payoffs vary across states due to the domestic inflation risk, which induces a mean-preserving spread. With probability one-half, the rate of return is given by $R^{LT,r} = 1.21/1.1 - 1 = 10\%$ and otherwise by $R^{LT,r} = 1.21/0.9167 - 1 = 32\%$.

Strategy: Short-term

Also, the one period forward rate at $t = 0$ remains the same as in Section 3.1.1, since $F_{01} = S_0 \times E[1 + \tilde{\iota}^r_{01}]/(1 + i^*_{01}) = 9.17\ SEK/USD$. The same is true for the $t = 1$ spot rates and one-period forward rates. While the AM's expected rate of return at $t = 2$ remains the same, that is, $E[\tilde{R}^{ST,r}] = 21\%$, we now have four cases to consider. We first look at the case where the spot exchange rate is $S_1 = 10.17\ SEK/USD$ and a domestic inflation of 10%, where the rate of return is

$$R_A^{ST,r}(x) = 31.10\% - \frac{(1 + i_{12}) \times \frac{x}{1 + i^*_{12}}}{10(1 + \pi_{02,1})},$$

with $\pi_{02,1} = 10\%$ and $R_A^{ST,r}(1.2) = 21.10\%$. Instead, if the domestic inflation is -8.33% then the rate of return can be derived as

$$R_B^{ST,r}(x) = 37.32\% - \frac{(1 + i_{12}) \times \frac{x}{1 + i^*_{12}}}{10(1 + \pi_{02,2})},$$

with $\pi_{02,2} = -8.33\%$ and $R_B^{ST,r}(1.2) = 25.32\%$. Using the same logic, we look at the case where the realization of the spot exchange rate is $S_1 = 8.17\ SEK/USD$. Now the rate of return is $R_C^{ST,r}(1.2) = 17.14\%$ if the domestic inflation is 10% and $R_D^{ST,r}(1.2) = 20.57\%$ if the domestic inflation is -8.33%.

Taken together, the results with domestic inflation risk differ drastically from our baseline model in Section 3.1.1. Both, the long- and short-term FX hedging strategies now deliver a risky payoff. Moreover, we can see that the introduction of domestic inflation risk is particularly harmful for the long-term FX hedging strategy, which now performs considerably worse than the short-term strategy.