Central bank digital currency in an open economy*

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Abstract

We examine the open-economy implications of the introduction of a central bank digital currency (CBDC). We add a CBDC to the menu of monetary assets available in a standard two-country DSGE model with financial frictions and consider a broad set of alternative technical features in CBDC design. We analyse the international transmission of standard monetary policy and technology shocks in the presence and absence of a CDBC and the implications for optimal monetary policy and welfare. The presence of a CBDC amplifies the international spillovers of shocks to a significant extent, thereby increasing international linkages. But the magnitude of these effects depends crucially on CBDC design and can be significantly dampened if the CBDC possesses specific technical features. We also show that domestic issuance of a CBDC increases asymmetries in the international monetary system by reducing monetary policy autonomy in foreign economies.

Keywords: Central bank digital currency, DSGE model, open-economy, optimal monetary policy, international monetary system.

JEL Codes: E50, F30

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Non-technical summary

Spurred by competition from innovative payment solutions developed by the private sector, central bank digital currency (CBDC) has received significant attention in both policy circles and academia. More recently, the Covid-19 pandemic has fanned public concerns that the virus could be transmitted by cash, thereby amplifying calls for developing contactless forms of payment (Auer et al. (2020)). The main objective of a CBDC is to enlarge access to central bank reserves beyond the small realm of commercial banks to the public at large.

Various recent studies have examined some of the macroeconomic and financial stability questions raised by the introduction of a CBDC and modelled their implications. However, these studies have focused mainly on the closed-economy dimension. This paper focusses instead on the open-economy implications of a CBDC which, as we show, are significant and far-reaching for policy and welfare.

To examine the open-economy implications of the introduction of a CBDC, we extend the two-country DSGE model with financial frictions of Eichenbaum et al. (2017) by adding a CBDC instrument to the menu of monetary assets available. In our model, the CBDC is a digital instrument: it is easily scalable and has no storage costs, unlike a physical instrument like cash. The CBDC is also a hybrid instrument: it is both a means of payment and a financial asset. It hence provides liquidity services, like cash, and unlike bonds. And it can be remunerated, like bonds, and unlike cash. Finally, the CBDC is a (super) safe asset: it is neither subject to duration risk, unlike bonds, nor to risks of bank runs or limited deposit insurance, unlike commercial bank deposits, nor to inflation risks — at least, when it is remunerated.

In our model, central bank liabilities, which serve as means of payment, unit of account and store of value, may therefore include cash and a CBDC, which can circulate simultaneously. We consider a broad set of technical features in CBDC design to instill as strong a degree of realism as possible to our model. Importantly, we allow the CBDC issued in the home country to be used in the foreign country, to an extent that depends on the CBDC's technical design.

We calibrate our model using the parameter assumptions of Eichenbaum et al. (2017) and examine two specific questions. First, we analyse the international transmission of standard monetary policy and technology shocks in light of two scenarios: one with a CBDC, and the other one without. In so doing, we analyze heterogeneity in the response to shocks arising from alternative technical designs of the CBDC. Second, we examine optimal monetary policy in the two economies and compare household welfare in the presence and absence of a CBDC with alternative design features.

Crucially, we show that the presence of a CBDC amplifies the international spillovers of shocks, thereby increasing international linkages. The key intuition is that the existence of a CBDC creates a new arbitrage condition that links together the interest rate differential, the exchange rate and the remuneration of the CBDC. Specifically, that arbitrage condition defines the risk-free rate in the foreign economy as a mark-up on the remuneration of the CBDC (i.e. the interest rate on the CBDC adjusted for exchange rate risk). This is quite intuitive as households, for the same remuneration, strictly prefer to hold CBDC relative to a foreign bond given that the CBDC provides liquidity services. This leads to stronger exchange rate movements in response to shocks in the presence of a CBDC — foreign agents rebalance much more into CBDC than they would into bonds, if the latter were the only internationally traded asset, because of the CBDC's hybrid nature.

We also show that the magnitude of these effects depend crucially on CBDC design and can be significantly dampened if the CBDC possesses specific technical features. For instance, tight restrictions on transactions in CBDC by foreigners or – even more importantly – adjusting the remuneration rate on the CBDC flexibly, e.g. using a Taylor rule, reduce international spillovers.

Finally, we find that the presence of a CBDC has significant effects on optimal monetary policy in the two economies and strengthens asymmetries in the international monetary system. In particular, issuance of a CBDC by the domestic economy curtails monetary policy autonomy in the foreign economy to an economically significant extent. It forces the foreign central bank to alter its monetary policy stance to mitigate the stronger international spillovers created by the CBDC. This suggests that introducing a CBDC sooner rather than later could give rise to a significant first-mover advantage to its issuer.

1 Introduction

Globalization and digitalization, in particular the emergence of innovative payment solutions from Fintech and Big Tech companies, such as PayPal, Alipay, WeChat Pay or Libra, have prompted central banks to consider whether to upgrade payment system infrastructures as well as the broader concept and provision of money. This has led interest in central bank digital currencies, henceforth CBDC, to grow considerably in policy-makers and scholarly circles alike. A recent survey suggest that central banks representing a fifth of the world's population are likely to issue a CBDC "very soon", while 80 percent of central banks worldwide are working on a CBDC (BIS (2020)). More recently still, the Covid-19 pandemic has fanned public concerns that the virus could be transmitted by cash, thereby amplifying calls for developing contacless forms of payment, such as CBDCs (Auer et al. (2020)).

A CBDC, in a nutshell, aims at enlarging access to central bank reserves beyond the small realm of commercial banks to the public at large – a concept sometimes referred to as "reserves for all". Cash would hence no longer be the only form of central bank money through which the public could transact and save. The same could be done with reserves deposited at the central bank.

This idea is admittedly a rather old one, which goes back at least to James Tobin in the 1980s (Tobin (1987)), if not even earlier.[1] But technological innovation has now significantly expanded the range of possibilities through which a CBDC can be introduced. Reflecting this, CBDCs can be designed in various ways and can vary according to a range of technical features. These features, as recalled e.g. in Auer and Boehme (2020), include whether the CBDC is provided directly to households and firms (retail CBDC) or indirectly via commercial banks (wholesale CBDC); whether it is supplied in fixed or variable quantity; whether it is remunerated or not; whether holdings of certain categories of economic agents, such as foreigners, are restricted or not; and whether the CBDC is

[1] It has been observed, for instance, that private persons held deposit accounts with some central banks – then in private hands – up to World War II, and that postal saving accounts might be considered as predecessors of CDBCs. See also Bordo and Levin (2017).

a token-based form of money (like cash) or an account-based one (like commercial bank deposits).

Various studies, such as Barrdear and Kumhof (2016), Agur et al. (2019), Andolfatto (2018), Brunnermeier and Niepelt (2019), Chiu et al. (2019), Fernandez-Villaverde et al. (2020) and Niepelt (2020), which we briefly review hereafter, have examined some of the macroeconomic and financial stability questions raised by the introduction of a CBDC and modelled their implications. However, these studies have focused squarely on closed-economy issues.[2]

The literature on the open-economy implications of a CBDC is thin, in contrast, although these can be sizeable and far-reaching for policy and welfare, as we also show below. George et al. (2018) adapt the framework of Barrdear and Kumhof (2016) to a small open-economy setting, where a set of reduced-form equations describe world demand and determine the exchange rate. But they do not consider situations where the CBDC circulates both at home and abroad, which are important in ongoing policy discussions about the potential international implications of a CBDC. Another paper (Benigno et al. (2019)) considers the global implications of a privately-issued global crytocurrency, like Libra – which differs markedly from a CBDC, as we also explain below.

Our paper aims to fill this gap and break new ground in the rapidly growing literature on CBDCs with a view to providing more rigorous analysis in discussions on this matter, not least among major central banks in relevant international fora.[3]

We examine the open-economy implications of the introduction of a CBDC. We extend the two-country DSGE model with financial frictions of Eichenbaum et al. (2017) by adding a CBDC instrument to the menu of monetary assets available. In our model,

[2] Brunnermeier and Niepelt (2019) provide a generic model of money and liquidity in general environments which could also include several currencies and, in this sense, include an international financial dimension, at least to some extent.

[3] For instance, an international group of central banks (including the Bank for International Settlements, the Bank of Canada, the Bank of England, the Bank of Japan, the European Central Bank, the Sveriges Riksbank and the Swiss National Bank), announced on 21 January 2020 that they would share experiences as they assess the potential use cases for CBDC in their domestic jurisdictions. The group will examine CBDC use cases; economic, functional and technical design choices, including cross-border interoperability; and the sharing of knowledge on emerging technologies. It will closely coordinate with the relevant institutions and forums, such as the Financial Stability Board (FSB) and the Committee on Payments and Market Infrastructures (CPMI).

the CBDC is a digital instrument: it is easily scalable and has no storage costs, unlike a physical instrument like cash. The CBDC is also a hybrid instrument: it is both a means of payment and a financial asset. It hence provides liquidity services, like cash, and unlike bonds. And it can be remunerated, like bonds, and unlike cash. Finally, the CBDC is a (super) safe asset: it is neither subject to duration risk, unlike bonds, nor to risks of bank runs or limited deposit insurance, unlike commercial bank deposits, nor to inflation risks — when it is remunerated.[4]

In the model, central bank liabilities, which serve as means of payment, unit of account and store of value, may therefore include cash and a CBDC, which can circulate simultaneously. We consider a broad set of technical features in CBDC design to instill as strong a degree of realism as possible to our model. Importantly, we allow the CBDC issued in the home country to be used in the foreign country, to an extent that depends on the CBDC's technical design.[5]

We calibrate our model using the parameter assumptions of Eichenbaum et al. (2017) and examine two specific questions.[6] First, we analyse the international transmission of standard monetary policy and technology shocks in light of two scenarios: one with a CBDC, and the other one without. In so doing, we analyze heterogeneity in the response to shocks arising from alternative technical designs of the CBDC. Second, we examine optimal monetary policy in the two economies and compare household welfare in the presence and absence of a CBDC with alternative design features.[7]

We show that the presence of a CBDC amplifies the international spillovers of shocks,

[4]The CBDC of our model is therefore different from other commonly mentioned safe assets like e.g. US dollar bonds or US dollar deposits, because it is a hybrid instrument which synthetizes characteristics of those assets. Hence it is different from a US Treasury bond, because it is immediately liquid and is not subject to market (duration) risk – a feature which is captured by the parameter Θ described below, which measures the extent of the liquidity services provided by the CBDC. It is also different from dollar deposits in a commercial bank, because it is not subject to bank runs or limited deposit insurance, since it is a liability of the central bank. It is hence the safest asset possible which, as we show below, is important for optimal monetary policy choice in the model.

[5]We do not allow the foreign country to issue its own CBDC as this is not necessary for the questions we examine in the paper (see below). Moreover, such an extension would conceivably require to model strategic interactions between the two countries in their decisions to introduce a CBDC, which would in turn make the model dauntingly more difficult to solve and simulate. But we plan to take up this specific question in future work.

[6]We also estimate the model using US and euro area data and show that our main results hold.

[7]Welfare is understood here as the sum of current and future discounted utility flows.

thereby increasing international linkages. The intuition behind this result is that the introduction of a CBDC creates a new arbitrage condition that links together interest rates, the exchange rate and the remuneration of the CBDC. Specifically, that arbitrage condition defines the risk-free rate in the foreign economy as a mark-up on the remuneration of the CBDC (i.e. the interest rate on the CBDC adjusted for exchange rate risk). This is quite intuitive as households, for the same remuneration, strictly prefer to hold CBDC relative to a bond given that the CBDC provides liquidity services, unlike bonds. This leads to stronger exchange rate movements in response to shocks in the presence of a CBDC — foreign agents rebalance much more into CBDC than they would into bonds, if the latter were the only internationally traded asset, because of the CBDC's hybrid nature. In simple and intuitive terms, the unique characteristics of a CBDC (scalability, liquidity, safety, (potentially) remuneration), create a new "super charged" Uncovered Interest Parity (UIP) condition if it is used internationally, which induces stronger international linkages in a quantitatively relevant way. As summarized in Table 1, only the CBDC ticks all boxes.

The mechanism implies several signature predictions: 1) there are larger exchange rate movements due to overshooting after a shock in the presence of a CBDC; 2) the risk-free rate in the foreign economy moves more strongly in the presence of a CBDC; it should (fall) rise if the foreign country's currency is expected to depreciate (appreciate); 3) the rise (fall) of the risk-free rate should lead to tighter financial conditions in the foreign country, with adverse (positive) consequences on real consumption and investment. These predicted effects emerge clearly from simulations of the model.

But we also show that the magnitude of the effects depends crucially on CBDC design and can be significantly dampened if the CBDC possesses specific technical features. For instance, tight restrictions on transactions in CBDC by foreigners or – even more importantly – adjusting the remuneration rate on the CBDC flexibly, e.g. through a Taylor rule, reduce international spillovers. Finally, we find that the presence of a CBDC has significant effects on optimal monetary policy in the two economies and strengthens asymmetries in the international monetary system. In particular, issuance of a CBDC by the domestic economy curtails monetary policy autonomy in the foreign economy to an economically significant extent. It forces the foreign central bank to alter its monetary policy stance to mitigate the stronger international spillovers created by the CBDC.

The rest of the paper is organized as follows. Section 2 provides a brief overview of the relevant literature. Section 3 introduces the model. Section 4 examines the model simulations on the international transmission of standard monetary policy and technology shocks in the presence or absence of a CBDC. Section 5 considers implications for optimal monetary policy in both economies. Section 6 concludes.

Table 1: Characteristics of the different monetary instruments in the model

	Scalability	Liquidity	Safety	Interest rate	International use
Cash		✓	✓		
Bonds	✓		✓	✓	✓
Deposits	✓			✓	
CBDC	✓	✓	✓	✓	✓

Notes: The table summarizes the salient characteristics of the different monetary instruments in the model. As we explain below, ξ is the parameter governing physical storage costs for cash; μ and σ determine the degree of liquidity services provided by each instrument; Ξ_t is risk on loan/deposits and ϕ determines the extent of restrictions on cross-border transactions.

2 Related literature

We can divide the existing literature in three strands: (i) papers introducing a CBDC in general; (ii) papers introducing a CBDC in DSGE models more specifically (since this is closer methodologically to what we do); (iii) and papers taking an open-economy perspective (like ours) to analyse crypto-currencies, albeit not specifically a CBDC. We review each strand of literature in turn.

Modelling CBDC, non-DSGE models. Many papers focus on the domestic implications of a CBDC in a stylised, often two-period model of the economy. These papers are hence different from ours, which focuses on the open-economy implications. Agur et al. (2019), for example, focus on design trade-offs and in particular on whether a CBDC should be made more similar to bank deposits or cash. One key insight is that design trade-offs ultimately hinge on the desirability of maintaining bank intermediation relative to the welfare gains of having more diversified payment instruments.

Another issue, as it has been often argued, is that introducing a CBDC could encourage depositor runs and threaten financial stability. But some recent contributions are more optimistic on the effects of a CBDC on bank intermediation and lending. For instance, Brunnermeier and Niepelt (2019) argue that alleged unwelcome effects on bank intermediation and financial stability would entirely depend on the monetary policy accompanying the issuance of a CBDC and on the strength of the central bank's commitment to serve as a lender of last resort. Under a strong commitment by the central bank, competition from the CBDC would give rise to an automatic substitution of one type of commercial bank funding (private deposits) by another one (central bank funding). In a related vein, Andolfatto (2018) shows in an overlapping generations model that introduction of an interest-bearing CBDC does reduce bank monopoly profit but does not necessarily lead to bank disintermediation. He points out that, in fact, a CBDC may lead to an expansion of bank deposits if competition forces incentivize banks to raise their deposit rates to attract deposits and outcompete the CBDC. Chiu et al. (2019) evaluate the general equilibrium effects of introducing a CBDC in a model where banks issue deposits and provide loans. In their framework (a monetary search model with a centralized and a decentralized market), a CBDC improves the efficiency of bank intermediation by limiting the market power of banks in the deposit market. Notably, a CBDC can boost lending in this model even if it is not used in equilibrium, by providing a viable outside option to depositors.

A somewhat contrasting view is Fernandez-Villaverde et al. (2020) who propose a model that shows that, during a panic, the rigidity of the central bank's contract with investment banks has the capacity to deter runs. Thus, the central bank is more stable than the commercial banking sector. Since depositors internalize this feature ex-ante, the central bank arises as a deposit monopolist, attracting all deposits away from the commercial banking sector, which might in turn endanger maturity transformation. Finally, Niepelt

(2020) propose an equivalence result according to which marginal substitution of outside money (e.g. a CBDC) for inside money (e.g. deposits) does not affect macroeconomic outcomes.

Modelling CBDC, DSGE models. A few papers have proposed to model a CBDC through the lenses of a DSGE model, much as we do. For instance, Barrdear and Kumhof (2016) propose a DSGE model calibrated on US data to study the domestic implications of CDBC issuance. They look in particular at both the steady state effects of a CBDC and the benefits of having a second monetary policy instrument which, as they show, improves substantially the central bank's ability to stabilise the economy. They also provide an overview of potential benefits and costs of a CDBC. Key differences between our paper, their paper and other related papers include our distinctive international perspective, and also the broad set of technical features in CBDC design we consider, which not only give greater realism to our model, but also allow us to draw concrete conclusions for policy.

Open-economy models. To our knowledge, only two papers develop open-economy models relevant to a CBDC. George et al. (2018) adapt the framework of Barrdear and Kumhof (2016) to a small open-economy setting, where a set of reduced-form equations describe world demand and determine the exchange rate. They find that a CBDC with a flexible remuneration improves domestic welfare if the CBDC is an imperfect substitute for bank deposits. Our paper is different insofar as we develop a full-fledged two-country model where cross-country flows are endogenous, which allows us to extend the analysis to situations where the CBDC circulates not only at home but also abroad. This feature is important in ongoing policy discussions about the potential international implications of a CBDC. In addition, we model explicitly the hybrid nature of a CBDC under alternative design features, derive a new arbitrage condition and the key economic mechanism through which a CBDC's global implications unfold, and pin down how a CBDC differs fundamentally from other monetary instruments, like cash or deposits, or standard safe assets, like a US dollar bond.

The other paper is Benigno et al. (2019) which proposes a two-country model with two national currencies and a global crypto-currency. The open-economy nature of the paper is very close to ours. But a key difference is their focus on a privately-issued global crypto-currency, like Libra, which is unlikely to match in safety and reputation a CBDC issued by a reputable central bank, and does not have a lender of last resort. Who would act as a lender of last resort in case of a run on the Reserve backing the circulation of Facebook's Libra project, for instance, remains unclear (see e.g. Eichengreen and Viswanath-Natraj (2020)). This presumably matters in the decision of agents to hold the CBDC in equilibrium. In line with this, a survey of more than 13,000 individuals across 13 advanced and emerging countries found that, in almost all countries, respondents indicated that they would feel most confident in digital money issued by the domestic monetary authority; in contrast, respondents globally expressed a lack of confidence in digital money issued by a tech or credit card company, particularly respondents from advanced economies (see OMFIF (2020) for further details). Still, as Benigno et al. (2019) show, a global crypto-currency can have powerful implications. In particular, the existence of the global crypto-currency restricts monetary policy autonomy in the two countries, thereby making monetary policy more synchronized and turning the traditional Mundellian trilemma between monetary policy autonomy, exchange rate flexibility and financial openness into a dilemma, where monetary policy is no longer an autonomous policy tool. Our finding that the presence of a CBDC strengthens asymmetries in the international monetary system and, in particular, reduces monetary policy autonomy in the foreign economy relative to the domestic economy, chimes with this conclusion.

3 Model

The model is in the spirit of Eichenbaum et al. (2017). It includes two economies: a home economy and a foreign economy. There are four types of agents: households, firms, financial agents and the government. Households consume, supply (differentiated) labour and can invest their holdings of liquidity. Households also require liquidity services to undertake transactions. There are four classes of monetary and financial assets: domestic and foreign bonds, which are remunerated at the risk-free rate, but provide no liquidity services; bank deposits, which are remunerated at the risky rate (more on this below) and provide no liquidity services (hence think of them as term deposits that are not readily available); cash, which is not remunerated, but provides liquidity services; and finally the CBDC which can be remunerated (depending on its design) and provides liquidity services.

Firms produce final goods, which are sold domestically and internationally. They have some degree of monopoly power and there is stickiness in price setting à-la Calvo (1983). Firm produce goods by combining capital and labour; entrepreneurs are penniless, they hence issue state-contingent claims against future earnings to finance capital in each period. The financial sector is populated by banks that intermediate funds (i.e. deposits) between households and firms. Banks collects deposits from households and issue loans to firms against claims on future profits. As stressed above, we think of bank deposits as term deposits; only the latter are used to finance investment, in line with Bernanke et al. (1999). Moreover, the returns on these loans are risky, hence returns on deposits are risky for households. This allows us to model the fact that a CBDC is safer than commercial bank deposits.[8]

The government consumes final goods, sets the policy rate, issues the CBDC and decides on its design. Design choice includes whether the CBDC is remunerated or not; whether there are restrictions to foreign holdings; and whether the CBDC is supplied in fixed or flexible quantity. We assume that the CBDC is issued in the home economy and that it can be purchased by agents in both economies conditional on potential restrictions faced by foreign users. Finally, as in Eichenbaum et al. (2017) we assume that financial markets are incomplete and that uncovered interest parity does not hold fully. Foreign

[8]Admittedly, bank deposits are insured in many advanced economies. However, insurance covers deposits only up to a certain limit (e.g. EUR 100,000 per bank per account in the euro area), which caps protection for economic agents with larger deposits, like firms. Moreover, in case of bank distress, it takes time before insurance payments are made, which is a (difficult-to-predict) cost for depositors. Finally, deposit insurance has not always prevented severe bank runs even in the recent past, for instance during the global financial crisis in e.g. Greece, Cyprus or even in the UK (Northern Rock). Taken together, these arguments suggest that households do not consider deposits as completely safe as there are intrinsic, non-insured, costs to bank default; see Angeloni and Faia (2009). For a similar modelling choice see e.g. Christiano et al. (2014).

agents face the same decision problem as domestic agents, with the exception of the CBDC; foreign variables are denoted by an asterisk (*).

3.1 Households

Households in both country derive utility from consumption and leisure. They need liquidity services to undertake transactions. To model this, we include cash and the CBDC in the utility function. Feenstra (1986) and Croushore (1993) show that the money-in-the-utility-function approach has intuitive micro-economic foundations and is equivalent to inserting liquidity constraints in the budget of households, to shopping-time models or to cash-in-advance constraints.

The intra-period utility of the representative household is:

$$U_t = \frac{\exp(e_t^C)(C_t - hC_{t-1})^{1-\sigma}}{1-\sigma} - \frac{\chi}{1+\varphi}\int_0^1 L(i)_t^{1+\varphi} di + \mu^{\$} \frac{(\frac{M_t}{P_t})^{1-\sigma^{\$}}}{1-\sigma^{\$}} + \mu^{DC} \frac{(\frac{DC_t}{P_t})^{1-\sigma^{DC}}}{1-\sigma^{DC}} \tag{3.1}$$

with C denoting consumption, L hours worked, M cash holdings, DC holdings of the CBDC and P_t the price level; χ, $\mu^{\$}$ and μ^{DC} are scaling parameters, while σ, $\sigma^{\$}$ and σ^{DC} are elasticities of substitution; finally, e_t^C is a consumption preference shock, which follows an AR(1) process.

The presence of a CBDC is the main difference between our model and that of Eichenbaum et al. (2017). We define the parameters $\mu^{DC} = \Theta\mu^{\$}$ and $\sigma^{DC} = \sigma^{\$} + (1-\Theta)\sigma^{\$}$ which capture the extent to which the CBDC relaxes liquidity constraint of households. In other words, Θ captures the preferences of agents as to whether the CBDC is closer to cash, to deposits or to a combination of the two, in terms of relaxing liquidity constraints, in the spirit of Agur et al. (2019).[9]. More specifically, if $\Theta = 1$, the CBDC provides the same liquidity services as cash ($\mu^{DC} = \mu^{\$}$, $\sigma^{DC} = \sigma^{\$}$); if $\Theta = 0$ the CBDC provides no liquidity services, like bank deposits (understood here again as term deposits) or bonds ($\mu^{DC} = 0$, hence $\frac{\partial U_t}{\partial DC_t} = 0$); finally if $\Theta > 1$ the CBDC provides better liquidity services

[9] Θ might also capture underlying characteristics of the CBDC that shape these preferences and would be otherwise difficult to model (for instance, if $\Theta > 1$ and the CBDC is seen as superior to cash, because it is appealing to use by e.g. digitally savvy users).

than cash, hence providing higher utility. We adjust the value of Θ to explore alternative configurations.[10]

There are several reasons why a CBDC might provide better liquidity services than cash. First of all, a CBDC being a digital monetary instrument, it is presumably less subject to theft than cash; second, some categories of agents (e.g. digitally savvy households) may regard it as a more appealing medium of payment; third, it is more easily scalable – in other words, agents quickly face physical storage, security and insurance costs if they want to hoard increasingly large amounts of cash, unlike a digital instrument like a CBDC.[11]

The budget constraint is:

$$P_tC_t + B_t^H + NER_tB_t^F + D_t + M_t + DC_t \leq \int_0^1 W(i)_tL(i)_t + \\ + R_tB_{t-1}^H + R_t^*NER_tB_{t-1}^F - \frac{\phi^B}{2}\left(\frac{NER_tB_t^F}{P_t}\right)^2 P_t + P_tR_t^DD_{t-1} + \xi^\$ M_{t-1} + \Pi_t + R_t^{DC}DC_{t-1} \tag{3.2}$$

Households use monetary instruments for consumption (C_t), purchases of risk-free domestic (B_t^H) and foreign (B_t^F) bonds multiplied by the nominal exchange rate NER_t (defined as units of domestic currency per unit of foreign currency, so that a fall means an appreciation of the domestic currency), to save in the form of deposits (D_t), and to hold cash (M_t) or CBDC (DC_t); there are cross-border costs to purchase foreign bonds ($\frac{\phi^B}{2}(\frac{NER_tB_t^F}{P_t})^2P_t$) which prevent uncovered interest parity to hold fully, in line with standard empirical evidence. Monetary instruments are acquired through the wage bill ($\int_0^1 W(i)_tL(i)_t$), returns on domestic (R_t) and foreign (R_t^*) bonds, and remuneration of both deposits and CBDC holdings. Cash is subject to linearly increasing storage costs ($\xi^\$$) (e.g. the costs of storing increasingly large amounts of cash in a vault).[12] The interest

[10]Notice that we assume separability between cash and CBDC in the utility function. Had we assumed that they are non-separable by using e.g. a CES aggregator à la Dixit-Stiglitz, holdings of cash and CDBC would depend in equilibrium on their relative price and on total demand for liquidity services. But then there would always be demand – even if it is very small – for both monetary instruments, which would prevent us from examining cases where there is neither CBDC nor demand for it, although this is our intended baseline.

[11]See below for a discussion of how we model storage costs for cash.

[12]In our baseline calibration, we assume that storage costs are zero for reasons of simplicity, i.e.

rate on the CBDC is distinct from the risk-free policy rate. Finally, Π_t denote profits (from banks and firms) net of lump-sum taxes. The first-order conditions are:

$$\exp(e_t^C)(C_t - hC_{t-1})^{-\sigma} - E_t\left[\beta h \exp(\epsilon_{t+1}^C)(C_{t+1} - hC_t)^{-\sigma}\right] = \lambda_t \tag{3.3}$$

$$E_t\left(\beta \frac{\lambda_{t+1}}{\lambda_t} \frac{R_t}{\pi_{t+1}}\right) = 1 \tag{3.4}$$

$$E_t\left(\beta \frac{\lambda_{t+1}}{\lambda_t} \frac{NER_{t+1}}{NER_t} \frac{R_t^*}{\pi_{t+1}}\right) = \left(1 + \phi^B NER_t B_t^F\right) \tag{3.5}$$

$$E_t\left(\beta \frac{\lambda_{t+1}}{\lambda_t} \frac{R_t^D}{\pi_{t+1}}\right) = 1 \tag{3.6}$$

$$\mu^\$ m^{-\sigma^\$} = \lambda_t - E_t \xi^\$ \left(\beta \frac{\lambda_{t+1}}{\pi_{t+1}}\right) \tag{3.7}$$

$$\mu^{DC} dc_t^{-\sigma^{DC}} = \lambda_t - E_t\left(\beta R_t^{DC} \frac{\lambda_{t+1}}{\pi_{t+1}}\right) \tag{3.8}$$

where $\{\lambda_k\}_{k=1}^{\infty}$ is the sequence of Lagrangian multipliers associated with the optimization problem. Equation (3.4) and Equation (3.5) are the bond holding conditions, under incomplete markets ($\phi^B > 0$) the uncovered interest parity condition does not hold. Equation (3.7) is the optimality condition for cash, with m_t denoting *real* cash holdings; demand for cash increases with its marginal utility, and decreases in holding costs and the inflation rate (i.e. the shadow cost of holding cash). Equation (3.8) defines domestic demand for the CBDC, with dc_t denoting the *real* holdings of CBDC. Demand increases in the utility of the CDBC and its interest rate, and decreases in the inflation rate.

The problem of households in the foreign economy is symmetrical to the problem of households in the home economy with one exception: the foreign economy does not issue a CBDC but can purchase it from the domestic economy subject to international frictions,

$\xi^\$ = 1$. We explore in Appendix C.2 alternative scenarios with higher storage costs, which have limited quantitative implications for our results.

captured by $\frac{\phi^{*,DC}}{2}(\frac{DC_t^*}{NER_t P_t^{*,2}})^2$. These frictions feature prominently in policy discussions on optimal CBDC design and on e.g. whether CBDC use by foreigners should be restricted e.g. through holding limits. In terms of monetary and financial assets, foreigners have access to foreign bonds, domestic bonds and the CBDC.

The first-order condition that defines the foreign demand for the domestic CBDC is:

$$\mu^{*,DC}\left(\frac{dc_t^*}{NER_t}\right)^{-\sigma^{DC}} - \phi^{*,DC}\lambda_t^*\frac{dc_t^*}{NER_t} - \lambda_t^* + E_t\left(\beta^*\lambda_{t+1}^*\frac{NER_t}{NER_{t+1}}\frac{R_t^{DC}}{\pi_{t+1}^*}\right) = 0 \quad (3.9)$$

Demand for the CBDC is driven by its usefulness as a payment instrument both domestically and abroad, by its interest rate, which is determined by the domestic monetary authority, by the inflation rate, the exchange rate and the costs of purchasing the CBDC in terms of consumption (λ_t). Notice that combining Equation (3.8), Equation (3.4) and Equation (3.5) suggests that decisions on CBDC holdings affect the domestic risk-free rate as well as the exchange rate. Moreover, equation Equation (3.9) shows that the same holds true for the foreign economy. Issuance of a CBDC creates an additional channel that connects domestic interest rates, the exchange rate and the marginal utility of households, as we explain in greater detail below.[13]

The CBDC is also a substitute for cash as a means of payment. Combining Equation (3.7) and Equation (3.8) it is possible to derive the demand for cash as a function of the marginal utility of CBDC holdings $U'(dc_t)$:[14]

$$m_t = \left\{\frac{1}{\mu^{\$}}\left[U'(dc_t) + \beta E_t\frac{\lambda_{t+1}}{\pi_{t+1}}\left(R_t^{DC} - \xi^{\$}\right)\right]\right\}^{-\frac{1}{\sigma^{\$}}} \quad (3.10)$$

intuitively, the demand for cash depends negatively on the marginal utility of the CBDC and positively on the extent of the liquidity services provided by cash ($\mu^{\$}$). More-

[13]Insofar as the CDBC relaxes the liquidity constraints of foreign agents, they can pay for goods with the CBDC. In other words, a model where foreigners can explicitly pay for imports with CBDC is equivalent to ours where the CBDC enters their utility function. In turn, from the foreign economy's perspective, the CBDC is not only an international safe asset, but also an international means of payment. Moreover, the conditions under which CBDC substitute for bonds are discussed in Appendix C.1.

[14]$U'(dc_t) \equiv \mu^{DC} dc_t^{-\sigma^{DC}}$.

over, when taking portfolio decisions, households also balance the physical storage costs of holding cash, captured by $\xi^{\$} \leq 1$, against the remuneration of CBDC, $R_t^{DC} \geq 1$. The larger is the difference between the two, the less households choose to hold cash, which is one dimension of the trade-off between physical and digital means of payment.

3.2 A CBDC and stronger international linkages: the key economic mechanism

The presence of a CBDC generates a new cross-currency asset pricing relationship which links together interest rates, the exchange rate and the remuneration of the CBDC, in the spirit of Benigno et al. (2019). Specifically, that relationship defines the risk-free rate in the foreign economy as a mark-up on the remuneration of the CBDC (i.e. the interest rate on the CBDC adjusted for exchange rate risk). This is quite intuitive as households, for the same remuneration, strictly prefer to hold CBDC relative to a bond given that the CBDC provides liquidity services, unlike bonds. This leads to stronger exchange rate movements in response to shocks in the presence of a CBDC – foreign agents rebalance much more into CBDC than they would into bonds, if the latter were the only internationally traded asset, because of the CBDC's hybrid nature.

The new arbitrage condition from the foreign country's perspective can be defined by combining Equation (3.9) with the Euler equation in the foreign country:[15]

$$R_t^{DC} \frac{NER_t}{E_t(NER_{t+1})} = R_t^* \left[1 + \phi^{*,DC} \frac{dc_t^*}{NER_t} - \frac{1}{\lambda_t^*} \mu^{*,dc} \left(\frac{dc_t^*}{NER_t} \right)^{-\sigma^{*,dc}} \right]$$

Consider first a simple case where there are no restrictions to the international use of CBDC ($\phi^{*,DC} = 0$) and rearrange to solve for R_t^*. The previous equation becomes:

$$R_t^* = R_t^{DC} \frac{NER_t}{E_t(NER_{t+1})} \left[1 - \frac{1}{\lambda_t^*} \mu^{*,dc} \left(\frac{dc_t^*}{NER_t} \right)^{-\sigma^{*,dc}} \right]^{-1} \tag{3.11}$$

$R_t^{DC} \frac{NER_t}{E_t(NER_{t+1})}$ is the remuneration of the CBDC from the foreign country's perspec-

[15] The Euler equation in the foreign economy is: $E_t \left(\beta^* \frac{\lambda_{t+1}^*}{\lambda_{t+1}^*} \frac{R_t^*}{\pi_{t+1}^*} \right) = 1$.

tive (e.g. the CBDC interest rate plus exchange rate valuation effects). It is worth noticing that $\left[1 - \frac{1}{\lambda_t^*}\mu^{*,dc}\left(\frac{dc_t^*}{NER_t}\right)^{-\sigma^{*,dc}}\right]^{-1} >> 1$ and can be exactly 1 only if the CBDC does not provide liquidity services ($\mu^{*,dc} = 0$). Equation (3.11), therefore, defines the foreign risk-free rate as a mark-up on the remuneration of the CBDC.

Whenever foreign households expect that it is profitable to hold CBDC, they rebalance their portfolios accordingly, and liquidate bonds to purchase CBDC. Crucially, price and quantity adjustments are stronger than what would occur if there were just domestic and foreign bonds in the model — i.e. assets which provide no liquidity services, with no or close-to-zero mark-up — because they are magnified by the mark-up, which capture the intrinsic value of the CBDC as a payment instrument.

In the case of bonds, the arbitrage condition would be: $R_t^* = \left[1 + \phi^* \frac{B_t^{*,F}}{N_t}\right]^{-1} \frac{NER_t}{NER_{t+1}} R_t$, with $\left[1 + \phi^* \frac{B_t^{*,F}}{N_t}\right] \approx 1$, which means that interest rates on domestic and foreign bonds have to be equal, once adjusted for exchange rate risk, with no mark-up. In other words, and as stressed above, agents rebalance much more into CBDC than they would into bonds because of the CBDC's hybrid nature — which is to be an internationally traded asset that also provides payment services.

To understand how international spillovers unfold, consider now the case of foreign households expecting a depreciation of the foreign currency relative to the domestic currency, i.e. $E_t(NER_{t+1})$ is smaller than (NER_t). That would imply that the domestic currency, i.e. the one in which the CBDC is denominated, is expected to appreciate. Hence foreign households expect positive valuation gains from holding CBDC. To capture those gains they sell other assets, like bonds, and purchase CBDC. But assuming that the interest rate on the CBDC is constant and cannot adjust (an assumption which we will relax below), interest rates on foreign bonds (R_t^*) have to increase to maintain equilibrium. This results in an immediate appreciation of the foreign currency relative to the domestic currency, in line with the standard exchange rate overshooting theory.

We will show that the mechanism predicted by Equation (3.11) is at play in model simulations. It is specific to the CBDC in two respects. First, since the CBDC is issued only in the domestic economy, the exchange rate plays no role when domestic households rebalance between domestic bonds and CBDC. Second, the interest rate on the CBDC is assumed to be fixed in the baseline simulation. As a consequence, when foreign households expect an exchange rate appreciation and demand CBDC to benefit from valuation gains, the foreign interest rate has to move to bear part of the adjustment. The induced movement in the foreign interest rate is larger than what is implied by standard arbitrage condition between domestic bonds and foreign bonds shown above as, in this case, the interest rate on domestic bonds would move, too, unlike the CBDC interest rate, thereby contributing to the adjustment process.

The mechanism implies several signature predictions: 1) there are larger exchange rate movements due to overshooting after a shock in the presence of a CBDC; 2) the risk-free rate in the foreign economy moves more strongly in the presence of a CBDC; it should (fall) rise if the foreign country's currency is expected to depreciate (appreciate); 3) the rise (fall) of the risk-free rate should lead to tighter financial conditions in the foreign country, with adverse (positive) consequences on real consumption and investment.

Constraints on international use of the CBDC might dampen those effects. For instance, if $\phi^{*,dc}$ is chosen such that $\phi^{*,DC}\frac{dc_t^*}{NER_t} = \frac{1}{\lambda_t^*}\left(\frac{dc_t^*}{NER_t}\right)^{-\sigma^{*,dc}}$ then spillovers are minimized. In Section 4.2 we explore different CBDC designs and quantify the role of $\phi^{*,dc}$, in particular.

3.3 The labour market

We assume that there is perfect consumption insurance within households. However, following Erceg et al. (2000), each household member is also member of a union that supplies a different type of labour (i) to firms and optimally negotiates wages. Labour varieties are combined by firms through an aggregator à-la Dixit and Stiglitz (1977):

$$L_t = \left(\int_0^1 L_t(i)^{\frac{\nu^L-1}{\nu^L}} di\right)^{\frac{\nu^L}{\nu^L-1}} \tag{3.12}$$

with the wage aggregator being $W_t = \left(\int_0^1 W_t(i)^{1-\nu^L} di\right)^{\frac{1}{1-\nu^L}}$. The optimal demand for labour variety i is proportional to the total demand for labour and the relative cost of each labour variety:

$$L_t(i) = \left(\frac{W_t(i)}{W_t}\right)^{-\nu^L} L_t \tag{3.13}$$

Unions choose the optimal price of labour, $\hat{W}(i)_t$ to maximise profits, taking demand for, and the marginal cost of, a given labour variety as given. We assume the existence of rigidities in the labour market, namely that unions can renegotiate wages with probability $1 - \xi_W$ in each period. The objective function of the union then is:

$$E_t \sum_{j=0}^{\infty} (\beta \xi_W)^j \lambda_{t+j} \left[\frac{\hat{W}(i)_t}{P_{t+j}} - MC(i)_{t+j}^W \right] L(i)_{t+j} \tag{3.14}$$

with the marginal cost of supplying one unit of labour being $MC(i)_t^W = \lambda_t^{-1} \chi (L(i)_t)^{\psi}$. Equation (3.14) is maximised under the constraint of Equation (3.13), the first-order condition is:

$$E_t \sum_{j=0}^{\infty} (\beta \xi_W)^j L_{t+j} \left[\frac{W_t}{W_{t+j}} \right]^{-\nu^L} \left[\lambda_{t+j} \frac{P_t}{P_{t+j}} \frac{\nu^L - 1}{\nu^L} \hat{w}_t^{1+\nu^L \psi} - \chi \left(\frac{1}{w_{t+j}} \frac{P_t}{P_{t+j}} \right)^{-\nu^L \psi} L_{t+j}^{\psi} \right] = 0 \tag{3.15}$$

with w_t and $\hat{w}_t$ as real wages. As all unions are equal, they all choose the same optimal wage if possible, hence we have dropped index i from Equation (3.15).[16] Aggregate wages are:

$$W_t = \left[(1 - \xi_W) \hat{W}_t^{1-\nu^L} + \xi_W W_{t-1}; 1 - \nu^L \right]^{\frac{1}{1-\nu^L}} \tag{3.16}$$

3.4 Firms

Firms in both economies produce final goods, using capital and labour, that are sold domestically and abroad. Each firm produces a specific variety of good, hence it has some degree of market power and final prices are higher than marginal costs. We additionally assume that firms face a friction à-la Calvo (1983) and are able to update prices with probability $1 - \xi$. Domestic and foreign goods are then bundled together by retailers (which operate in perfect competition) to create final consumption and investment goods.

[16]See Appendix A for a comprehensive derivation of the problem.

3.4.1 Production

Each firm (indexed by j) combines capital $K(j)_t$ and labor $L(j)_t$ to produce domestically traded $(Y(j)_{H,t})$ and exported $(X(j)_{F,t})$ goods using the technology:

$$Y(j)_{H,t} + X(j)_{F,t} = A_t K(j)_t^{\alpha} L(j)_t^{1-\alpha} \tag{3.17}$$

with A_t a common total factor productivity (TFP) shock which follows an AR(1) process. Cost-minimization leads to the following equilibrium conditions:

$$R_{K,t} = \int_0^1 \alpha MC(j)_t A_t K(j)_t^{\alpha-1} L(j)_t^{\alpha} dj \tag{3.18}$$

$$\frac{W_t}{P_t} = \int_0^1 (1-\alpha) MC(j)_t A_t K(j)_t^{\alpha} L(j)_t^{-\alpha} dj \tag{3.19}$$

where $MC(j)_t$ is the marginal cost.[17] Firms are penniless and need to rely on bank loans to finance new investments. The law of motion of capital is:

$$K_{t+1} = (1-\delta)K_t + I_t \left[1 - \frac{\phi^K}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right] \tag{3.20}$$

where δ is the depreciation rate of capital and $\left[1 - \frac{\phi^K}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right]$ are capital installation costs.

3.4.2 Price setting

Retailers have monopoly power and set optimally prices to maximise present and future (discounted) profits facing adjustment costs à-la Calvo (1983). The optimal domestic $(\hat{P}_{H,t})$ and foreign $(\hat{P}_{F,t})$ prices are:[18]

$$E_t \sum_{j=0}^{\infty} (\beta\xi)^j \lambda_{t+j} \left[\frac{\hat{P}_{H,t}}{P_{t+j}}\left(\frac{P_{H,t}}{P_{H,t+j}}\right)^{-\nu} Y_{H,t+j} - \frac{\nu}{\nu-1} MC_{t+j}\left(\frac{P_{H,t}}{P_{H,t+j}}\right)^{-\nu} Y_{H,t+j}\right] = 0 \tag{3.21}$$

[17] Goods aggregation is reported in Appendix A.
[18] Derivations are reported in Appendix A.

$$E_t \sum_{j=0}^{\infty} (\beta\xi)^j \lambda_{t+j} \left[\frac{NEER_{t+j}}{P_{t+j}} \hat{P_{F,t}} \left(\frac{P_{F,t}}{P_{F,t+j}} \right)^{-\nu} X_{F,t+j} - \frac{\nu}{\nu-1} MC^*_{t+j} \left(\frac{P_{F,t}}{P_{F,t+j}} \right)^{-\nu} X_{F,t+j} \right] = 0 \tag{3.22}$$

$Y_{H,t}$ and $X_{F,t}$ are domestic and foreign (exports) aggregate demand for the domestic goods. The aggregate price index for home goods sold domestically ($P_{H,t}$) and abroad ($P_{F,t}$) is the weighted average of the optimal price and the previous period's price:

$$P_{H,t} = \left[(1-\xi) \hat{P}_{H,t}^{1-\nu} + \xi P_{H,t+j}^{1-\nu} \right]^{\frac{1}{1-\nu}} \tag{3.23}$$

$$P_{F,t} = \left[(1-\xi) \hat{P}_{F,t}^{*1-\nu} + \xi P_{F,t+j}^{*1-\nu} \right]^{\frac{1}{1-\nu}} \tag{3.24}$$

3.4.3 Goods aggregation

We assume that there is a group of firms, called retailers, that purchase differentiated goods from (monopolistic) producers and bundle them together to produce final goods, with negligible costs. Final goods are created combining domestically produced and imported goods. For instance, final consumption goods C_t are created combining domestically produced goods, $C_{H,t}$, and imported goods, $C^*_{F,t}$ with the technology:

$$C_t = \left[\omega^{1-\rho} (C_{H,t})^{\rho} + (1-\omega)^{1-\rho} (C_{F,t})^{\rho} \right]^{\frac{1}{\rho}} \tag{3.25}$$

where ω is the home bias and ρ the elasticity of substitution between domestic and foreign goods. The demand for domestic and foreign goods depends on total consumption and on the relative price of both:[19]

$$C_{H,t} = \left(\frac{P_{H,t}}{P_t} \right)^{\frac{1}{\rho-1}} \omega C_t \tag{3.26}$$

[19]The demand functions are derived from the price maximization problem of retailers. See Appendix A.

$$C_{F,t} = \left(\frac{P_{F,t}}{P_t}\right)^{\frac{1}{\rho-1}} (1-\omega)C_t \tag{3.27}$$

with the price aggregator being:

$$P_t = \exp e_t^{\pi} \left[\omega \left(P_{H,t}\right)^{\frac{\rho}{\rho-1}} + (1-\omega)\left(P_{F,t}\right)^{\frac{\rho}{\rho-1}}\right]^{\frac{\rho-1}{\rho}} \tag{3.28}$$

e_t^{π} aggregate price shock. Government consumption goods and investment goods have similar problems with the same degree of elasticity between domestic and foreign goods and home bias. These problems are reported in Appendix A for convenience.

3.5 Financial sector

Banks intermediate funds between households and firms. Firms demand new investments but need loans to finance them. The financial system provides those loans in the form of state contingent claims on firms' capital.[20] To issue loans, banks receive liquidity from households in the form of deposits. In each period, banks sell deposits, with returns R_t^D, and finance investments of firms. Banks acquire all returns on capital which are then used to pay depositors. We assume that there is an exogenous default rate on investment projects Ξ_t. Profits from banking activity are rebated to households with lump-sum transfers. Formally, banks maximise profits:

$$E_t \sum_{j=0}^{\infty} \beta^j \lambda_{t+j} \left[(P_{t+j} K_{t+j} \Xi_{t+j} R_{K,t+j}) - D_{t+j} R_{t+j}^D\right] \tag{3.29}$$

subject to capital demand from firms, Equation (3.20), and liquidity constraint of banks. Total loans issued are equal to new investments, i.e. $L_t = I_t$. First-order conditions give the supply for new loans:

$$\lambda_t = Q_t \left\{\left[1 - \frac{\phi^K}{2}\left(\frac{L_t}{L_{t-1}} - 1\right)^2\right] - \frac{L_t}{L_{t-1}}\phi^K\left(\frac{L_t}{L_{t-1}} - 1\right)\right\} + \beta E_t\left[Q_{t+1}\phi^K\left(\frac{L_{t+1}}{L_t} - 1\right)\left(\frac{L_{t+1}}{L_t}\right)^2\right] \tag{3.30}$$

[20] In this way, there is no agency problem between banks and firms.

with $Q_t = \beta E_t \left[Q_{t+1}(1-\delta) + \lambda_{t+1} \exp(\Xi_{t+1}) R_{K,t+1}\right]$. Xi_t is assumed to follow the process $\Xi_t = \rho_\Xi \Xi_{t-1} + \varepsilon_t^\Xi$. In perfect competition, deposits are remunerated according to the marginal return on loans, $\exp(\Xi_{t+1}) R_{K,t+1}$.

3.6 Government

The central bank follows a Taylor-type rule to set the nominal interest rate:

$$\ln R_t = (1-\varrho) \ln R_{t-1} + \varrho \left[R_{ss} + \theta_\pi \ln \pi_t + \theta_y \left(\ln Y_t - \ln Y_{t-1}\right)\right] + e_t \tag{3.31}$$

with $e_t = \rho_r e_{t-1} +_t^R$. Government spending is exogenous.

3.7 Alternative designs for CBDC supply

We assume that the central bank in the domestic economy can issue a CBDC. We consider three scenarios.

CBDC with a fixed interest rate. First, the central bank issues a CBDC with a perfectly elastic supply and fixed (zero) interest rate. Under this design, the central bank accommodates all demand (domestic and foreign) for the CBDC keeping the interest rate constant. This is our baseline assumption on CBDC issuance.

CBDC supplied with a quantity-based rule. In the second scenario, the central bank issues a CBDC in fixed quantity and lets the market determine its price, which we refer to as a quantity-based CBDC. The issuance of CBDC in this case is modelled as a (fixed) share of GDP:

$$DC_t^{supply} = \Omega Y_t \tag{3.32}$$

with $\Omega \in [0,1]$ determines the supply of CBDC as share of GDP. The CBDC price P_t^{DC} in this case is determined by market forces:

$$DC_t^{supply} = P_t^{DC} \left(DC_t + DC_t^*\right) \tag{3.33}$$

remuneration on CBDC holdings then is $R_t^{DC} = \frac{P_t^{DC}}{P_{t-1}^{DC}}$.

CBDC with a flexible (Taylor-rule-type) interest rate. A third scenario we consider is when the central banks sets the interest rate on the CBDC flexibly using a Taylor rule and, for this interest rate, supplies quantities elastically. In this case, interest on the CBDC is:

$$\ln R_t^{DC} = (1 - \varrho_{DC}) \ln R_{t-1}^{DC} + \varrho_{DC} \left[R_{ss}^{DC} + \theta_\pi^{DC} \ln \pi_t + \theta_y^{DC} (\ln Y_t - \ln Y_{ss}) \right] \tag{3.34}$$

notice that when $\theta_\pi^{DS} = 0$ and $\theta_y^{DS} = 0$ the CBDC has a fixed interest rate, which can also be negative.

3.8 Aggregation

Aggregation across firms leads to:

$$Y_t^{tot} = \int_0^1 Y(i)_{H,t} di + \int_0^1 X(i)_{F,t} di = A_t \int_0^1 K(i)_t^\alpha L(i)_t^{1-\alpha} di \tag{3.35}$$

bonds are zero in net supply, i.e.

$$B_t^H + B_t^{*,F} = 0 \tag{3.36}$$

Each economy has 9 exogenous shocks: total factor productivity (TFP), government spending, monetary policy, consumption preferences, CPI, wage markups, capital returns, capital quality and foreign demand. The foreign demand shock is modelled as an exogenous increase in export demand for each country. Processes are reported in Appendix A.

4 Model simulations

We use standard calibration assumptions to simulate the effects of shocks in our model. Our baseline simulations use the parameter values of Eichenbaum et al. (2017). For the welfare exercise, we use the parameters of shock processes estimated in de Walque et al. (2005). An overview of the calibrated values of all parameters is appended hereafter. Moreover, we estimate the model using US and euro area data and show that our main results hold (the estimated results are similarly appended hereafter).

4.1 Baseline simulations

Consider now the effect of a one standard deviation expansionary total factor productivity shock in the domestic economy in the absence of a CBDC. The effects are fairly standard. As expected, the shock leads to an expansion of domestic output on impact, which dissipates gradually over time (see the grey dotted line in the first chart on the top row of Figure 1).[21]

The output expansion comes with standard effects of positive supply shocks. Consumption and investment increase, while inflation falls as production becomes increasingly efficient, thereby leading to an easing in monetary policy (see the second and third charts on the top row of Figure 1). The expansion also leads to significant real international spillovers. Output expands in the foreign economy – but much less than in the domestic economy – in line with the increase in foreign exports to the domestic economy (see the first chart on the bottom row of Figure 1). In contrast, foreign consumption and investment decline. One reason for this is that the foreign central bank strives to offset the increase in foreign inflation arising from the domestic economy's expansionary shock with tighter monetary policy (see the second and third charts on the bottom row of Figure 1). The domestic economy's exports decline in tandem, hit by lower foreign consumption and investment (see Figure E.7). International financial spillovers are significant, too. Monetary policy accommodation in the domestic economy relative to the foreign economy leads to exchange rate overshooting: the exchange rate is weaker on impact, both in real and nominal terms, from the domestic economy's perspective, and then gradually appreciates over time (see the penultimate charts on both rows of Figure 1). Domestic residents purchase domestic bonds – whose value increase due to the local easing in mon-

[21]Further details can be found in Appendix E.2 which reports impulse response functions for a broader range of variables.

etary policy – while foreigners sell foreign bonds – whose value decline due to the local tightening in monetary policy (see Figure E.9).

The effects of the same total factor productivity shock in the presence of a CBDC leads to stronger spillover effects, therefore increasing international linkages. Consider first a CBDC with fixed (zero) interest rate, limited restrictions to foreign purchases ($\phi^{DC} = 0.001$) and perceived to be somewhat (i.e. 10 percent) better than cash in terms of liquidity services. A striking result is that the expansion of foreign output and exports, and the decline in foreign consumption and investment, are stronger (contrast the grey dotted lines and the black lines in Figure 1 and Figure E.8). The reason, is that the key economic mechanism by which the CBDC amplifies international linkages is now at play. The introduction of a CBDC creates a new arbitrage condition that links together interest rates, the exchange rate and the remuneration of the CBDC. As stressed above, that arbitrage condition defines the risk-free rate in the foreign economy as a mark-up on the remuneration of the CBDC (i.e. the interest rate on the CBDC adjusted for exchange rate risk), which leads to stronger exchange rate movements due to the CBDC's hybrid nature — which is to be not only an internationally traded asset but also an instrument that provides liquidity services.

In line with this, the mechanism predicted by Equation (3.11) implies that the initial exchange rate overshooting is stronger by about 50% in nominal terms (but also in real terms at longer horizons, due to more pronounced responses in relative core inflation rates). For foreigners, the stronger expected depreciation of the foreign currency relative to domestic currency over time implies that it becomes more attractive to buy CBDC denominated in domestic currency. To balance the desired increase in foreigners' holdings of CBDC, foreign bond yields have to increase to maintain equilibrium. Consistently with this, the increase in the risk-free rate in the foreign economy is 50 basis points stronger on impact than without CBDC, thereby hitting foreign consumption and investment also more strongly (see the last rows of Figure 1 and Figure E.7).[22]

Consider now the effect of a one standard deviation contractionary monetary policy

[22]The net effect on foreign output remains, however, positive as exports grow even more strongly due to booming consumption in the domestic economy, where monetary policy is eased.

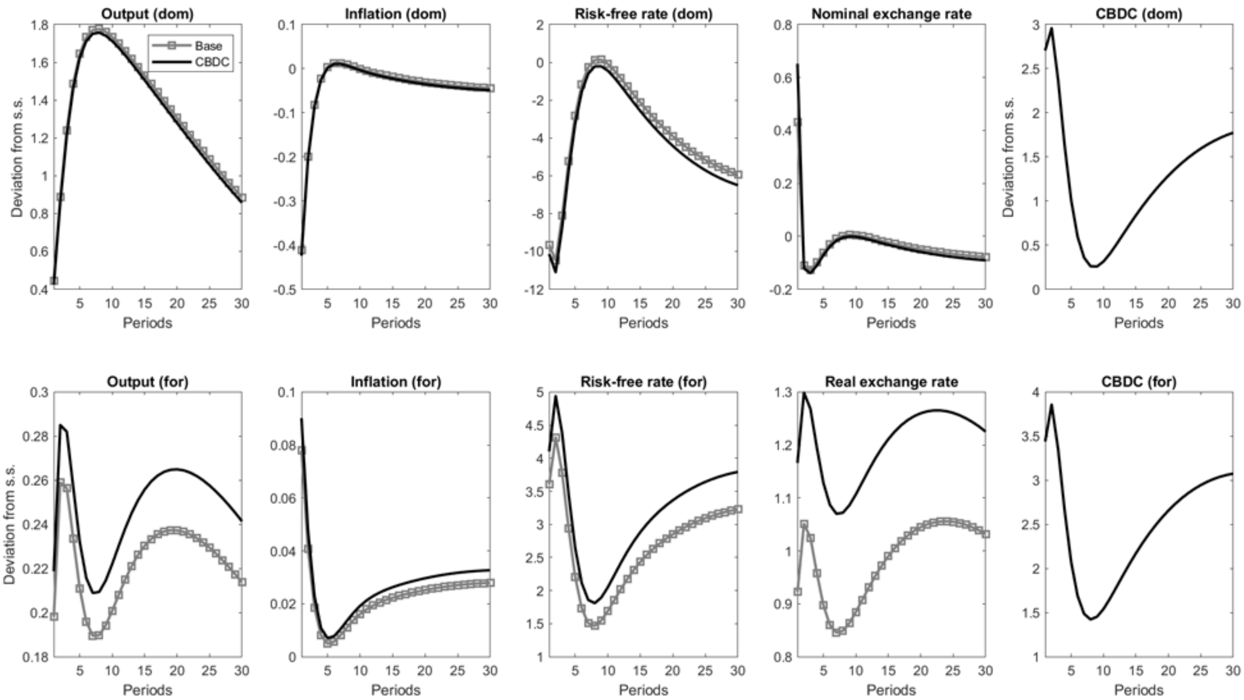

Figure 1: Response of selected variables to a one standard deviation expansionary total factor productivity shock in the domestic economy.
Notes: Responses are reported in deviations from the steady state.

shock in the domestic economy in the absence of a CBDC. Again, the effects are fairly standard. As expected the shock leads to a contraction of domestic output on impact, which dissipates gradually over time (see the grey dotted line in the first chart on the top row of Figure 2). The output contraction comes with standard recessionary effects: consumption and investment decline temporarily and inflation falls. But the recession also leads to significant real international spillovers. Output declines in the foreign economy – but much less than in the domestic economy – as foreign exports to the domestic economy fall (see the grey dotted line in the first chart on the bottom row of Figure 2). One reason why the foreign output contraction is more muted is the reaction of the foreign central bank which strives to offset the effects of the domestic economy's contractionary monetary policy shock by cutting its own policy rate (see the grey dotted line in the second chart on the top row of Figure 2). As a result, foreign consumption and foreign investment increase. The domestic economy's exports increase in tandem, unlike the foreign economy's exports, which are hit by the recession in the domestic economy; see Figure E.12. International financial spillovers are significant, too. Tighter monetary policy in the domestic economy relative to the foreign economy leads to a stronger exchange rate, both in real and nominal terms, from the domestic economy's perspective (see the penultimate chart of Figure 2). Domestic residents sell domestic bonds – whose value decline due to the local tightening in monetary policy – while foreigners buy foreign bonds – whose value increase due to the local easing in monetary policy.

The effects of the same monetary policy shock in the presence of a CBDC also leads to spillover effects. Assumptions on CBDC design are similar as in the previous simulations. A striking result is that the expansion of foreign consumption and investment is larger (contrast the grey dotted lines and the black lines in Figure E.11), although the response of output is overall unchanged.[23] The reason, is that the key economic mechanism by which the CBDC amplifies international linkages is again at play. The initial exchange rate overshooting is about 10 percent stronger in nominal terms. For foreigners, the stronger expected appreciation of the foreign currency relative to domestic currency over time implies that it becomes less attractive to buy CBDC denominated in domestic currency. To balance the desired decrease in foreigners' holdings of CBDC, foreign bond yields have to decrease to maintain equilibrium. Consistently with this, the decrease in the risk-free rate in the foreign economy is about 25 basis points stronger on impact than without CBDC, thereby boosting foreign consumption and investment also more strongly (see the last row of Figure E.11).[24]

[23]The reason is that the stronger positive response of domestic demand (i.e. consumption and investment) is offset by a stronger fall in exports in the presence of a CBDC.

[24]The net effect on foreign output remains, however, neutral as exports decline even more strongly due to weaker consumption in the domestic economy, where monetary policy is tightened.

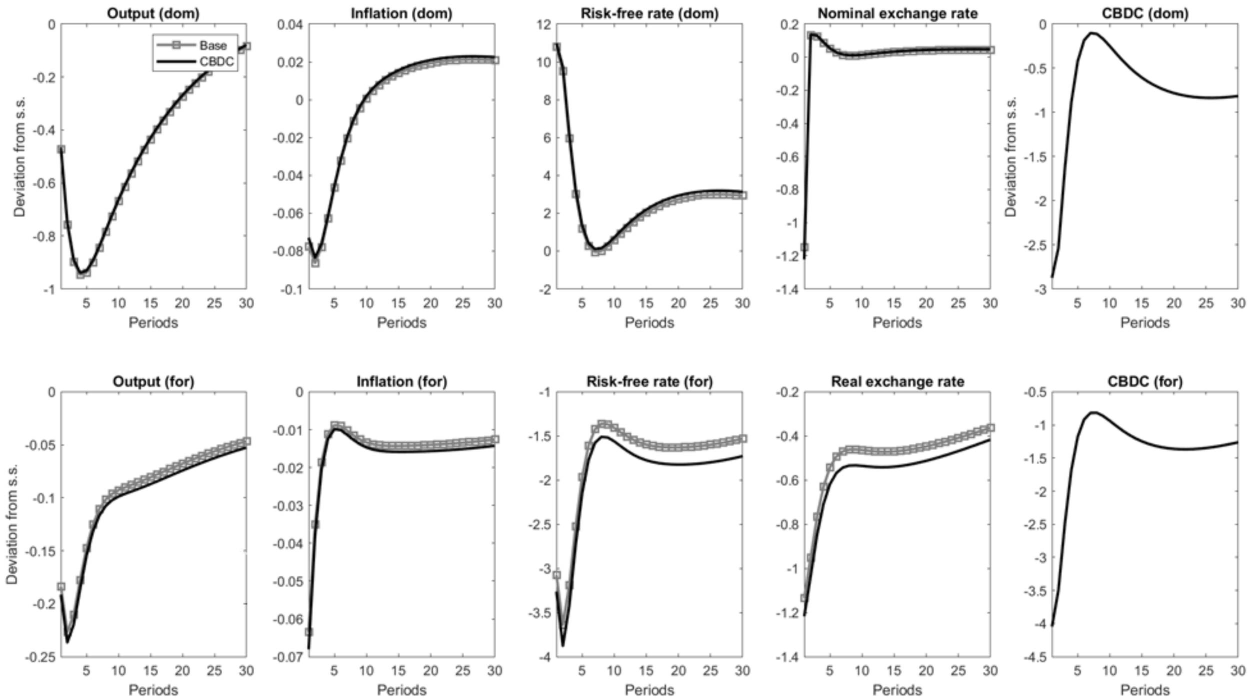

Figure 2: Response of selected variables to a one standard deviation contractionary monetary policy shock in the domestic economy.
Notes: Responses are reported in deviations from the steady state.

4.2 Simulations with alternative CBDC designs

The magnitude of the aforementioned effects depends crucially on CBDC design, notably on whether the supply of CBDC is fixed or elastic; whether the remuneration is fixed of flexible; and on the extent of the restrictions to foreign transactions, captured by parameter ϕ^{DC}, and the value of the liquidity services provided by the CBDC captured by parameter Θ.

CBDC with a fixed interest rate. In the baseline simulation, issuance of CBDC is fully elastic to demand, which enables its interest rate to remain constant (at zero in the baseline). But the effects of the shocks depend crucially on the value of Θ. In particular, spillover effects on foreign output increase with Θ — i.e. when the CBDC's liquidity services become increasingly valuable (see the first chart on the bottom of Figure 3). The higher is Θ the higher is the mark-up in Equation (3.11), which leads to stronger rebalancing forces into CBDC relative to bonds and stronger increases in foreign bond yields in equilibrium. The extent of restrictions on CDBC transactions by foreigners matter, too, but mostly for capital flows. When ϕ^{DC} is high and restrictions tight, foreign capital inflows into CBDC are more muted than when ϕ^{DC} is low and restrictions loose (contrast the black and grey lines of the second chart on the bottom row of Figure 3). But these restrictions make no significant differences for the economic magnitude of international output spillovers.

CBDC supplied with a quantity-based rule. An alternative design is to supply the CBDC with a quantity-based rule where the central bank issues a fixed quantity of CBDC relative to GDP and lets the market set its price (with the CBDC (domestic) remuneration being defined by Equation (3.33)). The response to a total factor productivity shock is broadly similar as in the baseline. Spillover effects on foreign output increase with Θ (see the first chart on the bottom of Figure 4). When ϕ^{DC} is high and restrictions tight, foreign capital inflows into CBDC are more muted than when ϕ^{DC} is low and restrictions loose (contrast the black and grey lines in the second chart on the second row of Figure 4). The

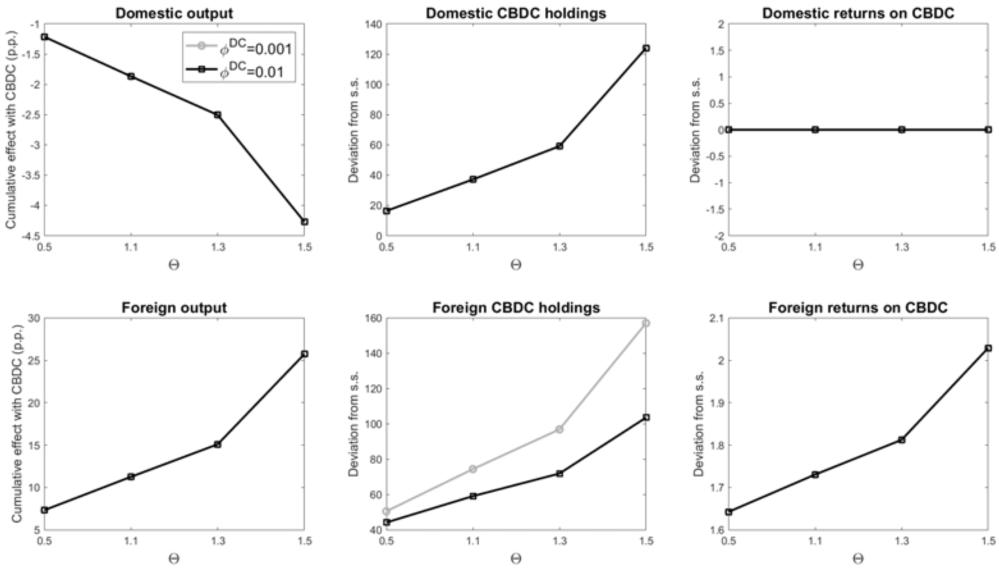

Figure 3: Cumulative responses to a total factor productivity shock in the domestic economy of key macroeconomic variables in the presence of a CBDC with fixed interest rate.

Notes: Cumulative responses in percentage difference relative to the model simulations without CBDC for output and cumulative deviations from the steady state for CBDC holdings and returns. A broader set of results is reported in Appendix E.3.

difference is that spillovers on foreign output also increase with the extent of restrictions on foreign transactions ϕ^{DC} (especially at high values of Θ; contrast the black line and the grey line of the first chart on the bottom row of Figure 4). The reason is that with a fixed supply of CBDC, CBDC remuneration is flexible. And since the latter enters international arbitrage relations, it also feeds directly into the degree of exchange rate adjustment, portfolio rebalancing and, ultimately, domestic and foreign demand.

CBDC with a flexible (Taylor-rule-type) interest rate. A third design is when the domestic central bank supplies CBDC depending on a flexible Taylor rule as in e.g. Equation (3.34). The most striking results is that real international spillovers are then considerably dampened (notice on the y-axis of the charts showing the responses of domestic and foreign output that the magnitude of the cumulative differences relative to the baseline simulations are up to 10 times smaller than in Figure 3 and Figure 4). The reason is that since the interest rate on the CBDC follows a Taylor rule, the interest rate in question then also co-moves strongly with the risk-free rate on domestic bonds, r_t^H. In turn, the international arbitrage relation on the CBDC is very similar to Equation (3.5),

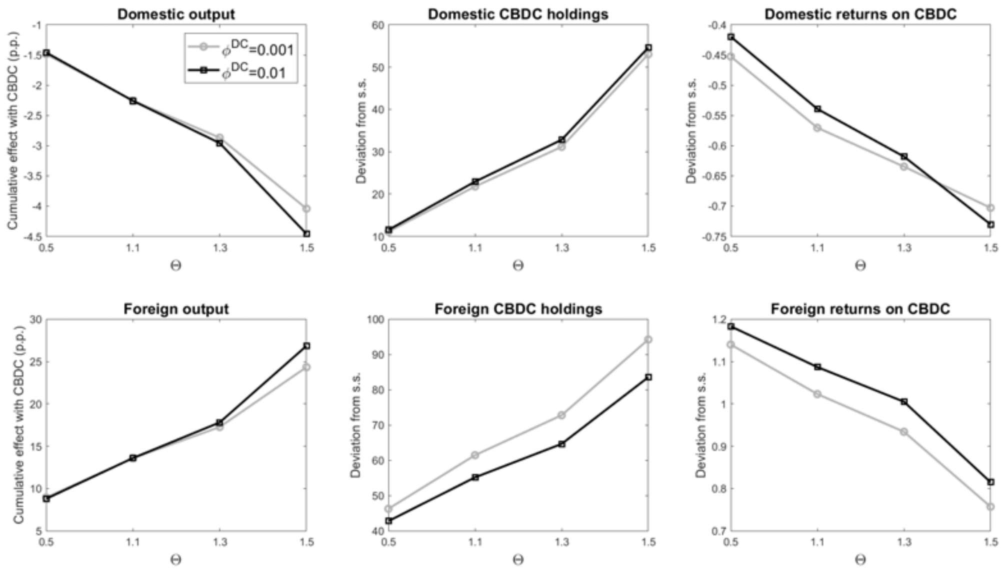

Figure 4: Cumulative responses to a total factor productivity shock in the domestic economy of key macroeconomic variables in the presence of a CBDC supplied in a fixed quantity.
Notes: Cumulative responses in percentage difference relative to the model simulations without CBDC for output and cumulative deviations from the steady state for CBDC holdings and returns. A broader set of results is reported in Appendix E.3.

which limits rebalancing between CBDC and bonds, and reduces movements in the exchange rate. Ultimately, the need for foreign bond yields to adjust is less strong, since the interest rate on the CBDC can adjust, which dampens the impact on foreign demand.

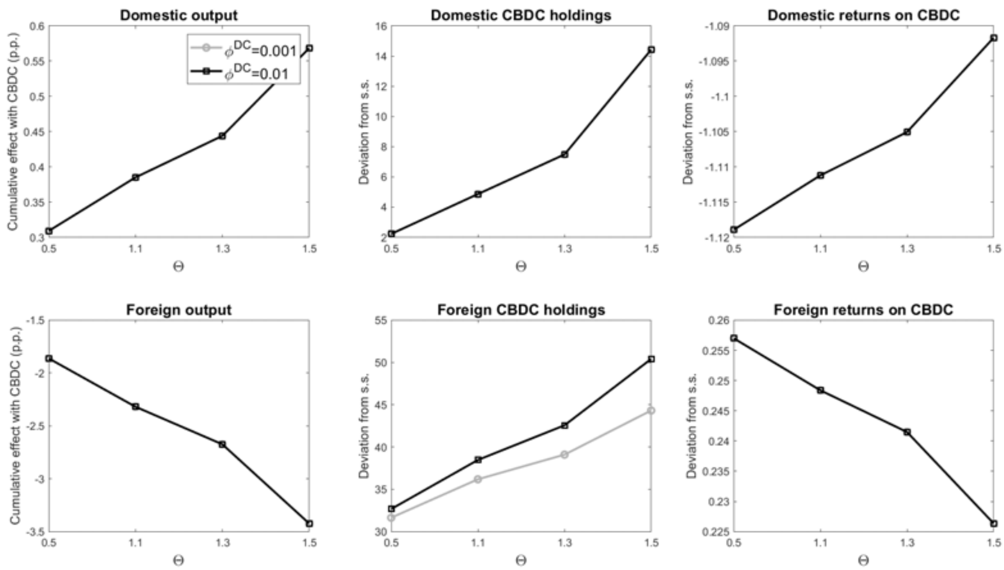

Figure 5: Cumulative responses to a total factor productivity shock in the domestic economy of key macroeconomic variables in the presence of a CBDC with a flexible (Taylor-rule-type) interest rate.
Notes: Cumulative responses in percentage difference relative to the model simulations without CBDC for output and cumulative deviations from the steady state for CBDC holdings and returns. A broader set of results is reported in Appendix E.3.

4.3 Further evidence on the mechanism

That the presence of a CBDC opens a new channel through which pressure on the exchange rate potentially unfolds implies another signature prediction of the model: uncovered interest parity (UIP) deviations – defined as exchange rate movements not accounted for by changes in bond yields – should become larger.[25]

One way to test that prediction is to estimate a standard UIP equation similar to Verdelhan (2018) on simulated data:

$$e_{t+k} - e_t = \alpha_k + \beta_k \left[r_t - r_t^* \right] + \varepsilon_{t+k} \tag{4.1}$$

where e_t is the log of the exchange rate and $r_t - r_t^*$ is the interest differential between domestic and foreign bonds. If UIP holds perfectly, the R^2 from that regression should equal exactly unity, i.e. exchange rate movements should be fully explained by changes in interest rate differentials. R^2s estimated at different horizons are reported in Figure 6 for the baseline model without CBDC and for three possible CBDC designs (fixed remuneration, quantity-based and flexible remuneration). In the model without CBDC, UIP does not hold by construction because of the presence of cross-border transaction costs; therefore it is unsurprising that the R^2 is lower than 1 (see the white bars of Figure 6). But when a CBDC is introduced, changes in interest rate differentials explain yet a lower share of exchange rate movements, as shown by the black and grey bars of Figure 6. Finally, under the flexible remuneration design, the remuneration on the CBDC moves in tandem with the domestic bond interest rate. As a result, interest rate differentials pick-up a share of the stronger exchange rate movements induced by the presence of a CBDC, hence leading to a higher R^2; see Figure 6.

The mirror image to the falling explanatory power of the interest rate differential, is that the remuneration on the CBDC should explain exchange rate changes. This hypothesis can be tested by estimating Equation (4.1) with the CBDC remuneration as

[25] UIP deviations are a well established empirical regularity of exchange rate markets, see Bansal (1997). Froot and Thaler (1990) shows that the standard UIP relation fails for a broad set of currencies; Verdelhan (2018) expands that framework adding global and US factors to the baseline UIP regression.

explanatory variable; Figure E.19 shows that indeed the exchange rate is driven by r_t^{dc}. Consider in particular the case of a CBDC with a flexible-interest-rate design: interest rate differentials explain almost as high a share of exchange rate movements –i.e. UIP holds– as the model without CBDC of Figure 6 (i.e. more than 40%). The reason is that the CBDC interest rate can then move in tandem with domestic bond yields. This in turn reduces incentives of foreign households to rebalance between CDBC and bonds after a shock. And this is one reason why international spillovers in the presence of a CBDC with this particular design are also minimal.

This finding is also evident from Figure 7, where we plot the simulated series for the domestic bond interest rate against the CBDC-flexible interest rate; both move almost one-to-one (the dots cluster nicely around the 45-degree line).

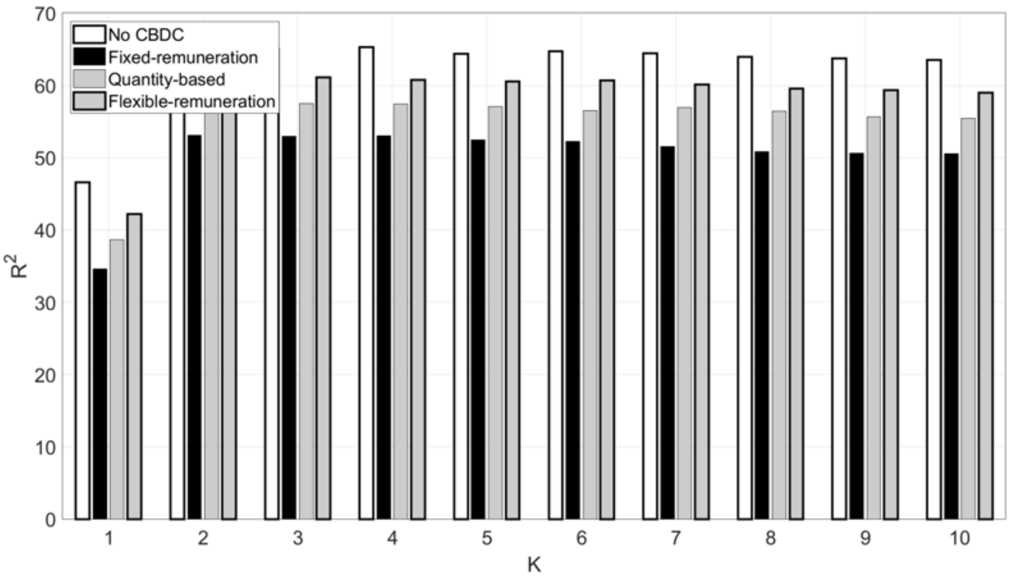

Figure 6: R^2 of the UIP regression $e_{t+k} - e_t = \alpha_k + \beta_k [r_t - r_t^*] + \varepsilon_{t+k}$ for different horizons.
Notes: The UIP regression is estimated separately on simulated data for the model without CBDC and three possible CBDC designs(fixed interest rate, quantity-based and flexible (Taylor-rule-type) interest rate).

4.4 Optimal monetary policy in the presence of a CBDC

How does a CBDC affects optimal monetary policy in the home economy and foreign economy and the gains from international coordination?

To address this question, we now turn to the analysis of the *systematic* component

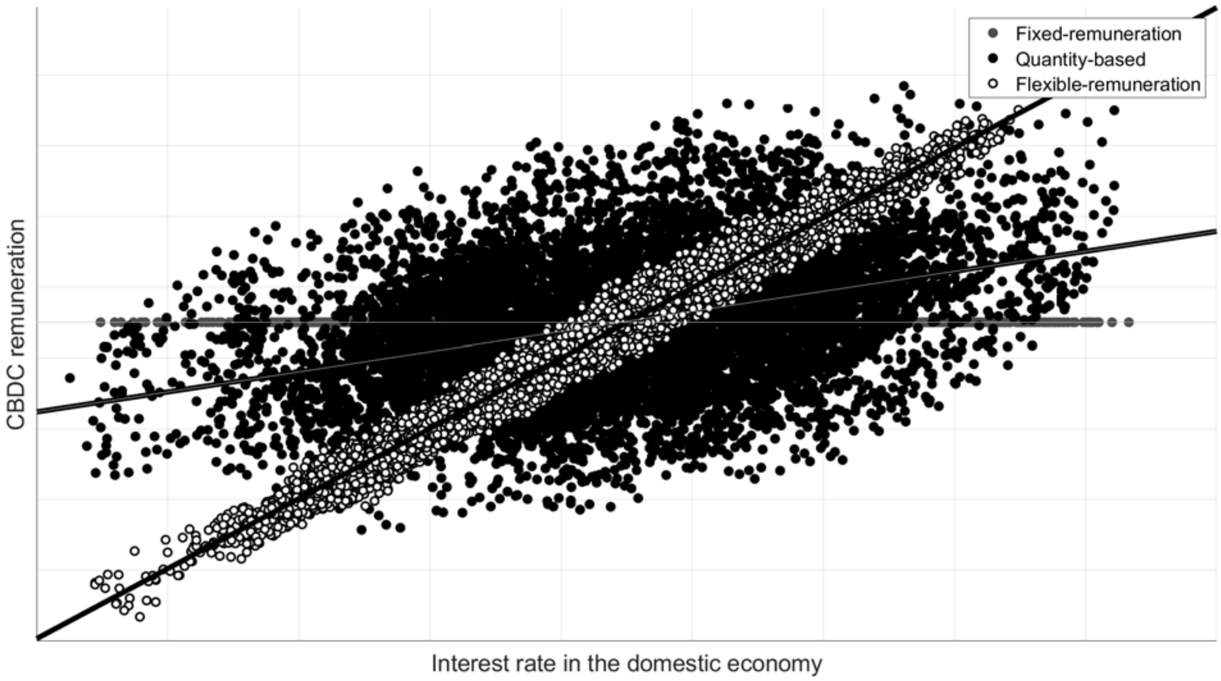

Figure 7: Correlation between the domestic bond interest rate and CBDC interest rate. **Notes:** the chart plots the simulated series for the domestic bond interest rate and the CBDC interest rate for the three possible CBDC designs (fixed interest rate, quantity-based and flexible (Taylor-rule-type) interest rate).

of monetary policy. We compute the parameters of the monetary policy rule described in Equation (3.31) that maximise household welfare (the sum of their current and future utility flows i.e. $W_t = U_t + \beta E_t(W_{t+1})$) for the model solved at the second order with pruning. Simulation results are reported in Table 2, where welfare is normalized to 1 in the model without CBDC.[26]

The main finding which emerges is that domestic issuance of a CBDC increases asymmetries in the international monetary system by reducing monetary policy autonomy in the foreign economy. In the absence of a CBDC, optimal monetary policy is strikingly similar in the home and foreign economies, with differences being driven by slight differences in the calibration of the shock processes. Both central banks should respond to inflation more aggressively than to output, and implement some degree of interest rate smoothing (see column (1) of Table 2). In the presence of a CBDC, however, optimal monetary policy in the two economies differ significantly. For the domestic economy,

[26]Technically, we optimize the parameters of the Taylor rule in each country to maximise average unconditional welfare under a second-order solution of the model. When doing so, we keep the other country's monetary policy parameter constant at the baseline calibration.

differences are limited relative to the simulation without CBDC. Depending on CBDC design, optimal monetary policy warrants a marginally stronger reaction to inflation and a weaker response to output (see columns (2) and (3) of Table 2). The foreign economy, however, is affected much more. Independently from CBDC design, optimal monetary policy warrants a central bank response to inflation and output that is between 50% to 100% stronger than in the baseline simulation (with the exception of the flexible remuneration CBDC design; see column (4)). The introduction of a CBDC by the domestic central bank therefore weighs on monetary policy autonomy of the foreign central bank. It forces the foreign central bank to alter its monetary policy stance to try and mitigate the stronger international spillovers created by the presence of the CBDC. In the case of a CBDC with a flexible remuneration, in contrast, optimal monetary policy warrants higher interest rate smoothing, which mitigates the need for central bank activism. However, in this case the foreign economy is also worst off from a welfare perspective (see the last entry in the bottom row of Table 2) .

Table 2: Optimal monetary policy

	No CBDC	Fixed-interest rate design	Quantity-based design	Flexible-interest rate design
		Domestic economy		
γ	0.50	0.50	0.52	0.53
θ_π	1.10	1.15	1.15	1.10
θ_y	0.45	0.10	0.05	0.45
$(1-\gamma)\theta_\pi^*$	0.55	0.58	0.55	0.52
$(1-\gamma)\theta_y^\star$	0.23	0.05	0.02	0.21
Welfare	1.00	0.91	1.05	0.92
		Foreign economy		
γ	0.58	0.74	0.55	0.80
θ_π	1.10	2.45	2.20	2.50
θ_y	0.60	1.40	1.35	0.45
$(1-\gamma)\theta_\pi^*$	0.46	0.64	0.99	0.50
$(1-\gamma)\theta_y^\star$	0.25	0.36	0.61	0.09
Welfare	1.00	-0.84	-1.20	-1.31

Notes: Optimal parameters of the monetary policy rule for different CBDC designs: col. (1) baseline model (without CBDC), col. (2) CBDC with fixed interest rate, col. (3) CBDC with fixed supply, col. (4) CBDC with a flexible (Taylor rule) interest rate. The key parameters optimized are interest rate persistence (γ), sensitivity to inflation (θ_π) and to output (θ_y). Welfare is computed as the stochastic mean of the welfare function $W_t = U_t + \beta(W_{t+1})$ at the second order and is normalized to 1 for the the model without CBDC shown in column (1).

5 Conclusion

A CBDC does not only have domestic macroeconomic and financial implications for the issuing economy. It has also implications for the rest of the world.

We have shown in this paper that the presence of a CBDC amplifies the international spillovers of standard macroeconomic shocks to a significant extent, thereby increasing international linkages. We have also shown that the magnitude of these effects depends crucially on CBDC design. They can be significantly dampened if the CBDC possesses specific technical features, for instance if there are tight restrictions on CBDC transactions by foreigners or if the CBDC is supplied with a flexible interest rate that would follow e.g. a Taylor rule. And we have shown that domestic issuance of a CBDC increases asymmetries in the international monetary system by reducing monetary policy autonomy in foreign economies. In our simulations, the foreign central bank needs to be up to twice as more reactive to inflation and output in the presence of a CBDC – depending on the latter's design.

These conclusions are the implications that follow from a stylized DSGE model and should therefore be viewed with caution. The model abstracts from CBDC issuance in the foreign economy. It includes only two economies and does not model explicitly the rest of the world. And it does not have much to say on the role of proceeds from CBDC purchases by the public. At the same time, the model provides a distinctive open-economy perspective on CBDC. It has a rich, micro-founded general-equilibrium structure. And it offers a strong degree of realism by incorporating explicitly the most salient aspects of a CBDC's technical design.

These features of our model allow us to draw specific conclusions for policy. Our findings suggest that if a CBDC is available to non-residents, additional volatility in capital flows, exchange rates and interest rates resulting from its presence can be mitigated through holdings limits on transactions by foreigners or through flexibility in the CBDC's remuneration rate. Of the two policy tools, our simulations suggest, the latter is the most powerful – price flexibility dominates quantitative restrictions. And that a

CBDC increases asymmetries in the international monetary system by reducing monetary policy autonomy in foreign economies, but not domestically, suggests that introducing a CBDC sooner rather than later could give rise to a significant first-mover advantage. These findings may be therefore relevant to inform upcoming international policy discussions, such as those of the international group of central banks which share experiences as they assess potential use cases for CBDC in their domestic jurisdictions.

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Appendix

A Derivations

A.1 Wage setting

Substituting Equation (3.13) into Equation (3.14), the objective function for the union becomes:

$$E_t \sum_{j=0}^{\infty} (\beta \xi_W)^j \lambda_{t+j} \left[\frac{\hat{W}(i)_t}{P_{t+j}} - MC(i)_{t+j}^W \right] \left(\frac{\hat{W}(i)_t}{W_{t+j}} \right)^{-\nu^L} L_{t+j} \tag{A.1}$$

the derivative w.r.t. $\hat{W}(i)_t$ is:

$$E_t \sum_{j=0}^{\infty} (\beta \xi_W)^j \left[(1-\nu^L) \frac{L_{t+j}}{P_{t+j}} \left(\frac{\hat{W}_t}{W_{t+j}} \right)^{-\nu^L} + \nu^L \lambda_{t+j} MC(i)_{t+j}^W L_{t+j} \left(\frac{\hat{W}_t}{W_{t+j}} \right)^{-\nu^L} \frac{1}{\hat{W}_t} \right] \tag{A.2}$$

substituting the definition of $MC(i)_t^W$, collecting $Ł_{t+j} \left(\frac{\hat{W}_t}{W_{t+j}} \right)^{-\nu^L}$ and dividing by $-\nu^L$:

$$E_t \sum_{j=0}^{\infty} (\beta \xi_W)^j Ł_{t+j} \left(\frac{\hat{W}_t}{W_{t+j}} \right)^{-\nu^L} \left[\lambda_{t+j} \frac{\hat{W}_t}{P_{t+j}} \frac{\nu^L - 1}{\nu^L} - \chi \left(Ł_{t+j} \left(\frac{\hat{W}_t}{W_{t+j}} \right)^{-\nu^L} \right)^{\psi} \right] \tag{A.3}$$

dividing by the price level, one gets the optimality condition in *real* wages:

$$E_t \sum_{j=0}^{\infty} (\beta \xi_W)^j Ł_{t+j} \left(\frac{\hat{W}_t}{W_{t+j}} \right)^{-\nu^L} \left[\lambda_{t+j} \frac{P_t}{P_{t+j}} \frac{\nu^L - 1}{\nu^L} \hat{w}_t - \chi \left(Ł_{t+j} \left(\frac{\hat{w}_t}{w_{t+j}} \frac{P_t}{P_{t+j}} \right)^{-\nu^L} \right)^{\psi} \right] \tag{A.4}$$

rearranging the terms of the previous equation it is possible to obtain Equation (3.15). Equation (3.15) can be also written in recursive form:

$$F_t^W \hat{w}_t^{1+\nu^L \psi} = K_t^W \tag{A.5}$$

with

$$F_t^W = L_t \lambda_t \frac{\nu^L - 1}{\nu^L} + \beta \xi_W E_t \left[\left(\frac{w_{t+1}}{w_t} \pi_{t+1} \right)^{\nu^L} \frac{F_{t+1}^W}{\pi_{t+1}} \right]$$

$$K_t^W = \chi L_t^{1+\psi} w_t^{\nu^L \psi} + \beta \xi_W E_t \left[\left(\frac{w_{t+1}}{w_t} \right)^{\nu^L} \pi_{t+1}^{\nu^L (1+\psi)} K_{t+1}^W \right]$$

A.2 Price setting

Domestic monopolist maximise profits under monopolistic competition and Calvo pricing. Profits are the sum of profits from domestic sales and imported goods, which are purchased from foriegn producers:

$$\begin{aligned} E_t \sum_{j=0}^{\infty} (\beta \xi)^j \lambda_{t+j} \Bigg[\left(\frac{\hat{P}(i)_{H,t}}{P_{t+j}} - MC_{t+j} \right) \left(\frac{\hat{P}(i)_{H,t}}{P_{H,t+j}} \right)^{\nu} Y_{H,t+j} + \\ + \left(\frac{NER_{t+j}}{P_{t+j}} \hat{P}(i)_{F,t} - MC^*_{t+j} \right) \left(\frac{\hat{P}(i)_{F,t}}{P_{t+j}} \right)^{-\nu} X_{F,t+j} \Bigg] \end{aligned} \tag{A.6}$$

A.3 Goods aggregation

Domestic output is used for consumption, government purchases and investments:

$$C_{H,t} + G_{H,t} + I_{H,t} = Y_{H,t} \tag{A.7}$$

similarly, in the foreign economy, the demand for goods exported from the domestic economy is:

$$C^*_{F,t} + G^*_{F,t} + I^*_{F,t} = X_{F,t}$$

(A.8)

in a similar way, the foreign economy demands foreign domestic goods and exports to the domestic economy:

$$C^*_{H,t} + G^*_{H,t} + I^*_{H,t} = Y^*_{H,t} \tag{A.9}$$

$$C_{F,t} + G_{F,t} + I_{F,t} = X^*_{F,t} \tag{A.10}$$

Final aggregate domestic output and exports are produced by perfectly competitive firms, through a CES aggregator. These firms buy (differentiated) goods from monopolists and produce a single final good with negligible costs. Aggregate domestically sold and exported goods for the domestic economy are:

$$Y_{H,t} = \left[\int_0^1 Y(i)_{H,t}^{\frac{\nu-1}{\nu}} di\right]^{\frac{\nu}{\nu-1}} \tag{A.11}$$

$$X_{H,t} = \left[\int_0^1 X(i)_{H,t}^{\frac{\nu-1}{\nu}} di\right]^{\frac{\nu}{\nu-1}} \tag{A.12}$$

with the demand for each variety i depending on the price of variety i relative to the aggregate price of domestic goods and on total demand:

$$Y(i)_{H,t} = \left(\frac{P(i)_{H,t}}{P_{H,t}}\right)^{-\nu} Y_{H,t} \tag{A.13}$$

similarly for exported goods:

$$X(i)_{F,t} = \left(\frac{P(i)_{F,t}}{P_{F,t}}\right)^{-\nu} X_{F,t} \tag{A.14}$$

perfect competition implies that:

$$P_{H,t} = \left(\int_0^1 P(i)_{H,t}^{1-\nu} di\right)^{\frac{1}{1-\nu}} \tag{A.15}$$

$$P_{F,t} = \left(\int_0^1 P(i)_{F,t}^{1-\nu} di\right)^{\frac{1}{1-\nu}} \tag{A.16}$$

conditions for the foreign economy are totally symmetric.

The optimal bundle of consumption goods is chosen maximize the profits of retailers, which are defined as:

$$P_tC_t - P_{H,t}C_{H,t} - P_{F,t}C_{F,t} \tag{A.17}$$

with $C_t = [\omega^{1-\rho}(C_{H,t})^\rho + (1-\omega)^{1-\rho}(C_{F,t})^\rho]^{\frac{1}{\rho}}$. Optimality conditions are given by Equation (3.26) and Equation (3.27). The aggregator for government consumption is

$$G_t = \left[\omega^{1-\rho}(G_{H,t})^\rho + (1-\omega)^{1-\rho}(G_{F,t})^\rho\right]^{\frac{1}{\rho}} \tag{A.18}$$

notice that the parameters for home bias and the elasticity of substitution between goods are identical to the consumption goods problem. As a consequence, the final price of government consumption goods is also P_t. Profit maximization leads to:

$$\begin{aligned} G_{H,t} &= \left(\frac{P_{H,t}}{P_t}\right)^{\frac{1}{\rho-1}} \omega G_t \\ G_{F,t} &= \left(\frac{P_{F,t}}{P_t}\right)^{\frac{1}{\rho-1}} (1-\omega)G_t \end{aligned} \tag{A.19}$$

Final investment goods are produced with a similar technology:

$$I_t = \left[\omega^{1-\rho}(I_{H,t})^\rho + (1-\omega)^{1-\rho}(I_{F,t})^\rho\right]^{\frac{1}{\rho}} \tag{A.20}$$

also in this case, as technology parameters are the same, the price of final investment goods is P_t. Demand curves, derived from profit maximization, are:

$$\begin{aligned} I_{H,t} &= \left(\frac{P_{H,t}}{P_t}\right)^{\frac{1}{\rho-1}} \omega I_t \\ I_{F,t} &= \left(\frac{P_{F,t}}{P_t}\right)^{\frac{1}{\rho-1}} (1-\omega)I_t \end{aligned} \tag{A.21}$$

A.4 Shock processes

Table A.1 shows the processes for all exogenous shocks of the domestic economy used in the estimation. Shocks for the foreign economy are exactly symmetric. The price mark-up shock is defined as a shock to the elasticity of substitution ν that defines the steady state price mark-up; a headline inflation shock is a shock to the price aggregator Equation (3.28); capital quality shocks are introduced as shocks to the law of motion of capital, Equation (3.20). Finally there is one global shock, a foreign demand shock to Equation (A.8) and Equation (A.10).

Table A.1: Shock processes

Shock	Process
TFP	$A_t = \rho_A A_{t-1} + \varepsilon_t^A$
Government spending	$\ln G_t = \ln G_{ss} + \rho_G \left(\ln G_{t-1} - \ln G_{ss}\right) + \varepsilon_t^G$
Monetary policy	$e_t = \rho_R e_{t-1} + \varepsilon_t^R$
Consumption preference	$e_t^C = \rho_C e_{t-1}^C + \varepsilon_t^C$
CPI shock	$e_t^\pi = +\rho_\pi e_t^\pi + \varepsilon_t^\pi$
Capital quality	$\psi_t = \rho_\psi \psi_{t-1} + \varepsilon_t^\psi$
Capital returns	$\Xi_t = \Xi_{ss} + \rho_\Xi \left(\Xi_{t-1} - \Xi_{ss}\right) + \varepsilon_t^\Xi$
Global demand	$Y_t^w = Y_{ss}^w + \rho_{yw} \left(Y_t^w - Y_{ss}^w\right) + \varepsilon_t^{yw}$

B Additional tables

Table B.2: Calibrations

Domestic economy			Foreign economy		
Parameter	Description	Value	Parameter	Description	Value
β	Discount factor	0.9926	β	Discount factor	0.9926
σ	Elasticity of consumption	1	σ	Elasticity of consumption	1
h	Habit persistence	0.65	h	Habit persistence	0.65
φ	Labor supply elasticity	1	φ	Labor supply elasticity	1
$\mu^{\$}$	Weight of cash	1	$\mu^{\$}$	Weight of cash	1
$\sigma^{\$}$	Elasticity of cash	10.62	$\sigma^{\$}$	Elasticity of cash	10.62
ϕ^B	Cross-border bond holding costs	0.001	ϕ^B	Cross-border bond holding costs	0.001
ϕ^K	Capital adj. costs	1.728	ϕ^K	Capital adj. costs	1.728
δ	Depreciation rate of capital	0.025	δ	Depreciation rate of capital	0.025
ν	Elasticity of demand	6	ν	Elasticity of demand	6
ρ	Elasticity of substitution	0.333333	ρ	Elasticity of substitution	0.333333
ω	Home bias	0.9	ω	Home bias	0.9
	Calvo pricing	0.6		Calvo pricing	0.6
α	Technology	0.3	α	Technology	0.3
γ	Int. rate smoothing	0.75	γ	Int. rate smoothing	0.75
θ_π	Sensitivity to inflation	1.2	θ_π	Sensitivity to inflation	1.2
θ_y	Sensitivity to output	0.6	θ_y	Sensitivity to output	0.6
$^{\$}$	Cash storage costs	1	$^{\$}$	Cash storage costs	1
$\frac{G_{ss}}{Y_{ss}}$	S.s. gov. spending to output ratio	0.2	$\frac{G_{ss}}{Y_{ss}}$	S.s. gov. spending to output ratio	0.2
ν^L	Elasticity of labor substitution	21	ν^L	Elasticity of labor substitution	21
$_{W}$	Calvo pricing for wages	0.65	$_{W}$	Calvo pricing for wages	0.65
ρ_R	Persistance of mon. policy shocks	0.36	ρ_R	Persistance of mon. policy shocks	0.06
ρ_A	Persistance of TFP shocks	0.96	ρ_A	Persistance of TFP shocks	0.95
ρ_G	Persistance of gov. spending shocks	0.83	ρ_G	Persistance of gov. spending shocks	0.95
ρ_C	Persistance of preference shocks	0.81	ρ_C	Persistance of preference shocks	0.8
ρ_ψ	Persistance of capital quality shocks	0.92	ρ_ψ	Persistance of capital quality shocks	0.97
ρ_Ξ	Persistance of risk shocks	0.5	ρ_Ξ	Persistance of risk shocks	0.5
ρ_π	Persistance of price shocks	0.95	ρ_π	Persistance of price shocks	0.84
ρ_{yw}	Persistance of global demand shocks	0.5	ρ_{yw}	Persistance of global demand shocks	0.5
σ_R	Volatility of mon. policy shocks	0.16	σ_R	Volatility of mon. policy shocks	0.24
σ_A	Volatility of TFP shocks	0.78	σ_A	Volatility of TFP shocks	0.44
σ_G	Volatility of gov. spending shocks	0.36	σ_G	Volatility of gov. spending shocks	0.53
σ_C	Volatility of preference shocks	0.49	σ_C	Volatility of preference shocks	0.73
σ_ψ	Volatility of capital quality shocks	0.53	σ_ψ	Volatility of capital quality shocks	0.55
σ_Ξ	Volatility of risk shocks	0.01	σ_Ξ	Volatility of risk shocks	0.01
σ_π	Volatility of price shocks	0.14	σ_π	Volatility of price shocks	0.15
σ_{yw}	Volatility of foreign demand shocks	0.01	σ_{yw}	Volatility of foreign demand shocks	0.01
CBDC parameters					
Θ	Preference for CBDC	1.1	Θ	Preference for CBDC	1.1
γ^{DC}	Persistance of CBDC returns	0.7	ϕ^{DC}	CBDC cross-boarder costs	0.001
θ_y^{DC}	Sensitivity to output	0.6			
θ_π^{DC}	Sensitivity to inflation	1.2			
$\frac{DC^{Supply}}{Y}$	CBDC supply (in terms of domestic GDP)	0.2			

C Extensions

C.1 A note on the substitution of bonds

Households hold CBDC for two main reasons. First, the CBDC is used for payments and hence relaxes the liquidity constraints of households (in terms of consumption utility this is picked up by $\frac{\mu^{DC}}{\lambda_t} DC_t^{-\sigma^{DC}}$. Second, the CBDC is also a financial asset. As an asset, the returns on CBDC holdings affect portfolio allocation decisions and, through foreign demand for CBDC, the equilibrium exchange rate. Both the "payment" function and the "asset" function influence demand for CBDC. The presence of the CBDC generates an additional no-arbitrage condition between the remuneration on the CBDC, which also includes the value of the liquidity services it provides, and all other assets.[27] That in turns changes how interest rates and the exchange rate react to shock, thus changing intertemporal consumption and investment decisions.

The demand for CBDC in the domestic economy can be derived combining Equation (3.4) and Equation (3.8):

$$dc_t = \left[1 - \frac{R_t^{DC}}{R_t}\right]^{-\frac{1}{\sigma^{DC}}} \left(\frac{\mu^{DC}}{\lambda_t}\right)^{\frac{1}{\sigma^{DC}}} \tag{C.1}$$

σ^{DC} and μ^{DC} define how useful the CBDC as a payment instrument is or, equivalently, how much the CBDC is effective in relaxing the liquidity constraints of households (in terms of consumption utility). Notice also that when households consider whether to buy CBDC, they compare the benefit of holding it against its shadow opportunity cost, i.e. purchasing the same amount of government bonds. Equation (C.1) thus creates a connection between the remuneration on the CBDC and the risk-free rate. In our baseline calibration, we assume that the CBDC is 10% more efficient than cash, thus resulting in μ^{DC} being 10% higher than $\mu^{\$}$ and σ^{DC} being 10% lower than $\sigma^{\$}$. We define the efficiency ratio between the CBDC and cash as Θ.

[27] We show below that introducing a CBDC affects the exchange rate and cross-country asset correlations; see Section 4.3.

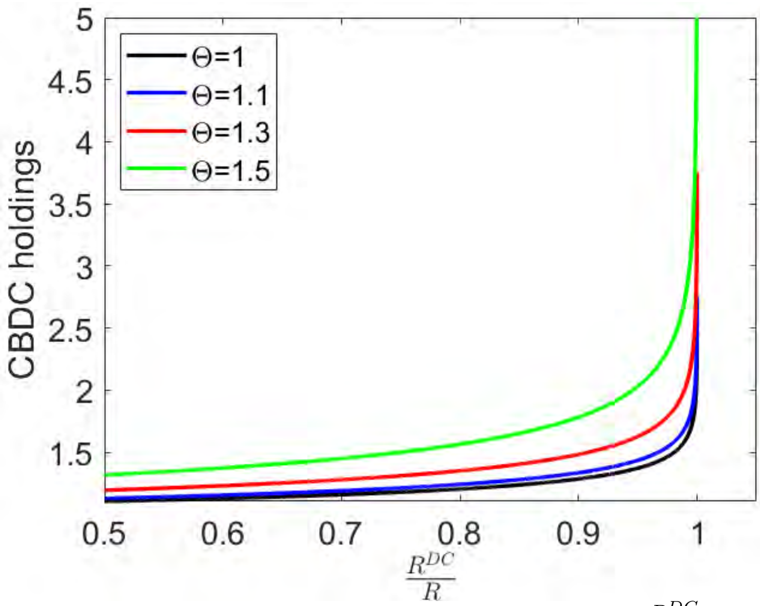

Figure C.1: CBDC demand for different levels of the spread $\frac{R^{DC}}{R}$ and usefulness as a payment instrument (Θ)

Equation (C.1) shows that demand for CBDC depends positively on the spread between returns on the CBDC and the risk-free rate as well as on the usefulness of the CBDC for payment services. Clearly, Equation (C.1) reaches an asymptotic value when $\frac{R^{DC}}{R} \to 1$. If returns on the CBDC are higher than the risk-free rate, households convert their holdings of bonds into CBDC. In the steady state, however, this condition never holds. Equation (C.1), however, defines an additional endogenous constraint on the path of domestic interest rates. Figure C.1 reports simulations for different levels of Θ and the spread between returns.[28] In particular, the more useful the CBDC is, the faster bond holdings are substituted with CBDC holdings. This result is economically intuitive as households derive utility from the CBDC beyond its remuneration since, differently from bonds, a CBDC also provides liquidity services.

Consider now the foreign CBDC holding condition. Combining Equation (3.5) and Equation (3.8) leads to:

$$\frac{\left[1 - \lambda_t^{-1} \mu^{DC} dc_t^{-\sigma^{DC}}\right]}{\Gamma_t} = \frac{R_t^{DC}}{R_t^*} E_t S_{t+1}^{-1} \tag{C.2}$$

[28] Figure E.6 shows additional combinations of CBDC efficiency and spreads.

where Γ_t are cross-border costs[29] and S_t denotes the exchange rate change. This relationship makes explicit two important economic mechanisms in the presence of a CBDC. First, ceteris paribus, when the foreign interest rate rises, the exchange rate has to depreciate and, conversely, when the remuneration rate on CBDC rises, the exchange rate has to appreciate. This is nothing else than a standard international no-arbitrage condition, which comes on top –and possibly in some cases against– the standard no-arbitrage conditions between relative interest rates on other monetary and financial assets. Notably, the existence of a CBDC interacts (and possibly interferes with) uncovered interest parity, i.e. one of the main mechanisms pinning down the relation between interest rates and the exchange rate in standard open-economy macro models. Second, when domestic households hold CBDC (i.e. the left hand side of Equation (C.2) *decreases*) the exchange rate has to appreciate in the absence of interest rate movements. Notice that this channel is stronger when the usefulness of the CBDC as a payment instrument increases.

The problem for the foreign economy is broadly similar:

$$dc_t^* = \left[1 + \Gamma_t^{DC} - \frac{R_t^{DC}}{R_t^*} S_{t+1}^{-1}\right]^{-\frac{1}{\sigma^{DC}}} \left(\frac{\mu^{DC}}{\lambda_t^*}\right)^{\frac{1}{\sigma^{DC}}} \tag{C.3}$$

Equation (C.3) is broadly similar to Equation (C.1) with two exceptions, namely the term Γ_t^{DC} (which picks up the costs faced by for foreigners for buying CBDC)[30] and the presence of the exchange rate in the equation. These two terms allow the spread between the remuneration rate on the CBDC and the foreign risk-free rate to be positive (i.e. up to $1+\Gamma_t^{DC} > 1$), without unleashing massive capital outflows from the foreign bond market. When the spread is larger than $1 + \Gamma_t^{DC}$, further increases in the remuneration rate on the CBDC can be accommodated only with an exchange rate depreciation in the foreign economy. Figure C.2 illustrates how transaction costs on foreign purchases of CBDC interact with the remuneration rate on the CBDC. The higher the transaction costs, the further away moves the threshold above which the foreign bond market is outcompeted by the CBDC. This is economically intuitive: tighter limits to foreign purchases of CBDC

[29] To keep notation compact we define here $\Gamma_t = 1 + \phi^B NER_t B_t^F$.

[30] Γ_t^{DC} is defined as $\phi^{DC} NER_t^{-1} dc_t^*$

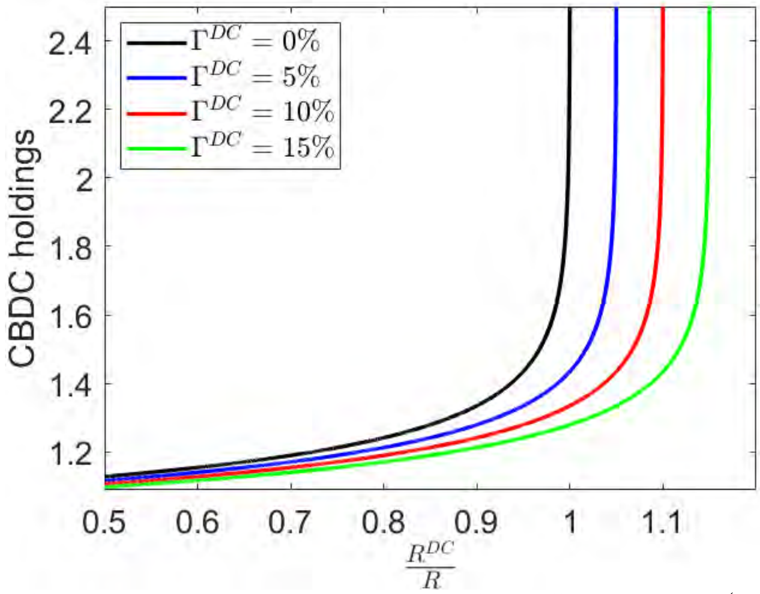

Figure C.2: CBDC demand in the foreign country for different levels (in percent of total) of transactions costs on foreign purchases assuming no changes in the exchange rate and $\Theta = 1.1$.

partially shield the foreign economy from large capital outflows when the CBDC has a more attractive remuneration rate.

C.2 Higher storage costs

In this section we explore the effects of higher storage costs. In our baseline calibration, there are no storage costs for cash (i.e $\xi^{\$} = 1$). We allow this parameter to vary such that households face a cost ranging from 5 to 20% per unit of cash.

Figure C.3 reports the response of output and CBDC holdings for a positive total factor productivity shock in the domestic economy. Higher storage costs reduce the demand for CBDC both domestically and abroad. This result is consistent across CBDC designs and translates in lower output deviations relative to the model without CBDC. The rational for this response is mechanical: cash holding costs tighten the budget constraint of households who, in turn, reduce purchases of all instruments, including CBDC; this is nothing else than a wealth effect. Lower CBDC purchases reduce substitution effects between bonds, deposits and the CBDC; additionally, in the foreign economy it reduces exchange rate fluctuations. Altogether, these dynamics mitigate the multiplicative effects of the CBDC on aggregate macroeconomic variables. When holding cash is costly, household also choose to hold less cash, but this substitution effect is second order relative to the wealth effect of storage costs. These results, however, should not be overstated. Cash holding costs have very limited (first order) effects on the model. Consider the case of fixed-remuneration design for CBDC. Moving from zero costs to a 20% holding costs would decrease output spillovers by 0.22 pp over a 30-period horizons and foreign output spillovers by 0.8 pp over the same period. These magnitudes are similar across different CBDC designs.

Figure C.4 reports the response to a domestic contractionary monetary policy shock. Also in this case increased storage costs reduce spillovers and CBDC holdings mainly through wealth effects. The order of magnitude, however, is very limited and significantly lower than 1 percentage point across CBDC designs over a 30-period horizon.

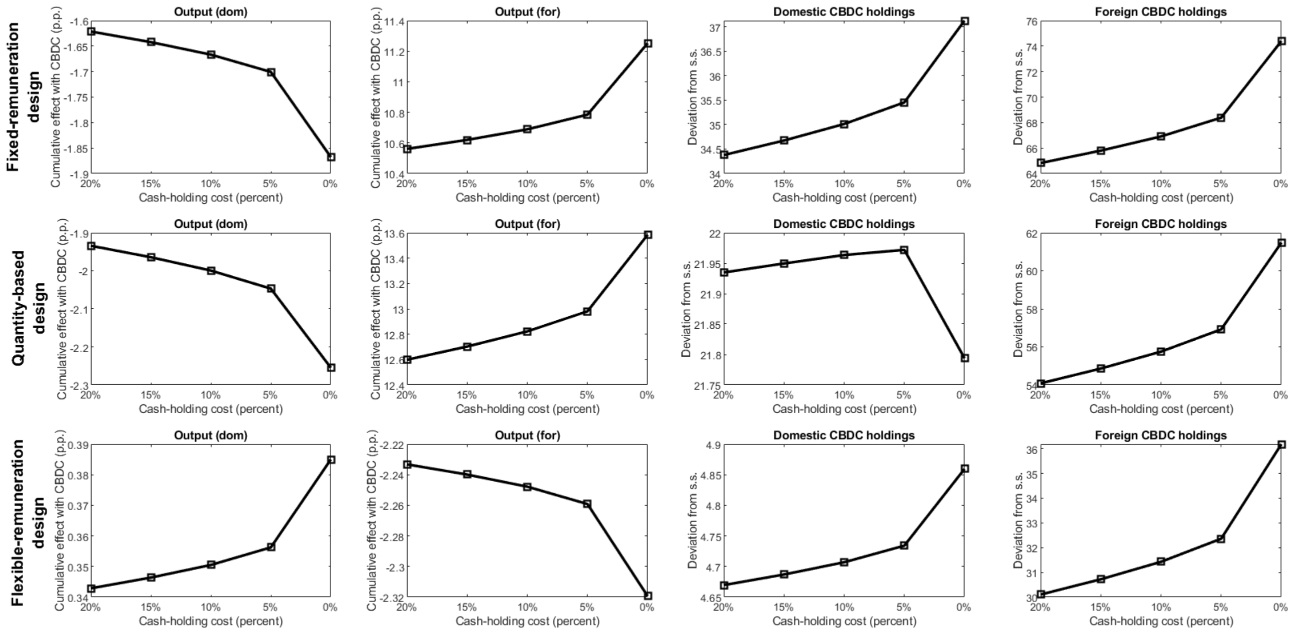

Figure C.3: Cumulative responses to a total factor productivity shock in the domestic economy of key macroeconomic variables for different levels of storage costs and CBDC designs.

Notes: Cumulative responses in percentage difference relative to the model simulations without CBDC for output and cumulative deviations from the steady state for CBDC holdings

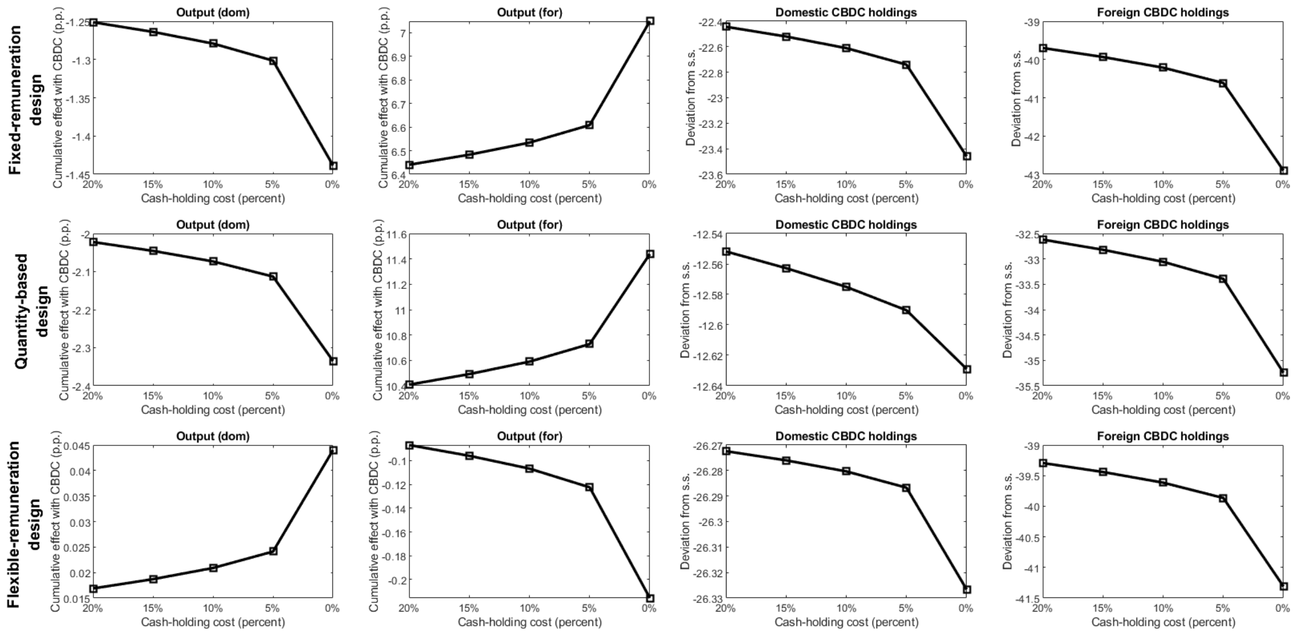

Figure C.4: Cumulative responses to a contractionary monetary policy shock in the domestic economy of key macroeconomic variables for different levels of storage costs abnd CBDC designs.

Notes: Cumulative responses in percentage difference relative to the model simulations without CBDC for output and cumulative deviations from the steady state for CBDC holdings

D Estimation

The log-linearized version of the model can be written in state-space form. Following Smets and Wouters (2007), Gerali et al. (2010) and Christiano et al. (2014), only a subset of the deep parameters of the model are estimated. In line with the standard practice, we estimate the parameters for preferences and the production function, but keep calibrated values for the depreciation rates and demand elasticities. This approach has the advantage of building on the (micro) empirical literature for those parameters.

The remaining deep parameters of the model are estimated as in Smets and Wouters (2007). The posterior distribution of the model parameters Ψ is approximated using the likelihood times the prior distribution, according to the well-known relation:

$$p\,(\Psi \mid Y) \propto \mathcal{L}\,(Y \mid \Psi)\, p\,(\Psi) \tag{D.1}$$

with $\Psi = \{\sigma^{\varepsilon^{A,H}}, \sigma^{\varepsilon^{A,F}}, \sigma^{\varepsilon^{R,H}}, \sigma^{\varepsilon^{R,F}}, \sigma^{\varepsilon^{C,H}}, \sigma^{\varepsilon^{C,F}}, \sigma^{\varepsilon^{\pi,H}}, \sigma^{\varepsilon^{\pi,F}}, \sigma^{\varepsilon^{\psi,H}}, \sigma^{\varepsilon^{\psi,F}}, \sigma^{\varepsilon^{yw,H}}, \sigma^{\varepsilon^{yw,F}},$ $\rho_{A,H}, \rho_{A,F}, \rho_{C,H}, \rho_{C,F}, \rho_{\pi,H}, \rho_{\pi,F}, \rho_{\psi,H}, \rho_{\psi,F}, \rho_{yw,H}, \rho_{yw,F}, \varrho_H, \varrho_F, \theta_{\pi,H}, \theta_{\pi,F}, \theta_{y,H}, \theta_{y,F}, \xi_H, \xi_F$ $\mu^{\$}_H,\ \mu^{\$}_F,\ \sigma^{\$}_H,\ \sigma^{\$}_F\}$. When estimating the model we deliberately exclude the government spending shock, which becomes colinear to a global demand shock in Equation (A.8) and Equation (A.10) and the risk shock, which – having to do with variance terms – would require a higher-order solution to be correctly estimated. Equation (D.1) does not have a closed form solution, hence we evaluate it with an MCM algorithm repeated for two chains with 1.5 million draws each.[31]

D.1 Data

The model is estimated using 10 quarterly macro-variables for the US and the euro area over the period 1999Q1 to 2019Q4: real GDP, consumption and investment, inflation and the short term rate[32] We restrict the sample to the post-1999 period to account for the

[31]We run two parallel chains of 1.5 million iterations each and discard the first 500,000 draws.

[32]We use the 3-month LIBOR rate for the US and the 3-month EURIBOR rate for the euro area. The short term money market rate is used, instead of the policy rate, to account for the effective lower bound period. As a robustness check we estimate the model using the Wu-Xia shadow rate and the policy rate; results do not change significantly.

fact that the creation of the euro in 1999 was a major structural break. Pre-1999 data, additionally, suffer for significant aggregation statistical challenges which make it difficult to create synthetic time series for the euro area in the 1970s and 1980s. Variables are transformed following Smets and Wouters (2007).

Observables are related to the model through the following measurement equations:

$$
\begin{aligned}
\Delta Y_t^{obs} &= 100 * (\ln Y_t - \ln Y_{t-1}) \\
\Delta C_t^{obs} &= 100 * (\ln C_t - \ln C_{t-1}) \\
\Delta I_t^{obs} &= 100 * (\ln I_t - \ln I_{t-1}) \\
\pi_t^{obs} &= 100 * \pi_t \\
R_t^{obs} &= R_t - 1
\end{aligned}
\tag{D.2}
$$

Table D.3 reports the estimated parameters, prior choices and posteriors. Relative to the estimates of de Walque et al. (2005) our data suggest a lower volatility of shocks while the estimates for the autoregressive components are broadly similar. Using the estimated parameters, the results are broadly similar as those of the baseline model simulations (see Figure D.5).

Table D.3: Estimated parameters

Param. Name	Prior distribution	Prior mean	Prior std	Posterior mean	Lower 10%	Upper 10%
$\sigma^{\varepsilon^{A,H}}$	Inv. Gamma	0.78	2	0.118	0.104	0.133
$\sigma^{\varepsilon^{A,F}}$	Inv. Gamma	0.44	2	0.073	0.063	0.084
$\sigma^{\varepsilon^{R,H}}$	Inv. Gamma	0.16	2	0.021	0.019	0.024
$\sigma^{\varepsilon^{R,F}}$	Inv. Gamma	0.24	2	0.031	0.028	0.033
$\sigma^{\varepsilon^{C,H}}$	Inv. Gamma	0.49	2	0.061	0.059	0.064
$\sigma^{\varepsilon^{C,F}}$	Inv. Gamma	0.73	2	0.216	0.186	0.255
$\sigma^{\varepsilon^{\pi,H}}$	Inv. Gamma	0.14	2	0.018	0.017	0.020
$\sigma^{\varepsilon^{\pi,F}}$	Inv. Gamma	0.15	2	0.018	0.018	0.019
$\sigma^{\varepsilon^{\psi,H}}$	Inv. Gamma	0.53	2	0.067	0.064	0.071
$\sigma^{\varepsilon^{\psi,F}}$	Inv. Gamma	0.55	2	0.343	0.306	0.379
$\sigma^{\varepsilon^{yw,H}}$	Inv. Gamma	0.01	2	0.007	0.004	0.123
$\sigma^{\varepsilon^{yw,F}}$	Inv. Gamma	0.01	2	0.009	0.004	0.019
$\rho_{A,H}$	Beta	0.95	0.1	0.960	0.957	0.963
$\rho_{A,F}$	Beta	0.95	0.1	0.999	0.998	0.999
$\rho_{R,H}$	Beta	0.36	0.1	0.488	0.472	0.504
$\rho_{R,F}$	Beta	0.06	0.1	0.206	0.196	0.218
$\rho_{C,H}$	Beta	0.81	0.1	0.934	0.930	0.939
$\rho_{C,F}$	Beta	0.8	0.1	0.990	0.990	0.999
$\rho_{\pi,H}$	Beta	0.95	0.1	0.998	0.997	0.999
$\rho_{\pi,F}$	Beta	0.84	0.1	0.719	0.707	0.732
$\rho_{\psi,H}$	Beta	0.92	0.1	0.694	0.675	0.711
$\rho_{\psi,F}$	Beta	0.97	0.1	0.948	0.942	0.956
$\rho_{yw,H}$	Beta	0.5	0.1	0.432	0.417	0.449
$\rho_{yw,H}$	Beta	0.5	0.1	0.467	0.462	0.473
ξ_H	Beta	0.6	0.1	0.703	0.691	0.718
ξ_F	Beta	0.6	0.1	0.719	0.676	0.687
$\mu_H^{\$}$	Normal	1	0.1	0.989	0.984	0.992
$\mu_F^{\$}$	Normal	1	0.1	1.210	1.194	1.223
$\sigma_H^{\$}$	Normal	10.62	2	9.347	9.081	9.608
$\sigma_F^{\$}$	Normal	10.62	2	10.436	10.251	10.645
ϱ_H	Beta	0.75	0.1	0.727	0.702	0.753
ϱ_F	Beta	0.75	0.1	0.858	0.851	0.865
$\theta_{\pi,H}$	Normal	1.2	0.25	1.001	1.000	1.002
$\theta_{\pi,F}$	Normal	1.2	0.25	1.319	1.302	1.343
$\theta_{y,H}$	Normal	0.8	0.1	0.541	0.524	0.559
$\theta_{y,F}$	Normal	0.8	0.1	0.755	0.748	0.764

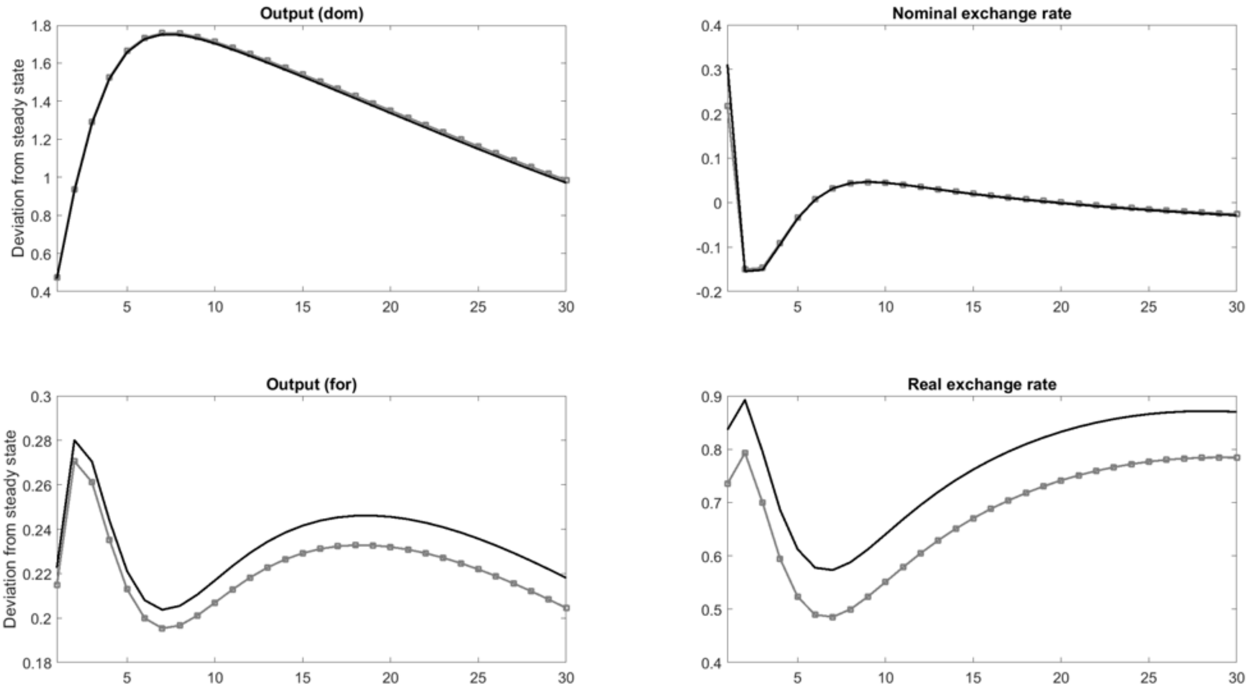

Figure D.5: Response to a total factor productivity shock in the domestic economy.
Notes: The chart reports the response to a positive total factor productivity shock in the domestic economy with the estimated parameters. The shock is calibrated to be the same as the total factor productivity shock used in the main section.

E Additional figures

E.1 Partial equilibrium exercises

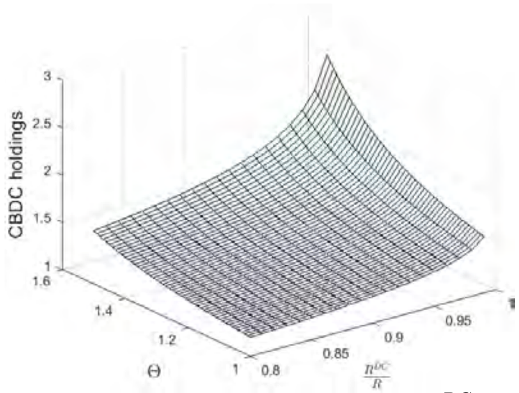

Figure E.6: CBDC demand for different levels of the $\frac{R^{DC}}{R}$ spread and of the value of liquidity services (Θ)

E.2 Baseline simulations - Full results

E.2.1 Total factor productivity (TFP) shock in the domestic economy

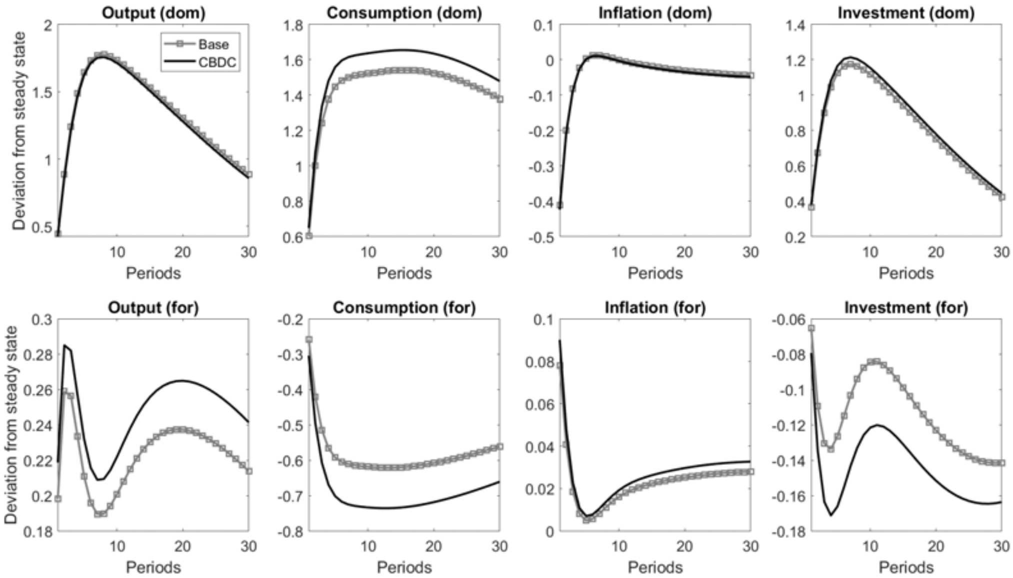

Figure E.7: Reaction of real variables to a 1 standard deviation TFP shock in the domestic economy.

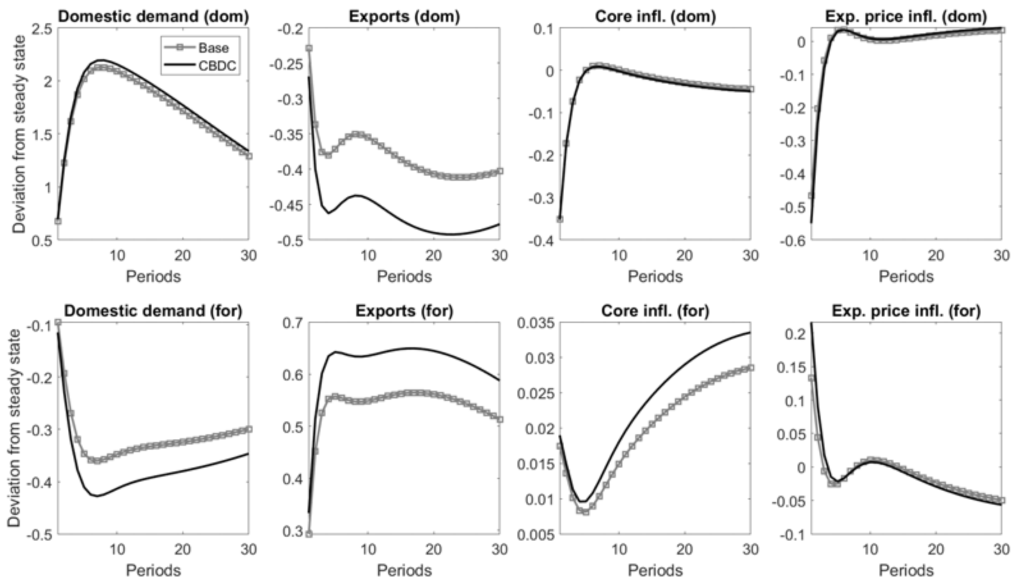

Figure E.8: Reaction of real variables to a 1 standard deviation TFP shock in the domestic economy (cont'd).

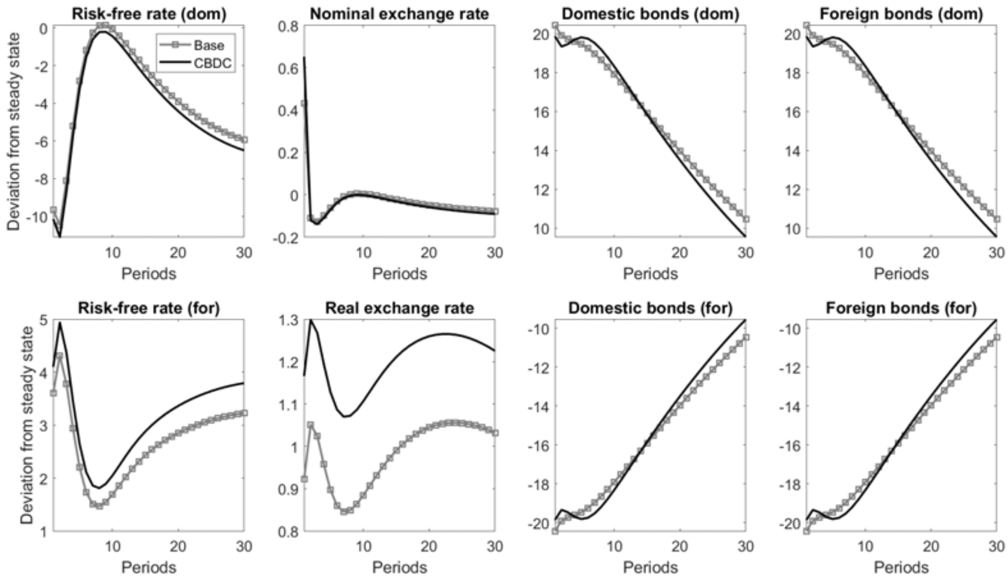

Figure E.9: Reaction of financial variables to a 1 standard deviation TFP shock in the domestic economy.

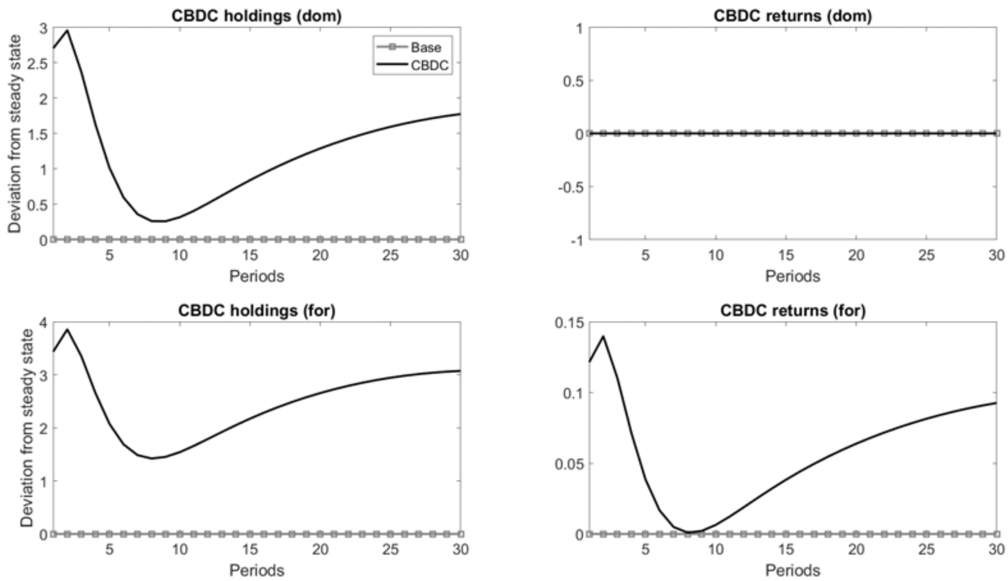

Figure E.10: Changes in CBDCs holdings and returns to a 1 standard deviation TFP shock in the domestic economy. Foreign returns on the CBDC are defined as the exchange-rate adjusted CBDC remuneration rate rate.

E.2.2 Monetary policy shock in the domestic economy

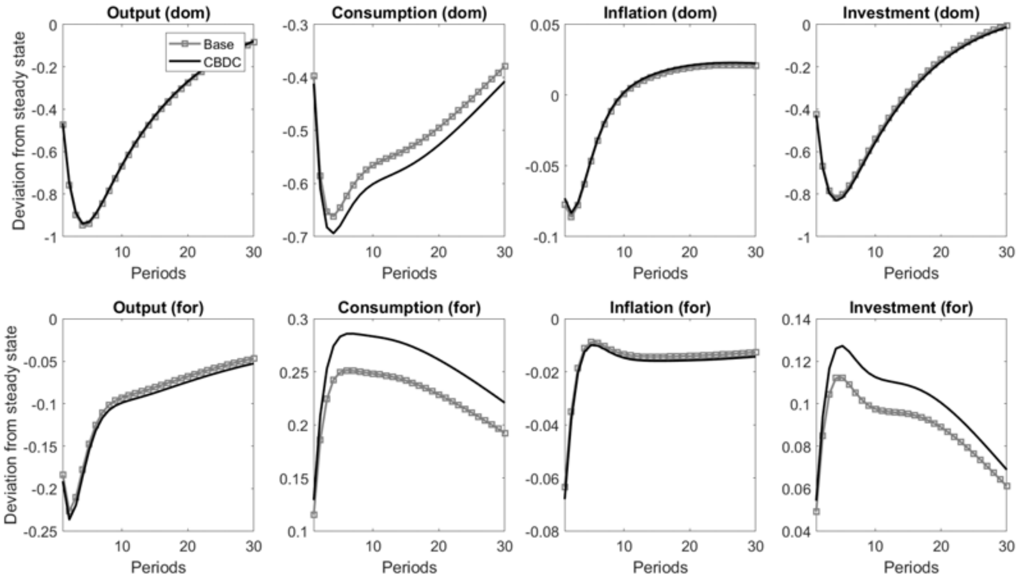

Figure E.11: Reaction of real variables to a 1 standard deviation monetary policy shock in the domestic economy.

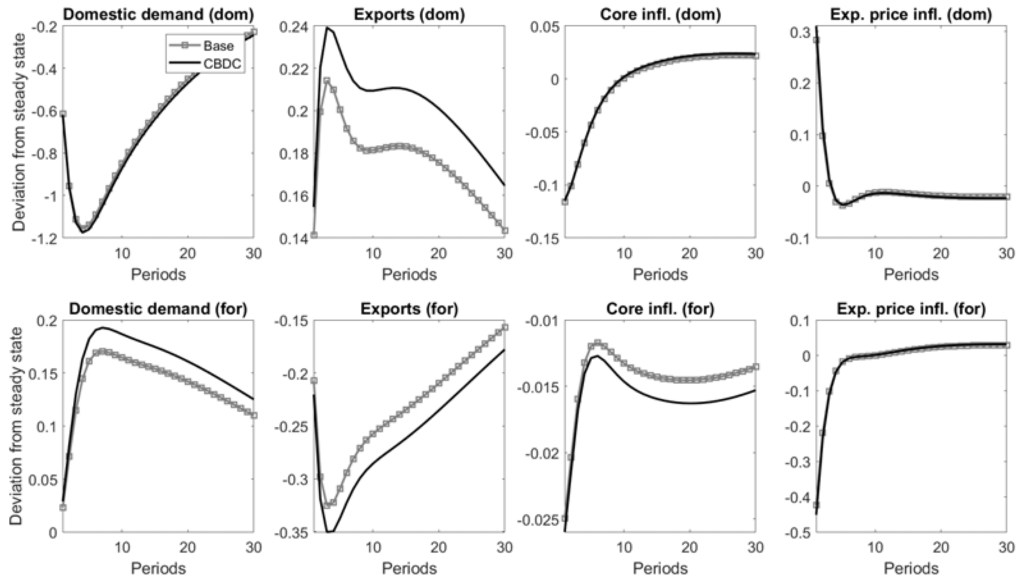

Figure E.12: Reaction of real variables to a 1 standard deviation monetary policy shock in the domestic economy (cont'd).

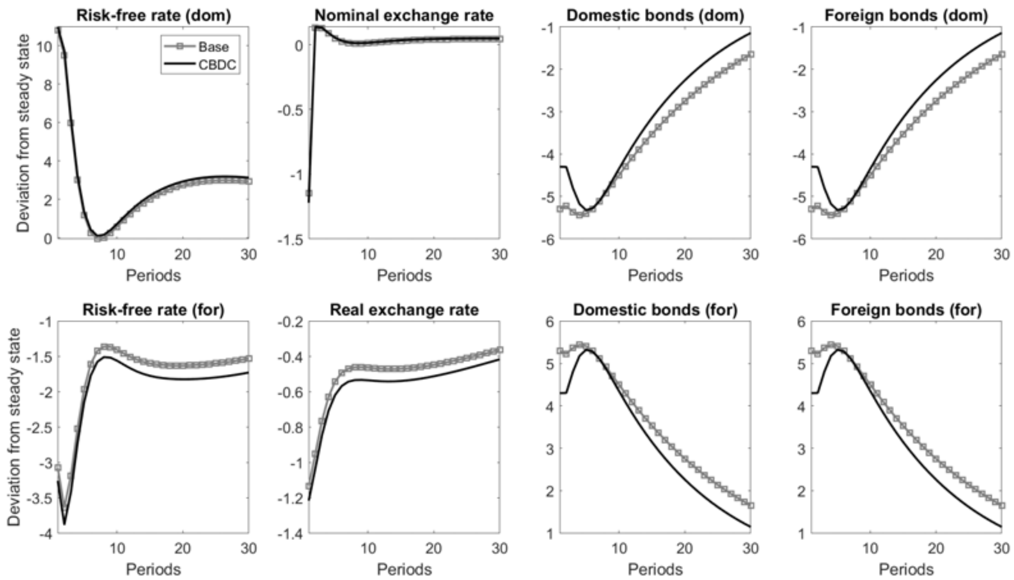

Figure E.13: Reaction of financial variables to a 1 standard deviation monetary policy shock in the domestic economy.

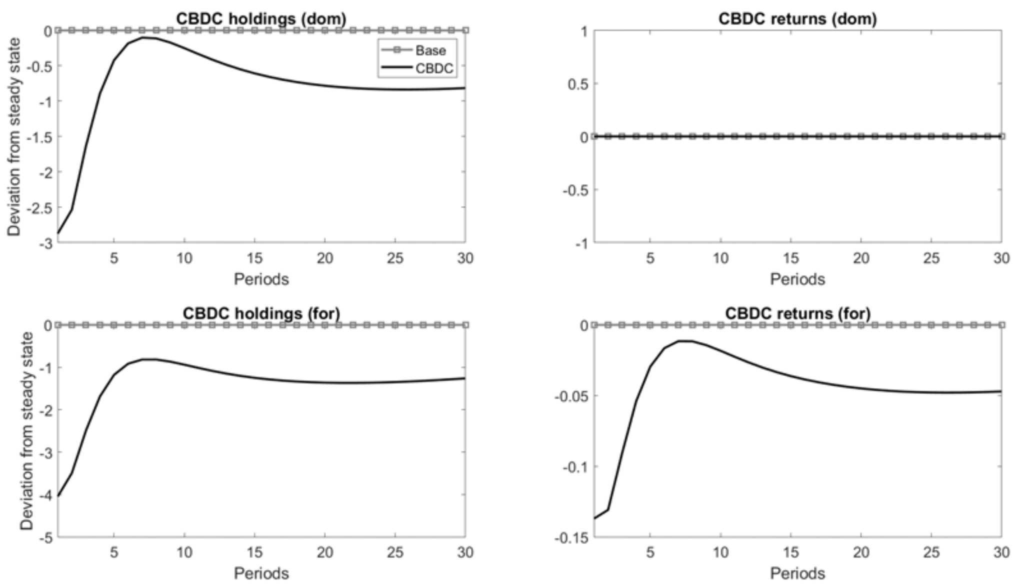

Figure E.14: Changes in CBDCs holdings and returns to a 1 standard deviation monetary policy shock in the domestic economy. Foreign returns on the CBDC are defined as the exchange-rate adjusted CBDC remuneration rate.

E.3 Alternative CBDC design features - further results

E.3.1 CBDC supplied in fixed quantity

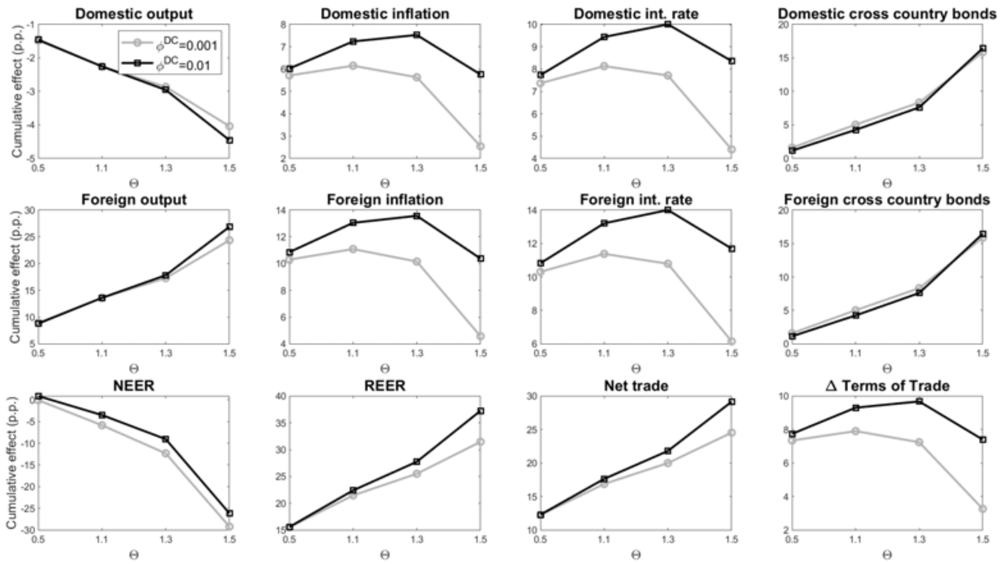

Figure E.15: Cumulative response to a TFP shock in the domestic economy under for a CBDC supplied in fixed quantity.
Notes: Cumulative responses in percentage difference relative to the model simulations without CBDC.

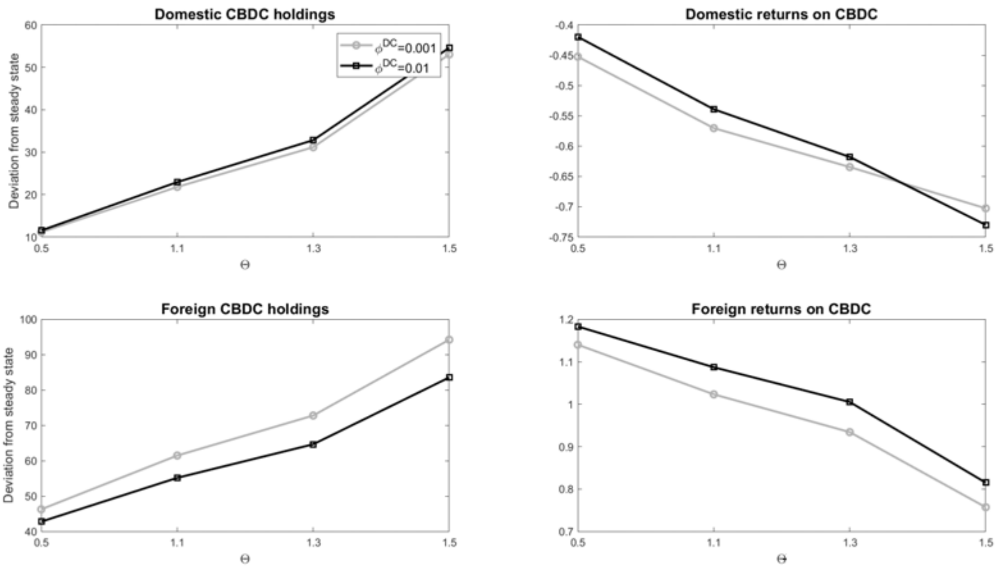

Figure E.16: Cumulative response to a TFP shock in the domestic economy under for a CBDC supplied in fixed quantity (cont'd).
Notes: Cumulative deviations from the steady state.

E.3.2 CBDC with a flexible (Taylor-rule-type) interest rate

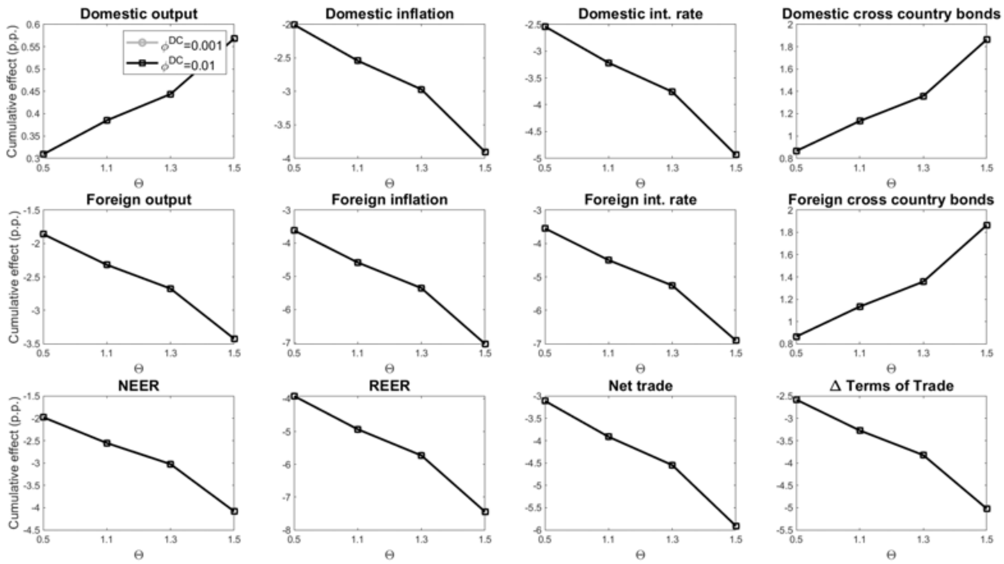

Figure E.17: Cumulative response to a TFP shock in the domestic economy for a CBDC with a flexible interest rate.
Notes: Cumulative responses in percentage difference relative to the model simulations without CBDC.

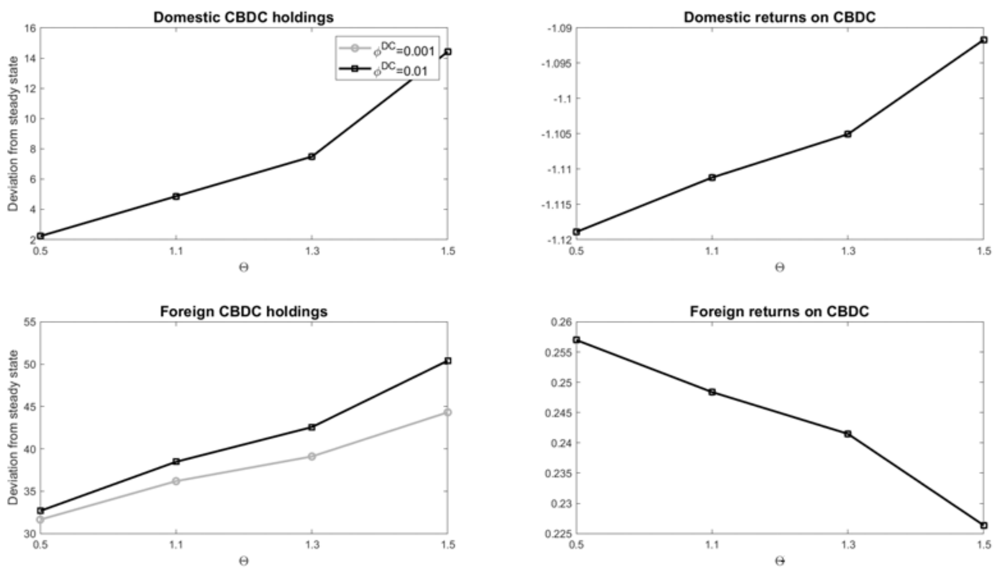

Figure E.18: Cumulative response to a TFP shock in the domestic economy for a CBDC with a flexible interest rate (cont'd).
Notes: Cumulative deviations from the steady state.

E.4 Regression results

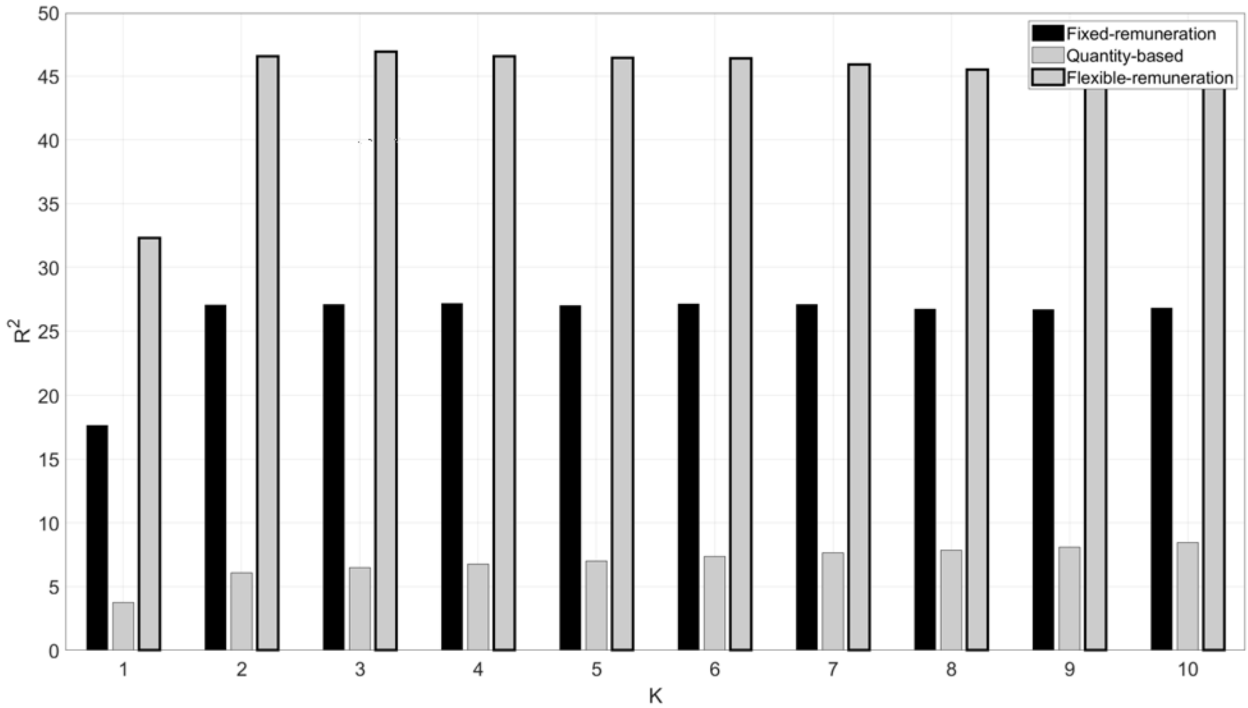

Figure E.19: R^2 of the UIP regression $e_{t+k} - e_t = \alpha_k + \beta_k \left[r_t^{dc} \right] + \varepsilon_{t+k}$ for different horizons. **Notes:** The regression is estimated separately on simulated data for the three possible CBDC designs(fixed remuneration, quantity-based and flexible remuneration).

E.5 Welfare analysis

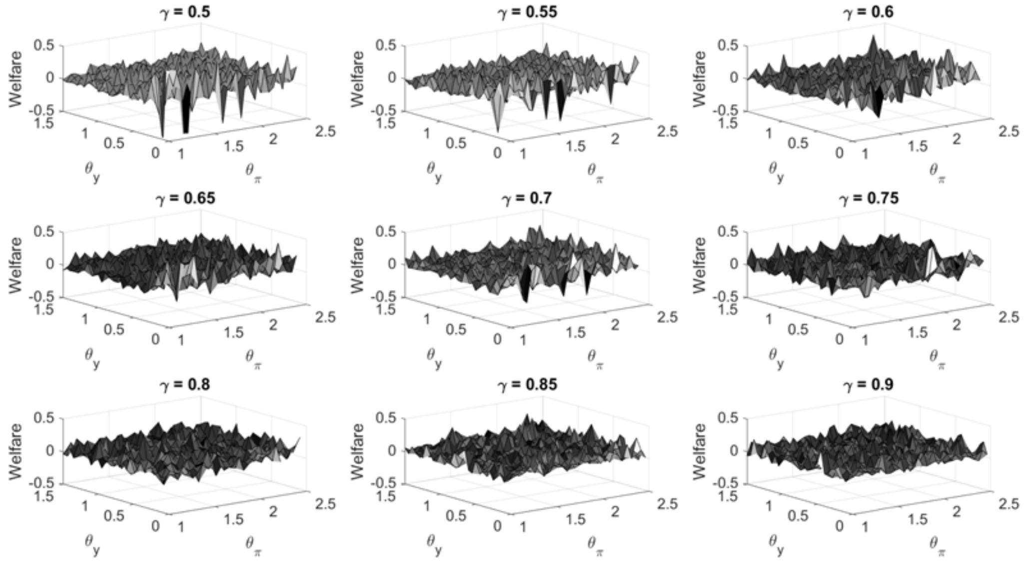

Figure E.20: Welfare values for different monetary policy rules in the domestic economy for the model with a CBDC with fixed remuneration.

Notes: Welfare is computed as the stochastic steady state of the welfare function ($W_t = U_t + \beta E_t(W_{t+1})$) at the second order. Values are reported in terms of steady state consumption $\times 10^4$.

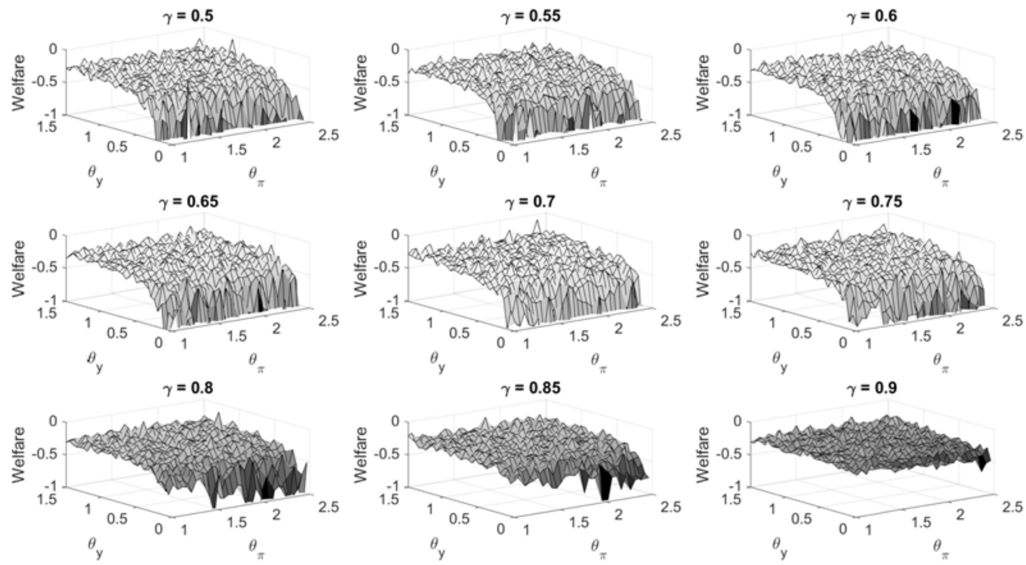

Figure E.21: Welfare values for different monetary policy rules in the foreign economy for the model with a CBDC with a fixed remuneration.

Notes: Welfare is computed as the stochastic steady state of the welfare function ($W_t = U_t + \beta E_t(W_{t+1})$) at the second order. Values are reported in terms of steady state consumption $\times 10^4$.